Electrical Engineering Texts

Samuel Zbell

ELECTRICAL ENGINEERING TEXTS

A COURSE IN

ELECTRICAL ENGINEERING

VOLUME I

DIRECT CURRENTS

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A COURSE IN ELECTRICAL ENGINEERING

VOLUME I

DIRECT CURRENTS

BY

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PREFACE TO THE SECOND EDITION

The wide use of this textbook among engineering colleges, since it first appeared seven years ago, has been most gratifying. It was intended primarily as a text of semi-elementary character to be used principally by engineering students who are not specializing in electrical engineering, and for men who study electrical engineering in industrial schools. Its almost immediate adoption by electrical engineering courses in the universities of the highest standing showed that it was meeting the needs of both teachers and students for the more advanced courses. The author has in the meantime received from fellow teachers constructive suggestions that would make the book even better adapted to their courses. As a result of such suggestions, some changes in the original material have been made, and considerable material of a more advanced nature has been added. None of the elementary and fundamental developments of the subject have been sacrificed, however.

The treatment of storage batteries has been made to conform to the latest practice, thanks to the coöperation of Mr. H. W. Beedle, engineer for the Electric Storage Battery Company. Considerable material has been added to Chapter VII, "Electrical Measurements," particularly in the discussion of potentiometers. The fundamental methods of making magnetic measurements have been added to Chapter VIII. In Chapter IX, electrostatics and dielectric phenomena are treated in considerably greater detail. The analysis of the characteristics of generators and motors in Chapters XI, XII, and XIII has been carried somewhat further than in the first edition.

As many teachers have said that a greater number of problems would add to the value of the book, an entirely new set of problems is included, the number being increased by more than one hundred. These problems offer a wide range in the matter of difficulty of solution.

The author is particularly indebted to Prof. H. E. Clifford of Harvard University, whose suggestions and review of the manuscript have been very helpful in the revision.

C. L. D.

Harvard University, Cambridge, Mass.
August, 1927.

PREFACE TO THE FIRST EDITION

For some times past the editors of the McGraw-Hill Electrical Engineering Texts have experienced a demand for a comprehenl sive text covering in a simple manner the general field of Electrica-Engineering. Accordingly, these two volumes were written at their request, after the scope and general character of the two volumes had been carefully considered.

As the title implies, the books begin with the most elementary conceptions of magnetism and current-flow and gradually advance to a more or less thorough discussion of the many types of direct and alternating current machinery, transmission devices, etc., which are met in practice. These two books are intended for Electrical Engineering students as a stepping stone to the more advanced Electrical Engineering Texts which are already a part of the series.

These two volumes should be useful also to students not planning to specialize in the electrical engineering field, who are taking courses in Electrical Engineering as a part of their general training. Such men often find difficulty in obtaining detailed and straightforward discussions of the subject in any one text and the brevity of their course does not give them time to assimilate fragmentary information obtainable only by consulting a number of references. Men taking foremen's and industrial courses in Electrical Engineering, which as a rule are carried on only in the evening, require text books sufficiently comprehensive, but at the same time not involving much mathematical analysis. Ordinarily, this type of student does not have ready access to reference libraries and is usually out of contact with his instructors except during the short time available for class-room work. In preparing this work the needs of the foregoing types of students have been carefully kept in mind and as a result, a liberal use of figures and illustrative problems has been made. Also frequent discussions of the methods of making measurements and laboratory tests are included.

In any course in Electrical Engineering, even though it be intended for non-electrical engineers, the author feels that the student gains little from a hurried and superficial treatment of the subject, as such treatment tends only to develop the memorizing of certain formulæ which are soon forgotten. Accordingly the attempt has been made in this text to develop and explain each phenomenon from a few fundamental and well-understood laws rather than to give mere statements of facts. Such treatment will develop the student's reasoning powers and give him training that will be useful in the solution of the more involved engineering problems that may arise later in his career.

Throughout the text, especially in the treatment of the more abstract portions, attempt has been made to show the ultimate bearing upon general engineering practice. The student takes more interest in the theory when he sees that it can be applied to the solving of practical problems. Because this work is not intended for advanced students in Electrical Engineering, little or no calculus is used and the mathematics is limited to simple equations.

The author is indebted to several of the manufacturing companies who have coöperated in the matter of supplying photographs, cuts and material for the text; and particularly to Professor H. E. Clifford of The Harvard Engineering School, for his many suggestions and for the care and pains which he has taken in the matter of editing the manuscripts.

C. L. D.

Harvard University, Cambridge, Mass.
January, 1920.

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CHAPTER I

MAGNETISM AND MAGNETS

1. Magnets and magnetism are involved in the operation of practically all electrical apparatus. Therefore an understanding of their underlying principles is essential to a clear conception of the operation of all such apparatus.

2. Magnetic Materials.—Iron (or steel) and its alloys is far superior to all other metals as a magnetic material, and is practically the only metal used for magnetic purposes. *Permalloy*, an alloy composed of 80 per cent. nickel and 20 per cent. iron has unusual magnetic characteristics. At present it can be made economically only in strips or rods of comparatively small cross-section (see Par. 156). Cobalt and nickel (and most of their alloys) possess magnetic properties, which are far inferior to those of iron. Liquid oxygen is also attracted to the poles of magnets.

3. Natural Magnets.—Magnetic phenomena were first noted by the ancients. Certain stones, notably at Magnesia, Asia Minor, were found to have the property of attracting bits of iron, hence the name *magnets* was given to these magic stones. The fact that such stones had the property of pointing north and south, if suspended freely, was not discovered until the tenth or twelfth century. The practical use of such a stone in navigation gave it the name of *Lodestone* or leading stone. Natural magnets

are composed of an iron ore known in metallurgy as magnetite, having the chemical composition Fe_3O_4.

4. Artificial Magnets.—If a piece of hardened steel be rubbed with lodestone, it will be found to have acquired a very appreciable amount of magnetism, which it will retain indefinitely. Such a steel magnet is called an *artificial magnet.* Artificial magnets commonly derive their initial excitation from an electric current as will be shown later. If a piece of soft steel or soft iron be similarly treated, it retains but a very small portion of the magnetism initially imparted to it.

These properties make it desirable to use hardened steel when a permanent magnet is desired and to use soft iron or steel when it is essential that the magnetism respond closely to changes of magnetizing force. It is found that even hardened steel ages or loses some of its magnetism with time. Where a high degree of permanency is desired, as in electrical instruments, or even in magnetos, the magnets are aged artificially.

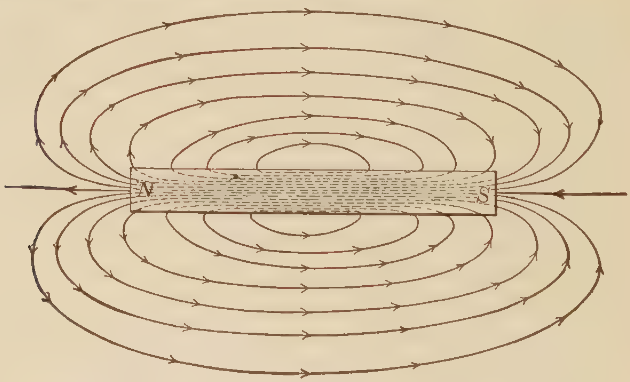


Fig. 1.—Magnetic field about a bar magnet.

5. Magnetic Field.—It is found that magnetism manifests itself as if it existed in lines, called *lines* of *magnetism* or *lines* of *induction*. The region in space through which these lines pass is called the *magnetic field.* Further, if the lines of induction of such a field be determined experimentally, it is found that they seem to emanate from one region of the magnet and enter some other region as shown in Fig. 1. These regions are called the *poles* of the magnet. The two poles are distinguished by

the position which they seek if suspended freely. The one which points north is called the *north-seeking pole* or *north pole* for short, and the other the *south-seeking pole,* or *south pole.* In practice it is assumed that the lines of induction leave the magnet at the north pole and re-enter it at the south pole. Within the magnet the lines of induction continue from the south to the north pole so that each line of induction forms a closed loop. The plane midway between the poles is the *neutral zone* or *equator* of the magnet. The entire path through which the lines of induction pass is called the *magnetic circuit.*

6. Effect of Breaking a Bar Magnet.—Neither a north pole nor a south pole can exist alone. For every north pole there

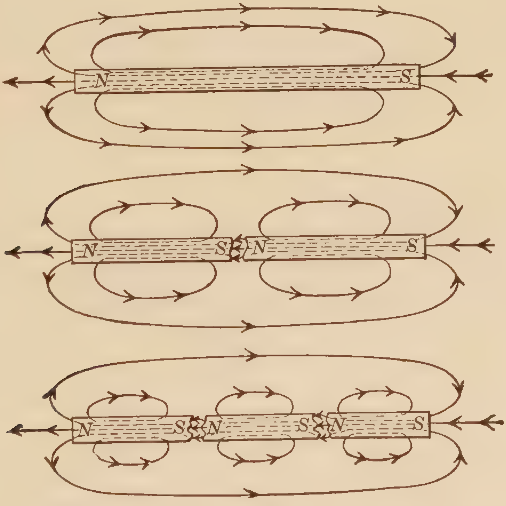
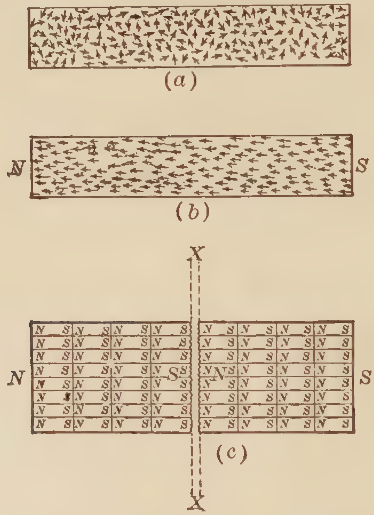


FIG. 2.—Effect of breaking a bar magnet.

exists an equal (but opposite) south pole. If an ordinary bar magnet be broken at the middle, or at various points, each fragment will constitute a bar magnet having its north and its south pole lying in the same respective directions as those of the original magnet. This phenomenon is easily explained by noting that the lines of induction still continue to pass from one fragment to the next adjacent one, and in so doing constitute north and south poles as shown in Fig. 2. In experimental work, this phenomenon may be easily illustrated by magnetizing a

highly-tempered steel knitting needle and breaking it at various points.

7. Weber's Theory.—An explanation of the appearance of north and south poles on breaking a magnet, and other phenomena occurring in the magnetization of iron, is offered by Weber's Theory which has been expanded by Ewing. The molecules of a magnet are assumed to be an indefinitely great number of very small magnets as shown in Fig. 3 (*a*). Under ordinary conditions these small magnets are arranged in a haphazard



(*a*) (*b*) (*c*)
FIG. 3.—Weber's molecular theory of magnets.

way, as shown at (*a*), so that the various north and south poles all neutralize one another, and no external effect is produced. Upon the application of a magnetizing force, however, the small magnets tend so to arrange themselves that their axes are parallel and their north poles are all pointing in the same general direction as the magnetizing force. This is shown in Fig. 3 (*b*). It is evident that if the magnet be cut along the line XX (Fig. 3 (*c*)) a new north and a new south pole will result, which, before the fracture took place, neutralized each other.

This theory is further substantiated by grinding a permanent magnet into very small particles. Each of the small particles

possesses the properties of the bar magnet, each having its own north and its own south pole. Further, the theory offers a rational explanation of saturation, hysteresis, etc., occurring in iron subjected to a magnetizing force. This will be considered later.

8. Consequent Poles.—Consequent poles are occasionally found in bar magnets where different portions have been rubbed

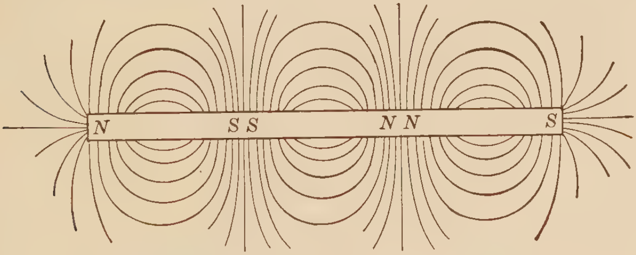


Fig. 4.—Consequent poles.

by a north pole, or a south pole, or when exciting coils, acting in opposition, have been placed upon the bar. Consequent poles are in reality due to the fact that the bar consists of two or more magnets arranged so that two north or two south poles exist in the same portion of the magnet. Consequent poles are illustrated in Fig. 4. The magnetic field shown in Fig. 12, page 12, is in a way illustrative of the field resulting from consequent poles. In this case, however, two bar magnets are used and a small air-gap exists between the adjacent north poles. Consequent poles are frequently produced on iron and steel bars due to their having come in contact with lifting magnets (see page 27).

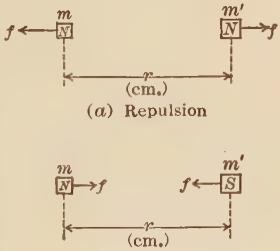


Fig. 5.—Repulsion and attraction between magnetic poles.

9. Magnetic Force.—When a freely suspended north pole is brought in the vicinity of another north pole, it is repelled, whereas, if a south pole is brought in the presence of a north pole, it is immediately attracted toward the north pole. South poles are also found to repel one another. From this it may be

stated that *like poles repel one another and unlike poles attract one another.*

10. Pole Strength.—The force of attraction (or repulsion) between two given poles is found to be inversely as the square of the distance between the poles, provided the dimensions of the poles are small compared with the distance between them. *A unit magnetic pole is one of such strength that if placed at a distance of one centimeter in free space from a similar pole of equal strength will repel it with a force of one dyne.*

Pole strength is measured by the number of unit poles which, if placed side by side, would be equivalent to the pole in question.

The force f, existing between poles in air may be formulated as follows:

$$f = \frac{mm'}{r^2} \text{ dynes,} \tag{1}$$

where m and m' are the respective pole strengths (in terms of a unit pole) of two magnetic poles, placed a distance r cm. apart, as shown in Fig. 5. This force may be attraction or repulsion according as the poles are unlike or like.

Example.—Two north poles, one having a strength of 500 units and the other a strength of 150 units, are placed a distance of 4 in. apart in air. What is the force in grams acting between these poles, and in what direction does it act?

$$4 \text{ in.} = 4 \times 2.54 = 10.16 \text{ cm.}$$

$$f = \frac{500 \times 150}{(10.16)^2} = \frac{75{,}000}{103.2} = 726 \text{ dynes.}$$

$$\frac{726}{981} = 0.740 \text{ g. Poles repel each other. } \textit{Ans.}$$

11. Lines of Force.—Thus far the magnetic field has been studied only with respect to the lines of magnetism or induction. If a single north pole be placed in such a field two effects will be observed.

1. This pole will be urged along the lines of induction.

2. The force urging this pole will be greatest where the lines of induction are the most dense, and, moreover, the force will be proportional to the number of lines per unit area taken perpendicular to the lines in the field in which the pole finds itself.

From these statements it can be seen that *lines of force,* similar to lines of induction, can be drawn, to represent the forces at the various points in the magnetic field. In much of the literature

on the subject lines of induction and lines of force are used indiscriminately. The fallacy of so doing is immediately apparent upon considering a solid bar magnet. Each line of induction is a closed line which passes completely through the solid metal of the magnet, leaving the magnet at the north pole and entering it at the south pole. Within the magnet the lines of induction go from the south to the north pole (see Fig. 1). On the other hand, the lines of force all originate at the north pole and terminate at the south pole (Fig. 6). Hence the lines of induction are all closed lines; the lines of force are not closed lines, but terminate at the poles. In air the lines of induction and the lines of force

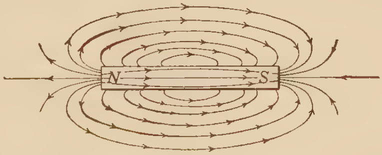


Fig. 6.—Lines of force associated with a bar magnet.

are equal and coincide. In magnetic media the lines of induction and the lines of force are not identical. They may even be in opposition, as in a bar magnet. These distinctions should always be kept in mind.[1]

12. Field Intensity.—It has been stated that the force acting upon a magnetic pole placed in a magnetic field is proportional to the density of the lines of force at that point. *Unit field intensity is defined as the field strength which will act upon a unit pole with a force of one dyne.* One *line of force* perpendicular to and passing through a square centimeter represents unit field intensity. Field intensity is usually represented by the symbol H. It is evident that if a pole of m units be placed in a field of intensity H, the force acting on this pole is

$$f = m \times H \text{ dynes.} \tag{2}$$

A pole placed in such a field must be of such small size that it will have no appreciable disturbing effect upon the magnetic field.

Example.—When a small pole having a strength of 25 unit poles is placed in a magnetic field it is acted on with a force of 200 dynes. What is the field intensity at this point?

The force per unit pole

$$H = \frac{200}{25} = 8 \text{ dynes per unit pole.} \quad \textit{Ans.}$$

[1] For a more complete discussion of the effects see "Principles of Electrical Engineering" by Harold Pender, page 56. (McGraw-Hill Book Company, Inc.)

It also follows that the density of the lines of force, taken for an area perpendicular to the direction of these lines is 8 lines per square centimeter or 8 gausses.

Example.—A total flux of 200,000 lines passes in air between two parallel pole faces, each 8 cm. square. The field is uniformly distributed. With what force (grams) will a pole, having a strength of 100 units, be acted upon if placed in this field?

Flux density $= \frac{200{,}000}{8 \times 8} = 3{,}120$ lines per square centimeter or 3,120 gausses. Being in air this value of flux density also equals the field intensity, H.

$$f = m \times H = 100 \times 3{,}120 = 312{,}000 \text{ dynes.}$$

$$\frac{312{,}000}{981} = 319 \text{ g.} \quad \textit{Ans.}$$

13. Flux Density.—Flux density is the number of lines of induction per unit area, taken perpendicular to the induction. In free space, flux density and field intensity are the same, numerically, but within magnetic material the two are entirely different. The two should not be confused. The unit of flux density (one line per square centimeter) is often called the *gauss*, but the expression "lines per square centimeter" and "lines per square inch" are often used in practical work when speaking of flux density.

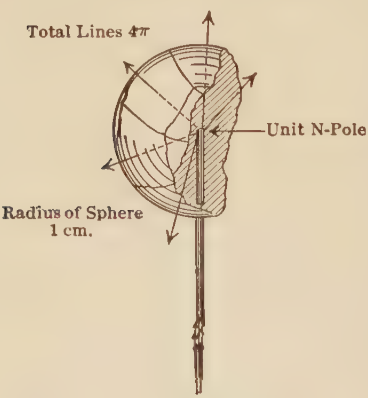


Fig. 7.—Lines of force emanating from a unit N-pole.

By definition the force exerted by a unit pole upon another unit pole at centimeter distance in air is always one dyne. The field intensity on a spherical surface of one centimeter radius and with a unit pole at its center must then be unity and can be represented by one line per square centimeter over the entire spherical surface as shown in Fig. 7.

Since there are 4π square centimeters on the surface of a sphere having one centimeter radius, each unit pole must have emanating from it 4π or 12.6 lines of force. Figure 7 represents a portion of a spherical surface of one centimeter radius and shows roughly the passage of one line of force through each square centi-

meter of surface, each line originating in the unit north pole. This also explains the appearance of the 4π term so often encountered in magnetic formulas. $4\pi m$ lines of force emanate from a pole having a strength of m units.

Example.—A pole having a strength of 400 units is placed at the center of a sphere having a radius of 3 cm. What is the flux density at the surface of the sphere and what force will be exerted on a pole of 10 units placed at the surface of the sphere?

Total lines emanating from pole $= 400 \times 4\pi = 5{,}027$ lines.

Area of surface of sphere $= 4\pi r^2 = 4\pi 9 = 113.1$ sq. cm.

Flux density $= \dfrac{5{,}027}{113.1} = 44.4$ gausses.

Force upon pole of 10 units $= 44.4 \times 10 = 444$ dynes. *Ans.*

As a check, the force may also be determined by the law of inverse squares (see Par. 10).

$$f = \frac{mm'}{r^2} = \frac{400 \times 10}{3 \times 3} = 444 \text{ dynes.}$$

14. The Compass Needle.—The compass consists of a hardened steel needle or small bar, permanently magnetized and accurately balanced upon a sharp pivot. The north-seeking end or north pole points north, and the south-seeking end points south. The north pole of the needle is usually colored blue or given some distinguishing mark. With the exception of a few used for lecture purposes, the needle is enclosed in an air-tight case for mechanical protection. Mariner's compasses are mounted carefully upon gimbals, so that they always hang level. Upon steel ships, heavy iron balls placed near the compass are necessary to neutralize the magnetic effect of the ship itself.

By means of the compass the polarity of a magnet is readily determined. The south pole of the compass points to the *north* pole of the magnet as shown in Fig. 8. Likewise, the *north* pole of the compass points to the *south* pole of the magnet. This action of the compass needle follows immediately from the law that like poles repel and unlike poles attract each other. This is very useful in practical work for it enables one to determine the polarity of the various poles of motors and generators and to show if the exciting coils are correctly connected.

Further, the compass needle always tends to set itself in the direction of the magnetic field in which it finds itself, the north end of the needle pointing in the direction of the lines of force or

magnetic lines. This is illustrated in Fig. 9. By placing a small compass at the various points in the region of a magnet, and drawing an arrow at each point, the arrow pointing in the same direction as the needle, the field around the magnet may be mapped out as shown in Fig. 9. In mapping out a field in

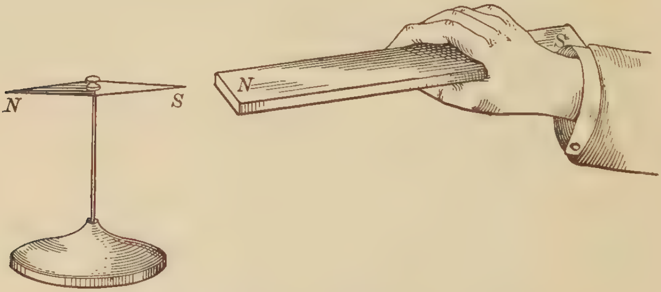


FIG. 8.—Compass needle and bar magnet.

this way it must be remembered that the earth's field may exert considerable influence on the compass needle in addition to the effect of the field being studied.

15. Magnetic Figures.—If a card be placed over a magnet and iron filings be sprinkled over the card, a magnetic figure is

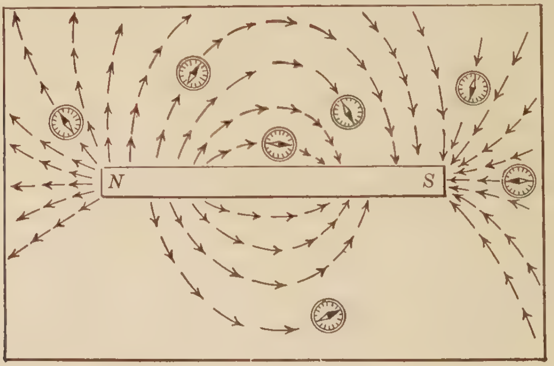


FIG. 9.—Exploring the field about a bar magnet with a compass.

obtained. The filings at each point set themselves in the direction of the line of force at that point, and the resultant figure shows in very close detail the character of the magnetic field. Figure 10 shows the magnetic field due to two bar magnets placed

side by side with unlike poles adjacent. On the other hand, Fig. 11 shows the field due to these same bar magnets when like poles are adjacent. It will be noted in Fig. 10, that the lines of

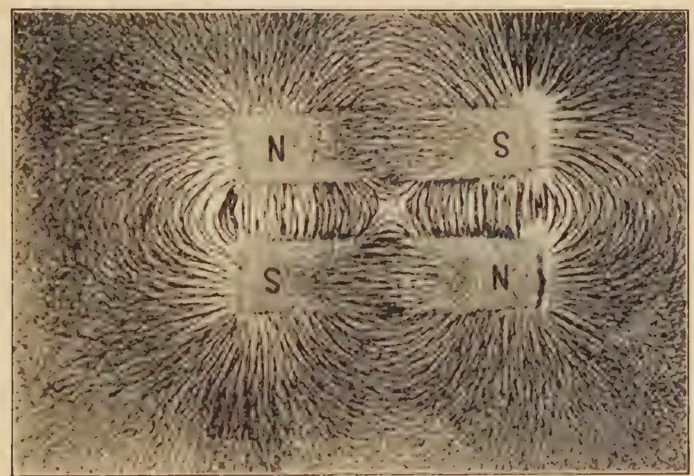


FIG. 10—Magnetic figure, unlike poles adjacent.

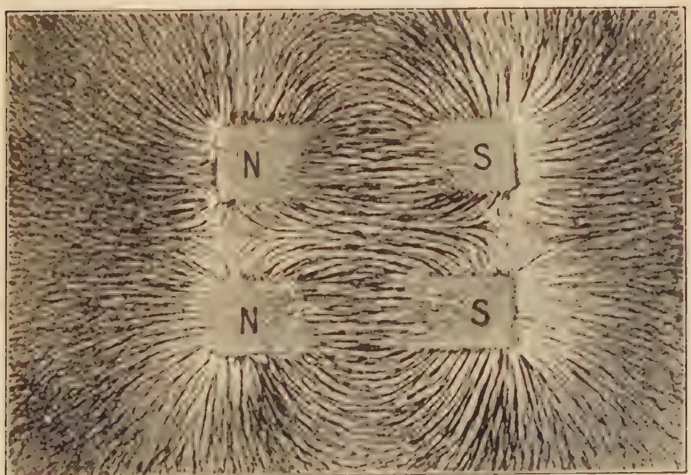


FIG. 11.—Magnetic figure, like poles adjacent.

force seem like elastic bands stretched from one pole to the other, acting to pull the unlike poles together. In Fig. 11 the lines of force from the two like poles appear to repel one another, indicat-

ing a state of repulsion between the poles. Figure 12 shows the field obtained by placing the bar magnets end to end, having the two north poles adjacent.

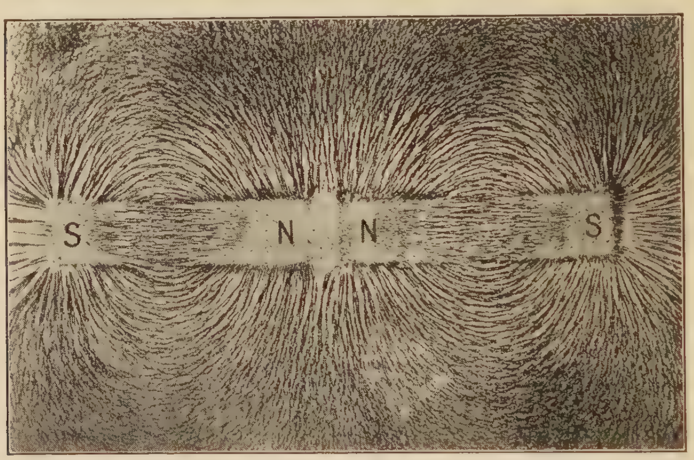


FIG. 12.—Magnetic figure, like N-poles adjacent.

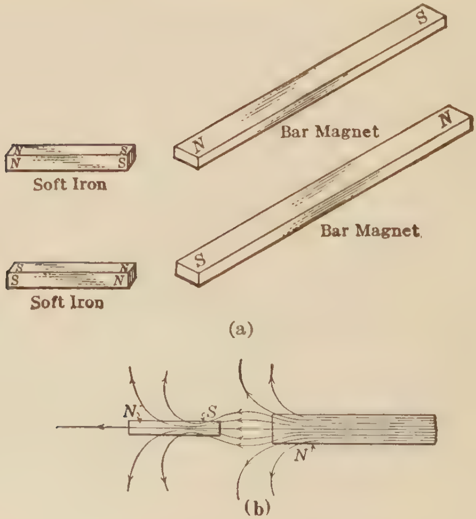


FIG. 13.—Poles produced by magnetic induction.

16. Magnetic Induction.—If a magnet is brought near a piece of soft, non-magnetized iron, the piece of iron becomes

magnetized by induction. If the north pole of the magnet is brought near the iron, a south pole is induced in that part of the iron nearest the inducing magnet, and if the south pole of the magnet is brought near the iron a north pole is similarly induced. This is illustrated in Fig. 13 (*a*).

The reason that magnetic poles are so induced is the distribution which the lines of magnetic induction (Fig. 13 (*b*)) have when a small, soft-iron bar is brought near the north pole of a bar magnet. The lines of induction, which leave the north pole of the bar magnet, will concentrate in the soft iron, because iron permits the passage of magnetic lines far better than air does. Since the magnetic lines which leave the north pole of the bar magnet must *enter* the soft iron at the end which is adjacent to

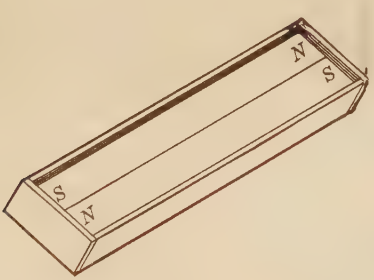


FIG. 14.—Proper method of "keeping" bar magnets.

the N-pole of the bar magnet, a S-pole is formed at that end of the soft-iron bar which is adjacent to the N-pole of the bar magnet. As lines of magnetic induction are continuous, they must also *leave* the soft-iron bar, and they do so at the end which is more remote from the magnetizing N-pole. Hence a N-pole is formed at the more remote end of the soft-iron bar.

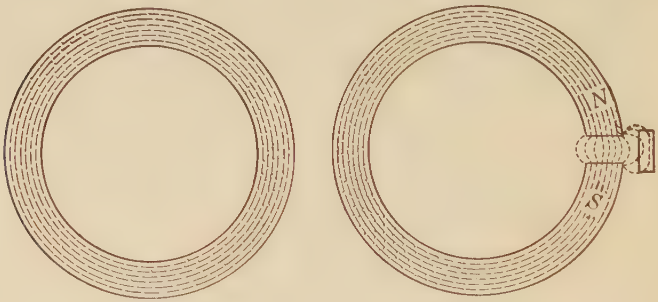
The inducing N-pole attracts the induced S-pole and repels the induced N-pole on the soft iron. As the induced S-pole is nearer to the inducing pole, attraction predominates.

It is sometimes noticed that if a comparatively weak north pole be brought into the vicinity of a strong north pole, attraction between the two results, rather than the repulsion which might be expected. This is no violation of the laws governing the attraction and repulsion of magnetic poles, but comes from the

fact that the strong north pole induces a south pole which overpowers the existing weak north pole and results in attraction. In this way it is easy to reverse the polarity of a compass needle by holding one end too close to a strong magnetic pole of the same polarity.

For a similar reason, when two bar magnets are put away in a box, the adjacent ends should be of opposite polarity, as shown in Fig. 14. They will retain their magnetism better under these conditions. When a horseshoe magnet is not in use a "keeper" of soft iron should be placed across the poles.

17. Law of the Magnetic Field.—*The magnetic field always tends so to conform itself that the maximum amount of flux is attained.* This offers further explanation of the attraction of iron to poles of magnets. The iron is drawn toward the magnet



(*a*) (*b*)

FIG. 15.—Ring magnets.

so that the magnetic lines may utilize it as a part of their return path, since iron conducts these lines much better than the air. This is illustrated in the horseshoe magnet of Fig. 16. The armature is drawn toward the poles of the magnet, and the return path through the air is materially shortened, so that the number of magnetic lines is materially increased. The maximum flux exists when the armature is in contact with the poles.

18. Other Forms of Magnets.—The simple bar magnet frequently is not suitable for practical work. For the same amount of material, other forms are more powerful and more compact. Figure 15 (*a*) shows a closed ring magnet. All the magnetic flux is contained in the ring and little external effect is noted. This type is not very useful. However, if the ring be cut as shown

in Fig. 15 (*b*), a north and a south pole are obtained. A piece of soft iron, if brought near this gap, will be strongly attracted and will tend to be drawn across the gap and thus shorten the length of the flux path.

The horseshoe magnet, shown in Fig. 16, is very useful, for two reasons. The two poles being near each other, a comparatively strong field exists. Further, if the function of the magnet is to exert a pull upon an armature, each pole is equally effective. Figure 123, Chap. VII, page 145 shows a horseshoe magnet such as is used in Weston direct-current instruments.

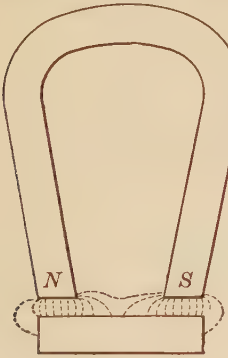
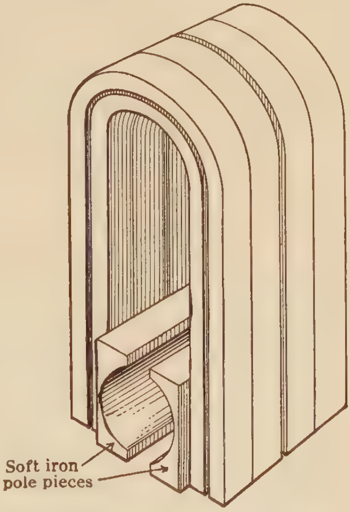


Fig. 16.—Horseshoe magnet attracting a soft-iron armature.

(*a*) Compound or laminated bar magnet.



(*b*) Compound horseshoe magnet used in magnetos.

Fig. 17.—Examples of laminated magnets.

19. Laminated Magnets.—It is found that thin steel magnets are stronger in proportion to their weight than thick ones. This is probably due to the more uniform hardening which can be obtained with the thinner metal during the quenching which follows heat treatment. Hence, for a given amount of material a magnet made up of several laminations, as shown in Figs. 17 (*a*) and 17 (*b*) is more powerful than one made of a single piece of metal of the same total mass. Figure 17 (*b*) shows the form of horseshoe magnet generally used for telephone and ignition magnetos.

20. Magnet Screens.—There is no known insulator for magnetic flux. No appreciable change in the flux or in the pull of a magnet is noticed if glass, paper, wood, copper, or other such material be placed in the magnetic field. However, it is often desirable to shield galvanometers and electrical measuring instruments from the earth's field and from stray fields due to

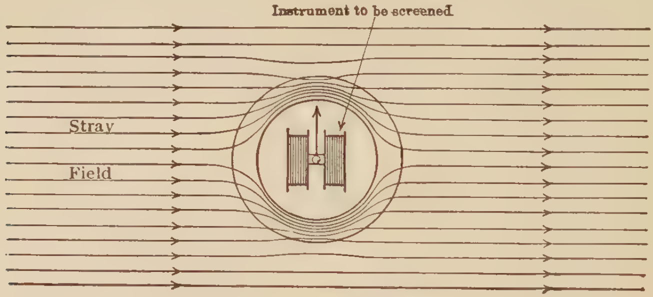


FIG. 18.—Magnetic screen.

generators, conductors carrying currents, etc. This is done by surrounding the instrument with an iron shell as shown in Fig. 18. This shell by-passes practically the entire flux and thus prevents it from affecting the sensitive portions of the instrument. The smaller the openings in the shell, the more effective the screening becomes. Three or four shells, with air spaces between, are found to be more effective than one shell of the same total thickness. Such, however, are used only in connection with the screening of the most sensitive galvanometers.

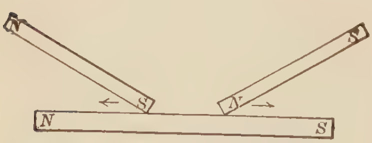


FIG. 19.—Divided touch method of magnetizing.

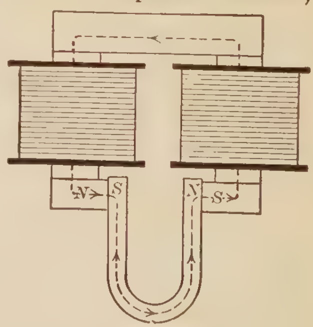


FIG. 20.—Magnetizing a horseshoe magnet with an electromagnet.

21. Magnetizing.—A magnet may be magnetized by merely rubbing it with another magnet. The resulting polarity at any point is opposite to that of the last pole which came in con-

tact with this point. Therefore, it is well to rub one end with the north pole of the inducing magnet and the other end with the south pole. This may be done simultaneously by the "divided touch" method shown in Fig. 19. It is advisable to rub both sides of the bar. Stronger magnets may be obtained by placing them between the poles of a very powerful electromagnet. Figure 20 shows this method of magnetizing a horseshoe magnet. An armature or "keeper" should be placed across the poles of the horseshoe magnet before removing it from the electromagnet. Magnetization may also be produced by inserting the magnet in a suitable exciting coil and allowing a heavy current to flow in the coil. A few turns of low resistance wire may be wound around the magnet and connected in series with a fuse to the supply mains. Upon closing the switch, an enormous current passes temporarily, but the fuse blows immediately and prevents damage to the electric circuit. The heavy rush of current is usually sufficient to leave the steel in a strongly magnetized condition.

22. The Earth's Magnetism.—The earth behaves as a huge bar magnet, the poles of which are not far from the geographical poles. The north magnetic pole (corresponding to the south pole of a magnet) is situated in Boothia Felix, about 1,000 miles from the geographical north pole. The south magnetic pole has never been located but experiment points to the existence of two south poles. Due to the non-coincidence of the geographical and magnetic poles and to the presence of magnetic materials in the earth, the compass points to the true north in only a few places on the earth's surface. The deviation from the true north is called the declination, and magnetic maps are provided showing the declination at various parts of the earth. At New York it is about 9° west. The declination undergoes a gradual variation from year to year, called the variation change. A careful record is kept of this secular variation and scientific measurements, such as are used in astronomy, surveying, and navigation, must be corrected correspondingly. The needle undergoes a very small daily variation and an annual variation, due possibly to the influence of the sun and the moon.

A freely suspended and balanced needle does not take up a position parallel to the earth's surface, when under the influence

of the earth's magnetism alone, but assumes a position making some angle with the horizontal. This angle is called the *dip* of the needle. At New York it is about 70° north. The dip undergoes changes similar to those in the variation. The field intensity (total, not horizontal) of the earth's field at New York is about 0.61 c.g.s. units, although this value changes slightly from time to time.

CHAPTER II

ELECTROMAGNETISM

23. Magnetic Field Surrounding a Conductor.—It had long been suspected that some relation existed between electricity and magnetism, but it remained for Oersted, in 1819, to show that this relation not only existed but that it was a definite relation.

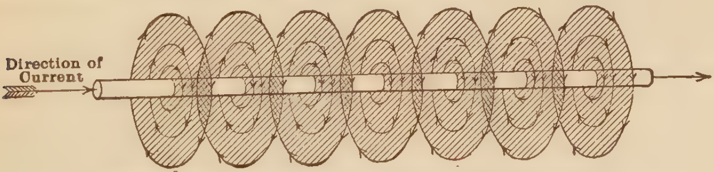


FIG. 21.—Magnetic field about a straight conductor.

If a compass be brought into the neighborhood of a single conductor carrying an electric current, the needle deflects, thus indicating the presence of a magnetic field. It is further observed that the needle always tends to set itself at right angles to the current. When it is held above the conductor, the

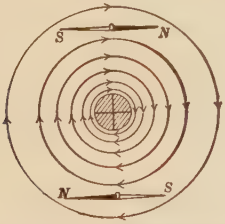


FIG. 22.—Lines of flux surrounding a cylindrical conductor—current inwards.

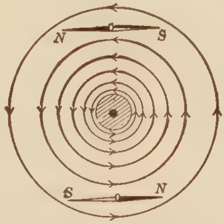


FIG. 23.—Lines of flux surrounding a cylindrical conductor—current outwards.

needle points in a direction opposite to that which it assumes when held beneath the conductor. Further investigation shows that the magnetic flux exists in circles about the conductor (if there is no other magnetic field in the vicinity) as shown in Figs. 21, 22, and 23. These circles have their centers at the

axis of the conductor and their planes are perpendicular to the conductor. If the current in the conductor be reversed, the direction in which the compass needle is deflected will be seen to reverse also, showing that the direction of this magnetic field is dependent upon the direction of the current. The relation of the two is shown in Fig. 21. The fact that the magnetic flux exists in circles perpendicular to the conductor explains the reversal of the compass needle when moved from a point above the conductor to a point beneath it, since the direction of the field above the conductor must be opposite to that beneath the conductor. This is illustrated in Figs. 22 and 23.[1]

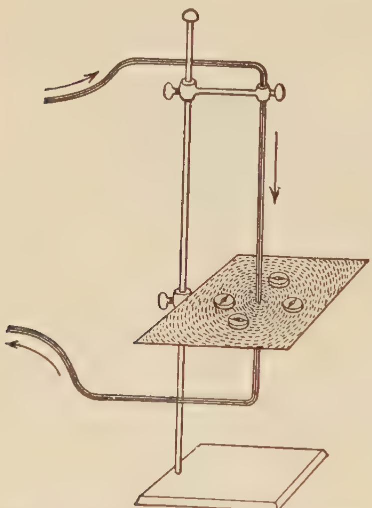


Fig. 24.—Investigation of the magnetic field surrounding a conductor.

The experiment shown in Fig. 24 is illustrative of this concentric relation of the flux to the conductor. A conductor carrying a current is brought vertically down through a horizontal sheet of cardboard. Iron filings sprinkled on the cardboard form concentric circles. (A current of about 100 amp. is necessary to obtain distinct figures.) If four or more compasses are arranged as shown in Fig. 24 they will indicate, by the direction in which their needles point, that the magnetic lines are circles having the axis of the wire as a center.

24. Relation of Magnetic Field to Current.—A definite relation exists between the direction of the current in a conductor and the direction of the magnetic field surrounding the conductor. There are two simple rules by which this relation may be remembered.

[1] A circle having a cross inside (⊕) indicates that the current is flowing into the paper, and represents the feathered end of an arrow. A circle having a dot at the center (⊙) indicates that the current is flowing out of the paper, and represents the approaching tip of an arrow.

Hand Rule.—Grasp the conductor in the right hand with the thumb pointing in the direction of the current. The fingers will then point in the direction of the lines of flux (Fig. 25).

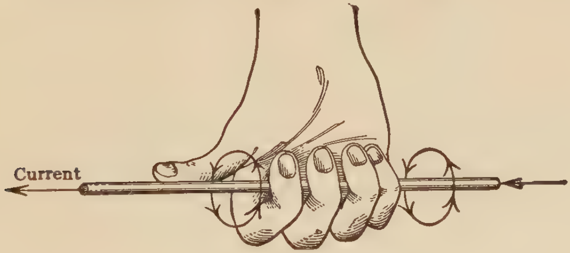


FIG. 25.—Hand rule.

Corkscrew Rule.—The direction of the current and that of the resulting magnetic field are related to each other as the forward travel of a corkscrew and the direction in which it is rotated.

This last rule is probably the most common and the most easily remembered. However, it must not be inferred from this rule that the magnetic field exists in spirals about the conductor. It exists actually in planes perpendicular to the conductor.

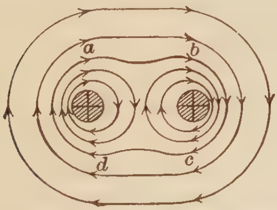


FIG. 26.—Magnetic field about two parallel conductors—current in same direction.

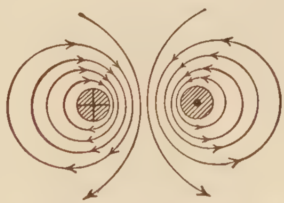


FIG. 27.—Magnetic field about two parallel conductors—current in opposite directions.

25. Magnetic Field of Two Parallel Conductors.—When each of two parallel conductors carries an electric current, flowing in the same direction, there is a tendency for the two conductors to be drawn together. The reason for this is obvious. In Fig. 26 the lines of force encircle each conductor in the same direction (corkscrew rule) and the resultant field is an envelope of lines tending to pull the conductors together. Further reason for this attraction is given by the rule of Par. 17 stating that the magnetic field tends so to conform itself that the number

of magnetic lines is a maximum. The pulling together of the conductors reduces the length of path *abcd* through which the lines must pass. The field due to each conductor separately is still circular in form but the resultant magnetic lines are no longer circular, as is shown in Fig. 26.

In Fig. 27 is shown the field which exists when two parallel conductors carry current in opposite directions. The magnetic lines are circles, but these circles are not concentric either with one another or with the conductor. The lines are crowded between the conductors and therefore tend to push the conductors farther apart. Again, when the conductors separate, the area through which the flux passes is increased, so that the magnetic circuit in this case also tends so to conform itself that the magnetic flux is a maximum.

From the foregoing, the following rules may be formulated. *Conductors carrying current in the same direction tend to be drawn together; conductors carrying current in opposite directions tend to be repelled from each other.*

All electric circuits tend to take such a position as will make their currents parallel and flowing in the same direction.

This effect is especially pronounced in modern large-capacity power systems. Bus-bars have been wrenched from their clamps; transformer coils have been pulled out of place and transformers wrecked by the forces produced by the enormous currents arising under short-circuit conditions.

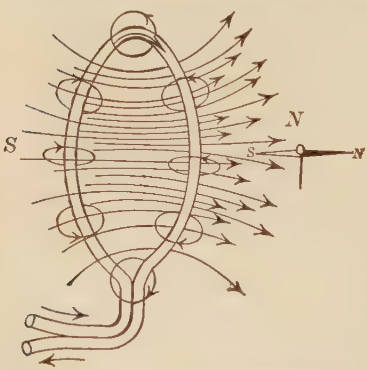


FIG. 28.—Magnetic field produced by a single turn.

26. Magnetic Field of a Single Turn.—If a wire carrying a current be bent into a loop a field similar to that shown in Fig. 28 results. This magnetic field has a north pole and a south pole which possess all the properties of similar poles of a short bar magnet. A compass needle placed in this field assumes the direction shown, the north pole pointing in the direction of the magnetic lines.

27. The Solenoid.—An electric conductor wound in the form of a helix and carrying current is called a *solenoid*. A simple solenoid and the magnetic field produced within it when current flows through the conductor is shown in Fig. 29. The solenoid may be considered as consisting of a large number of the turns

FIG. 29.—Magnetic field produced by a helix or solenoid.

shown in Fig. 28 placed together. The solenoid winding may consist of several layers as shown in Fig. 31.

The relation of the direction of the flux within the solenoid to the direction in which the current flows in the helix may be determined by the hand rule, or by the corkscrew rule of Par. 24.

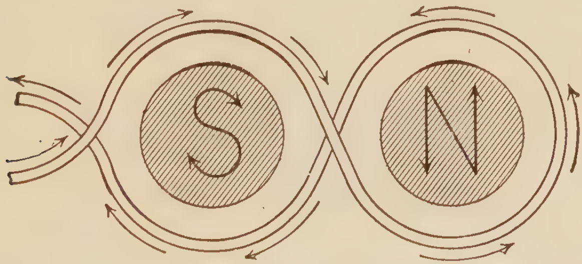


FIG. 30.—Relation of magnetic poles to direction of exciting current.

Another simple method is shown in Fig. 30, where the arrows at the ends of the *N* and the *S* show the direction of current in the coil. For example, when looking down upon a north pole the current direction in the coil will be counter-clockwise as shown by the *N*; when looking down upon a south pole the direction of the exciting current will be clockwise as shown by the *S*.

28. The Commercial Solenoid.—The solenoid is used in practice for tripping circuit breakers (Par. 269), for operating contactors in automatic motor starters (Par. 248), for operating voltage regulating devices (Par. 235), for arc lamp feeds (Chap. XIV, Vol II), for operating valves, and for numerous other purposes. In practically all instances a soft-iron (or steel)

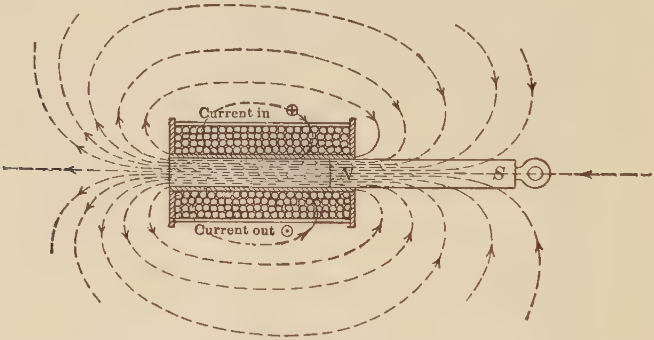


FIG. 31.—Simple solenoid and plunger.

plunger or armature is necessary to obtain the tractive pull required of the solenoid. The operation of a solenoid and plunger is indicated in Fig. 31. The flux due to the solenoid produces magnetic poles on the plunger. The pole nearer the solenoid will be of such sign that it will be urged along the lines of force

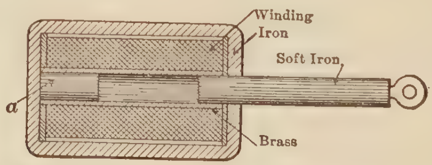


FIG. 32.—"Iron-clad" solenoid and plunger with stop.

(see Par. 11), and in such a direction as to be drawn within the solenoid.

A position of equilibrium is reached when the center of the plunger reaches the center of the solenoid (Fig. 31). Figure 32 shows an "iron-clad" solenoid commonly used for tractive work. The iron-clad feature increases the range of uniform pull and produces a very decided increase of pull as the plunger approaches the end of the stroke. When a stop a is used,

the solenoid becomes a *plunger electromagnet.* This changes the characteristics of the solenoid in that the maximum pull

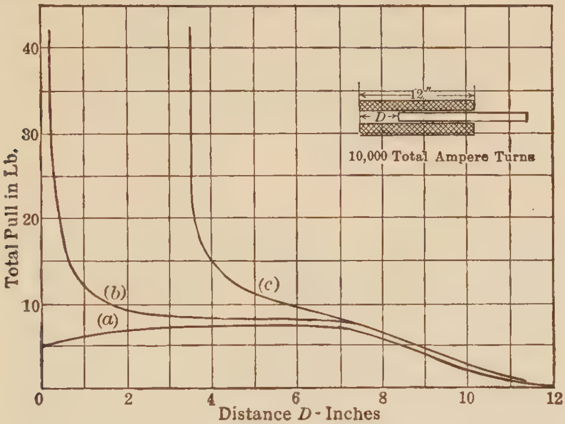


FIG. 33.—Pull of solenoid on plunger.

now occurs when the end of the plunger is near the stop. Figure 33 shows the results of solenoid tests made by C. R. Underhill.[1] Curve (*a*) is the pull upon the plunger of a simple solenoid like that of Fig. 31; curve (*b*) shows the pull when this solenoid is iron-clad as in Fig. 32 but without a stop; curve (*c*) shows the effect of the "stop" on the pull. It will be noted that the iron-clad feature and the stop have but little effect except near the end of the stroke.

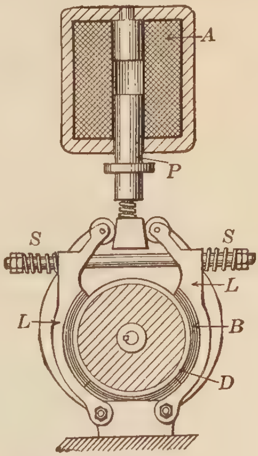


FIG. 34.—Plunger electromagnet operating a crane brake.

An important practical application of the solenoid occurs in the braking of elevators and cranes. When the power is removed from the lifting motor or when the power is interrupted due to a broken wire or other accident, the brake must be applied immediately. One method of accomplishing this is shown in Fig. 34. When the power, for any reason, is interrupted, the plunger P of the solenoid A drops, due partly to gravity and partly

[1] "Standard Handbook," Sec. 5.

to the action of the springs S. The springs S immediately force the levers L against the brake bands B, pressing these against the brake drum D, thus effecting the braking action. When the power is applied to the lifting motor, the plunger P is pulled up, thus releasing the brake. A plunger electromagnet is most suitable for this purpose because the stroke is short and the pull must be positive.

29. The Horseshoe Solenoid.—The use of an armature in connection with solenoids is well illustrated by the relay or the sounder used in telegraphy, and also by electric bells, buzzers, etc. To increase the effectiveness of such devices two solenoids

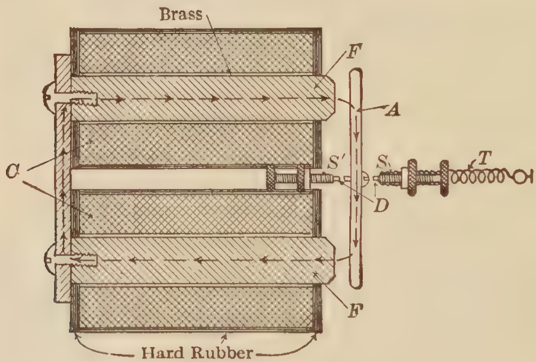


FIG. 35.—Telegraph relay.

are used, each being placed on one of the legs of a horseshoe or U-shaped magnet. When the coils C (Fig. 35) are excited, the iron armature A is attracted because of the tendency of the magnetic lines to make their path of minimum length. As a rule, the armature A is not allowed to close the magnetic circuit completely, for under these conditions the magnetic lines still exist after the excitation is removed, preventing rapid release of the armature. The stop S' prevents the armature making contact with the cores FF and thus completely closing the magnetic circuit. The two sets of contacts D close any secondary circuit that the relay may be operating. The spring T draws the armature back against a stop S when the excitation is removed.

30. The Lifting Magnet.—Lifting magnets are used commercially to handle iron and steel in various forms. A very considerable saving of time and labor is effected by their use, because

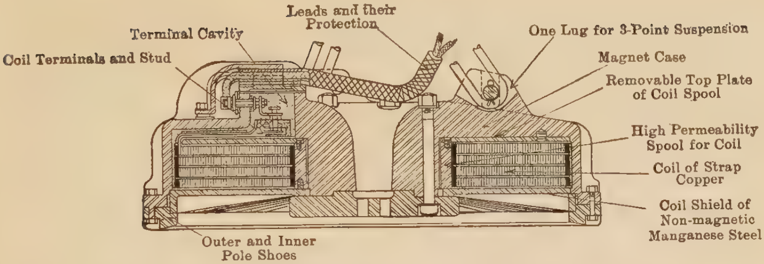


FIG. 36.—Cross-section of a lifting magnet.

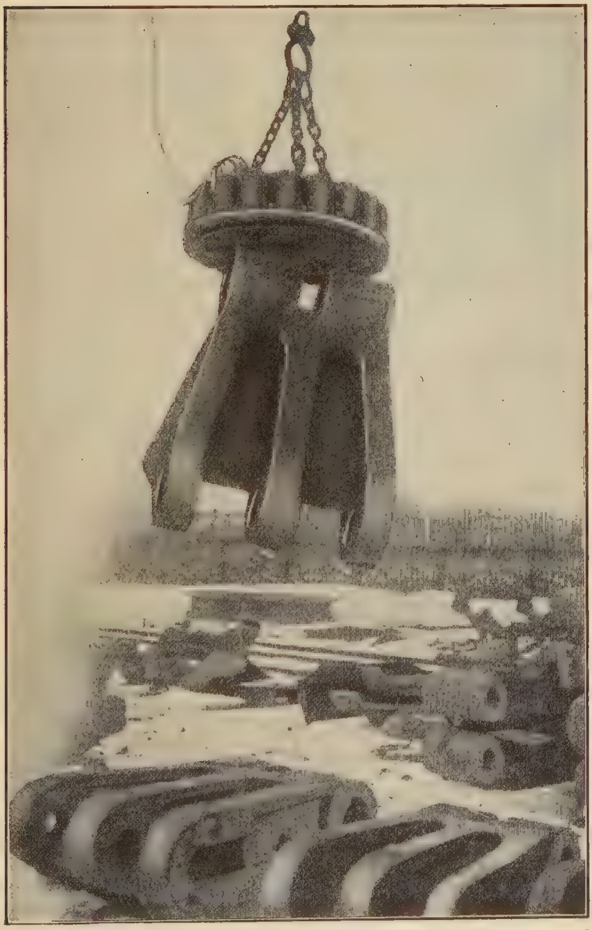


FIG. 37.—Cutler-Hammer 36-in. magnet, handling heavy castings.

chains and slings for holding the load are not necessary. They are very useful for handling steel billets in rolling mills, although the billets cannot be picked up when red hot as they lose their magnetic properties at this temperature. Magnets are especially useful in loading and unloading steel rails, for an entire layer may be picked up and laid down again without being disarranged. Lifting magnets effect a very great saving of labor when small pieces of iron, such as scrap iron, are handled, for they will pick up large quantities at every lift. Without such a magnet it would be necessary to move each individual piece by hand.

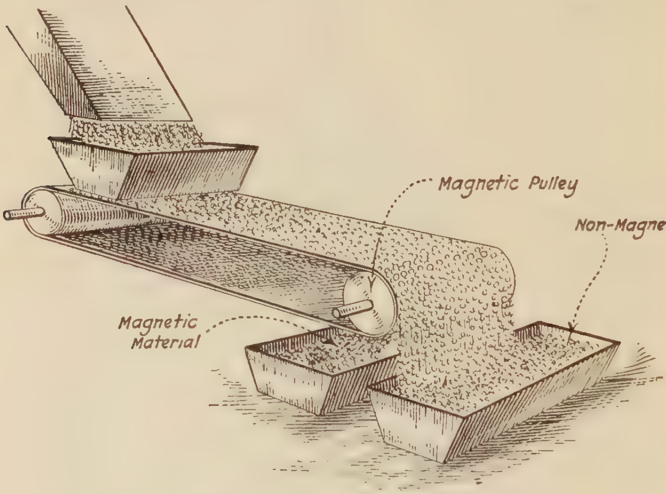


Fig. 38.—Magnetic separator.

Figure 36 shows in cross-section a typical Cutler-Hammer lifting magnet.

Figure 37 shows a lifting magnet in actual operation.

Formulæ for the holding force of electromagnets are given in Par. 164.

It should be understood that the magnet itself does little or no work in the lifting, but merely serves as a holding device. The actual work is performed by the engine or motor which operates the steel ropes or chains attached to the magnet.

31. Magnetic Separator.—Another important application of magnetic principles is found in the magnetic separator shown

in Fig. 38. It is especially designed to remove steel and iron from coal, rock, ore, etc., but it may be used for separating steel shot from molding sand, iron chips from machine shop turnings, etc. The material is fed on an endless belt running at a speed of about 100 ft. per minute. The belt passes over a magnetized pulley. The non-magnetic material immediately drops off into a hopper, but the magnetic material is held by the pulley until the belt leaves the pulley when the material drops into a second hopper. The pulley is magnetized by concentric exciting coils, to which current is carried by means of slip-rings.

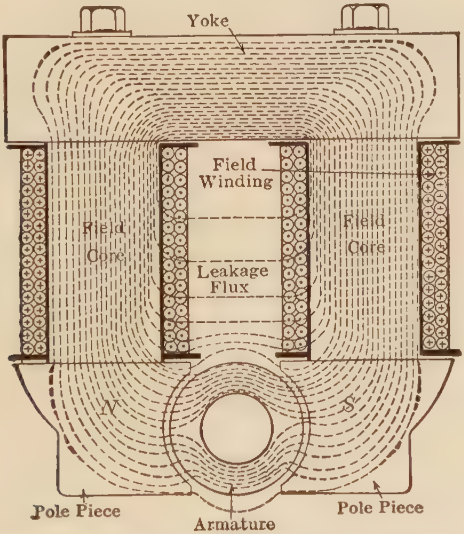


Fig. 39—Magnetic circuit and field windings of an Edison bipolar generator.

32. The Magnetic Circuits of Dynamos.—One of the most important uses of electromagnets is in the magnetic fields of motors and generators. An early and simple type of such a magnetic circuit is illustrated by the Edison bipolar generator, shown in Fig. 39. The type of magnetic circuit shown is very inefficient, because of its great length, in comparison with its sectional area. There results a considerable magnetic leakage which reduces, therefore, the amount of flux passing through the armature. Moreover, the flux in taking the shortest path tends

to crowd through the upper half of the armature. This tends to produce unsatisfactory commutation.

The magnetic circuit of a bipolar generator of modern design is shown in Fig. 40. Because of the symmetry of the magnetic circuit the flux divides evenly through the two sides of the armature. The long air path existing between the pole-shoes reduces the magnetic leakage to a minimum. It is to be noted that the flux in the cores divides as it passes into the yoke.

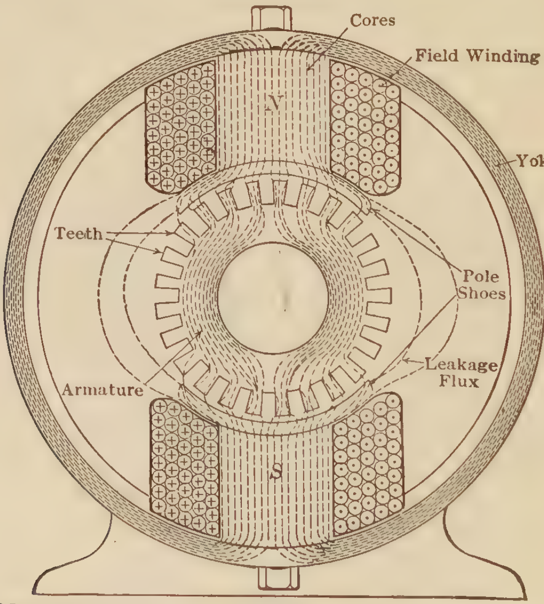


FIG. 40.—Magnetic circuit and field windings of a modern bipolar generator.

Ordinarily the yoke need be only one-half the cross-section of the field cores. Direct-current machines of the bipolar type are made usually in small units.

Figure 41 shows the more complex magnetic circuits of a multipolar generator having eight poles. It is to be noted that the poles are alternately north and south. The flux passing through the field cores divides, both on reaching the yoke and on reaching the armature path and the cross-section of the yoke need be only one-half that of the cores. In both Fig. 40 and Fig. 41 the magnetic leakage is very materially reduced

by placing the exciting ampere-turns as near the armature as possible. This result is not secured in the Edison bipolar

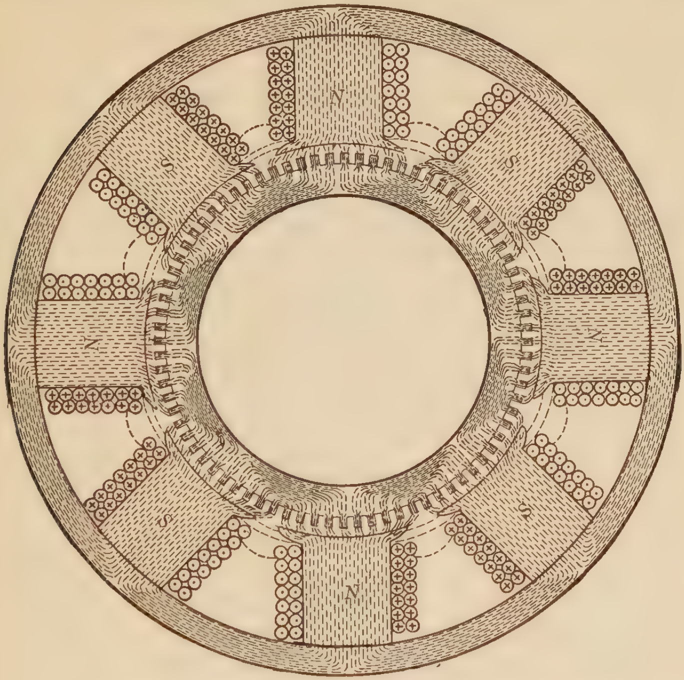


FIG. 41.—Magnetic circuits of a multipolar generator.

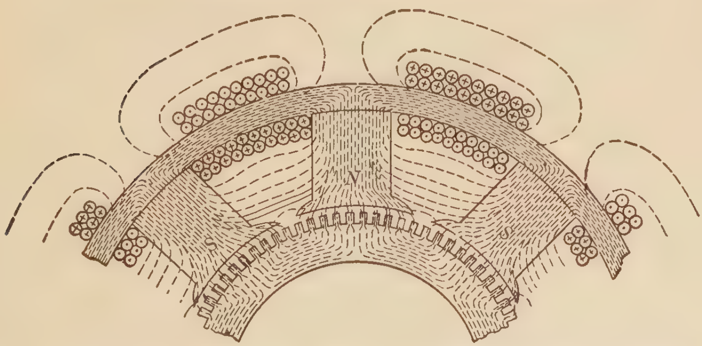


FIG. 42.—Magnetic leakage produced by incorrect position of exciting coils.

generator of Fig. 39. To illustrate, Fig. 42 shows the same generator as that in Fig. 41 but with the exciting coils placed

upon the yoke. They still act upon the magnetic circuits, but because of their remoteness from the armature, a large magnetic leakage exists around the outside of the yoke and through the interpolar space, resulting in a smaller percentage of the total flux passing through the armature.

It should be understood that of itself magnetic leakage does not lower the efficiency of a machine, since to maintain a constant magnetic field does not require an expenditure of energy. However, in order that the necessary number of magnetic lines may reach the armature both the yokes and the cores must have sufficiently large cross-sections to carry the leakage flux in addition to the useful armature flux. This in turn increases the amount of field copper. Therefore a large magnetic leakage results in a much heavier and more expensive machine than would otherwise be necessary.

CHAPTER III

RESISTANCE

33. Electrical Resistance.—The current flowing through an electric circuit depends not only on the electromotive force impressed on the circuit, but on the circuit properties as well. For example, if a copper wire be connected across the terminals of a battery a current will flow through this wire. If a poor contact be made at one of the battery terminals or at some other point in the circuit, the current will decrease, even with the electromotive force remaining constant. Also, heat will be dissipated at the point of poor contact. Likewise, if the copper wire be cut and a small incandescent lamp be inserted in the circuit, the lamp filament will be heated and may become incandescent. At the same time the current in the circuit will decrease in magnitude. In both cases heat is noticed at the points in the circuit where the poorer conducting medium is inserted. Also in each case a decrease in current accompanies the insertion of the poorer conducting medium, even with a constant electromotive force.

This property of tending to prevent the flow of current and at the same time causing electric energy to be converted into heat energy is called *resistance.*

Resistance in the electric circuit may be likened in its effect to friction in mechanics. For example, if a street car is running at a uniform speed on a straight, level track, friction tends to prevent the moving of the car. The power which is used in moving the car is converted by friction into heat. Friction tends to impede the flow of water in a pipe or in a flume, some of the energy of the water being expended in overcoming this friction. The loss of energy is represented by a loss of head. This energy loss is largely absorbed by the water and therefore careful measurements would show a slight increase in its temperature.

As will be shown in the next chapter, the energy loss which occurs when an electric current flows through a resistance, is directly proportional to the amount of resistance.

All[1] substances have resistance, but the resistance of some substances is many times greater than that of others. This leads to the classification of substances into either conductors or insulators. Even silver, one of the best conductors, has appreciable resistance, and glass or porcelain, among the best insulators known, will allow a small amount of current to flow and therefore are not perfect insulators. The best conductors are the metals, silver coming first and copper second. Carbon and ordinary water also may be classed as conductors. Distilled or pure water, however, is a good insulator. Oils, glass, silk, paper, cotton, ebonite, fiber, paraffin, rubber, etc., may be considered as nonconductors or good insulators. Wood, either dry or impregnated with oil, is a good insulator, but wood containing moisture is a partial conductor.

34. Unit of Resistance.—The ohm is the practical unit of resistance and is defined as that resistance which will allow one ampere to flow if one volt is impressed across its terminals (also see p. 54).

An ohm also has such a value that one ampere flowing through it for one second dissipates as heat one joule of energy.

The resistance of insulating substances is ordinarily of the magnitude of millions of ohms, so that it is awkward to express this resistance in terms of a unit as small as the ohm. The *megohm*, equal to 1,000,000 (10^6) ohms, is the unit ordinarily used under these conditions. (The prefix "mega" means million.)

On the other hand, the resistance of bus-bars and short pieces of metals may be so low that the ohm is too large a unit for conveniently expressing it. Under these conditions the *microhm* is used as the unit, and is equal to 1/1,000,000 of an ohm (10^{-6}). (The prefix "micro" means one millionth.)

[1] Professor Kamerlingh-Onnes of Leyden, in 1914, was able to produce a circuit in which an electric current showed no diminution in strength 5 hr. after the electromotive force had been removed. The current was induced magnetically in a short-circuited coil of lead wire at $-270°$ C. in the presence of liquid helium and the inducing source then removed. Liquid helium has the lowest temperature known, being in the neighborhood of absolute zero ($-273°$ C.). This experiment indicates that the resistance of the lead was practically zero at this extremely low temperature.

35. Resistance and Geometry of Conductors.—The resistance of a body of a given material depends both on its geometry and on the direction of flow of current. For example, consider the rectangular prism shown in Fig. 43, composed of two equal cubes a centimeter on edge and of the same conducting material. If the current I_1 flows from side A to side B, which is the side opposite A, it must flow successively through the two cubes. Assume the resistance between opposite faces of each cube to be two microhms. The current I_1 in going from side A to side B must therefore encounter the resistance of two cubes in series, or a total of 4 microhms.

On the other hand, if a current I_2 flows from side C to the opposite side D, the cross-section through which the current flows is

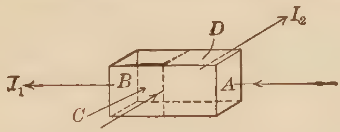


FIG. 43.—Rectangular prism as a conductor.

twice that of a single cube. Hence the resistance encountered by I_2 in each centimeter length of its path is only half that encountered by the current I_1, or is but 1 microhm.

From the foregoing it is obvious that the resistance of a body of a given material depends both on its geometry and the direction of the current flow through the body. In practice, the length of electrical conductors is usually large as compared with their cross-section and the direction of current flow is obvious.

36. Specific Resistance or Resistivity.—In Par. 35 the path for the current I_2 through the prism, has a cross-section twice that of the path for I_1 and a length of path half that for I_1. The resistance which I_2 encounters is *one-fourth* that encountered by the current I_1. From this and similar reasoning, it may be said that *the resistance of a homogeneous body of uniform cross-section varies directly as its length and inversely as its cross-section*, the length being taken in the direction of current flow and the cross-section perpendicular to the direction of current flow.

That is,

$$R = \rho \frac{L}{A} \tag{3}$$

where R is the resistance in ohms, L is the length in the direction of the current flow, A is the area at right angles to the current flow, and ρ is a constant of the material known as its *resistivity* or *specific resistance.*

If L is 1 cm. and A is 1 cm. square, the substance in question must have the form of a cube, 1 cm. on an edge and

$$R = \rho \frac{1}{1 \times 1}$$

or

$$R = \rho$$

ρ is called the *specific resistance* or the *resistivity* of the substance, in this case per cm. cube. ρ may be expressed in terms of an in. cube or in other units as will be shown later. The resistivity of copper is 1.724 microhms, or 1/580,000 ohm, per centimeter cube at 20° C. It is evident that the cube is a perfectly definite unit of resistivity for the resistance between any two opposite

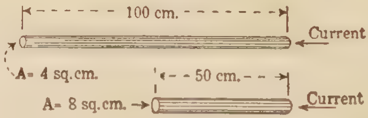


Fig. 44.—Brass rods having equal volumes but unequal resistances.

faces is the same. The resistivities of various substances are given in Par. 45. Knowing the specific resistance in terms of the centimeter cube, the resistance of a wire, bar, etc., may be readily computed from Eq. (3).

Example.—Determine the resistance of the two brass rods A and B (Fig. 44) the resistivity of the brass being 11.4 microhms per centimeter cube. Rod A is 100 cm. long and has a circular cross-section of 4 sq. cm.; rod B is 50 cm. long and has a circular cross-section of 8 sq. cm.

$$\text{Rod } A. \quad R = 11.4 \frac{100}{4} = 285 \text{ microhms. } \quad \textit{Ans.}$$

$$\text{Rod } B. \quad R = 11.4 \frac{50}{8} = 71.25 \text{ microhms. } \quad \textit{Ans.}$$

Although both rods have the same volume, rod A has four times the resistance of rod B, because its length in the direction of current flow is twice that of B and its cross-section, perpendicular to the direction of current flow, is one-half that of B.

Example.—Determine the resistance of 3,000 ft. of annealed 0000 copper wire having a diameter of 0.460 in., the specific resistance of copper being

1.724 microhms (= 0.000001724 ohm) per centimeter cube (20° C.) (see Par. 39).

$$3{,}000' = 3{,}000 \times 12 \times 2.54 = 91{,}500 \text{ cm.}$$

$$\text{Cross-section} = \frac{\pi}{4}(0.460 \times 2.54)^2 = 1.07 \text{ sq. cm.}$$

$$R = \rho\frac{L}{A} = (0.000001724) \times \left(\frac{91{,}500}{1.07}\right) = 0.1472 \text{ ohm.} \quad \textit{Ans.}$$

37. Volume Resistivity.—Since the volume of a body

$$V = LA$$

where L is its length and A its uniform cross-section, Eq. (3) may be written

$$R = \rho\frac{L}{A} = \rho\frac{L^2}{V} = \rho\frac{V}{A^2}. \tag{4}$$

That is:

The resistance of a conductor varies directly as the square of its length when the volume is fixed.

The resistance of a conductor varies inversely as the square of its cross-section when the volume is fixed.

Example.—A kilometer of wire having a diameter of 11.7 mm. and a resistance of 0.031 ohm is drawn down so that its diameter is 5.0 mm. What does its resistance become?

The original cross-section of the wire

$$A_1 = \frac{\pi}{4}11.7^2 = 107.5 \text{ sq. mm.}$$

The final cross-section

$$A_2 = \frac{\pi}{4}5.0^2 = 19.64 \text{ sq. mm.}$$

Applying Eq. (4)

$$R_1 = \rho\frac{V}{(107.5)^2} = 0.031 \text{ ohm}$$

$$R_2 = \rho\frac{V}{(19.64)^2}$$

Since the volume of the wire does not change during the drawing process and the resistivity constant ρ remains the same,

$$\frac{R_2}{R_1} = \frac{R_2}{0.031} = \frac{\rho\dfrac{V}{(19.64)^2}}{\rho\dfrac{V}{(107.5)^2}}.$$

$$R_2 = 0.031\frac{(107.5)^2}{(19.64)^2} = 0.031\frac{11{,}560}{386} = 0.93 \text{ ohm.} \quad \textit{Ans.}$$

Also see the example, Par. 36, page 36 (Fig. 44). The volume and resistivity of the two brass rods are the same. Their resistances are proportional to the square of their lengths. Likewise their resistances are inversely proportional to the square of their cross-sections.

38. Conductance.—Conductance is the reciprocal of resistance and may be defined as being that property of a circuit or of a material which tends to permit the flow of an electric current. The unit of conductance is the reciprocal ohm or *mho*. Conductance is usually expressed by g.

$$g = \frac{1}{R} \tag{5}$$

also

$$g = \gamma \frac{A}{L} \tag{6}$$

where γ is the *specific conductance* or the *conductivity* of a substance, A the uniform cross-section and L the length.

The conductivity of copper is 580,000 mhos per centimeter cube.

Example.—Determine the conductance of an aluminum bus-bar 0.5 in. thick, 4 in. wide, and 20 ft. long.

The conductivity of aluminum is 61 per cent. that of copper and copper has a conductivity of 580,000 mhos per centimeter cube.

The conductivity of aluminum is:

$$\gamma = 0.61 \times 580{,}000 = 354{,}000 \text{ mhos/cm. cube}$$

The cross-section of the bus-bar:

$$A = 0.5 \times 4 \times 2.54 \times 2.54 = 12.9 \text{ sq. cm.}$$

The length $L = 20 \times 12 \times 2.54 = 610$ cm.

The conductance:

$$g = 354{,}000 \times \frac{12.9}{610} = 7{,}490 \text{ mhos.} \quad \textit{Ans.}$$

39. Per Cent. Conductivity.—For some time the per cent. conductivity of copper was based upon results obtained in 1862 by Matthiesen, who made careful measurements of the resistance of supposedly pure copper. He found the resistivity to be 1.594 microhms per centimeter cube at 0° C. In view of the uncertainty of the quality of his copper, the Bureau of Standards[1] made a large number of measurements upon commercial copper. Its recommendation that the standard of resistivity be 1.724 microhms per centimeter cube at 20° C. has been adopted internationally. The per cent. conductivity for carefully refined copper may exceed 100. Comparison to obtain per cent. conductivity should be made at 20° C.

[1] See "Copper Wire Tables" *Circular* of the Bureau of Standards, No. 31, (1914).

Example.—A copper rod 4 ft. long and having a diameter of 162 mils has a resistance of 0.0016 ohm at 20° C. What is its per cent. conductivity?

$$\text{Cross-section} = \frac{\pi}{4}(0.162 \times 2.54)^2 = 0.133 \text{ sq. cm.}$$

$$\text{Length} = 4 \times 12 \times 2.54 = 122 \text{ cm.}$$

$$\text{Resistance per centimeter cube} = \frac{0.0016 \times 0.133}{122} = 0.000001744 \text{ ohm}$$
$$= 1.744 \text{ microhms.}$$

$$\text{Per cent. conductivity} = \frac{1.724}{1.744}\,100 \text{ or } 98.9 \text{ per cent.} \quad \textit{Ans.}$$

40. Resistances in Series and in Parallel.—If a number of resistances r_1, r_2, r_3, etc. (Fig. 45) are connected in series, that is, end to end, the total resistance of the combination is:

$$R = r_1 + r_2 + r_3 + \cdots \tag{7}$$

That is:

In a series circuit the total resistance is the sum of the individual resistances.

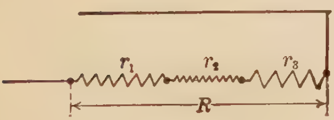


FIG. 45.—Resistances in series.

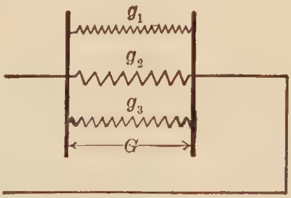


FIG. 46.—Conductances in parallel.

If a number of conductances g_1, g_2, g_3, etc., are connected in parallel (Fig. 46) the total conductance of this portion of the circuit must be equal to the sum of the individual conductances, that is,

$$G = g_1 + g_2 + g_3 + \cdots \tag{8}$$

Since,

$$G = \frac{1}{R}, \quad \text{Eq. (8) may be written}$$

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots \tag{9}$$

That is:

In a parallel circuit, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

For a circuit having two resistances in parallel, r_1 and r_2, the joint resistance

$$R = \frac{r_1 r_2}{r_1 + r_2}; \tag{10}$$

for three resistances in parallel the joint resistance is

$$R = \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1} \tag{11}$$

Example.—Determine the total resistance of a circuit having four branches, the individual resistances of which are 3, 4, 6, and 8 ohms, respectively.

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = 0.333 + 0.250 + 0.167 + 0.125$$

$$= 0.875 \text{ mho.}$$

$$R = \frac{1}{0.875} = 1.142 \text{ ohms.} \qquad \textit{Ans.}$$

41. The Circular Mil.—In the English and American wire tables the circular mil is the standard unit of wire cross-section.

The term *mil* means one-thousandth; for example, a millivolt = 1/1,000 volt. A mil is *one thousandth* of an inch. A

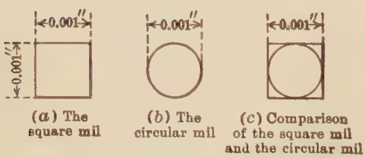


FIG. 47.—The square mil and circular mil.

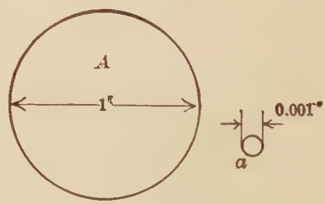


FIG. 48.—Cross-section expressed in cir. mils.

square mil is a square, each side of which is one mil (0.001 in.), as shown in Fig. 47 (*a*). The area of a square mil is $0.001 \times 0.001 = 0.000001$ sq. in.

A circular mil is the area of a circle whose *diameter* is *one mil* (0.001 in.) (Fig. 47(*b*)) and is usually written C.M. or *cir. mil*. As will be seen from Fig. 47(*c*), a cir. mil is a smaller area than a square mil. The area in square inches of a cir. mil $= \pi/4(0.001)^2 = 0.0000007854$ sq. in.

The circular mil is the unit with which the cross-section of wires and cables is measured, just as the square foot is the unit by which larger areas such as floors, land, etc. are measured. The advantage of the circular mil as a unit is that circular areas

measured in terms of this unit bear a very simple relation to the diameters.

In Fig. 48, A represents the cross-section of a wire having a diameter of 1 in. Required: to determine its area in cir. mils.

The area, A $= \frac{\pi}{4}(1)^2$ sq. in.

The area, a, of a cir. mil $= \frac{\pi}{4}(0.001)^2$ sq. in.

The ratio of $\frac{A}{a}$ obviously gives the number of cir. mils in A

Therefore

$$\frac{A}{a} = \frac{\frac{\pi}{4}(1)^2}{\frac{\pi}{4}0.000001} = 1{,}000{,}000 \text{ cir. mils.}$$

The general relation may be written:

$$\text{Cir. mils} = \frac{D_1{}^2}{(0.001)^2} = 1{,}000{,}000\ (D_1)^2 = D^2 \qquad (12)$$

where

D_1 is the diameter of the wire in *inches*.
D is the diameter of the wire in *mils*.

The matter may be summed up in two rules:

To obtain the number of circular mils in a solid wire of given diameter express the diameter in mils and then square it.

To obtain the diameter of a solid wire having a given number of circular mils, take the square root of the circular mils and the result will be the diameter of the wire in mils.

Example.—00 wire (A.W.G.) has a diameter of 0.3648 in. What is its cir. milage?

$$0.3648 \text{ in.} = 364.8 \text{ mils}$$
$$(364.8)^2 = 133{,}100 \text{ cir. mils.} \quad \textit{Ans.}$$

Example.—A certain wire has a cross-section of 52,640 cir. mils. What is its diameter?

$$\sqrt{52{,}640} = 229.4 \text{ mils} = 0.2294 \text{ in.} \quad \textit{Ans.}$$

42. The Circular-mil-foot.—Another convenient unit of resistivity, especially in the English system, is the resistance of a cir.-mil-foot. This unit is the resistance of a wire having a cross-section of 1 cir. mil and a length of 1 ft., as shown in Fig. 49.

The resistance of a cir.-mil-foot of copper at 20° C. is 10.37 ohms. (In practical work this resistance may frequently be taken as 10 ohms.) Knowing this resistivity, the resistance of any length and size of wire may be determined by Eq. (3).

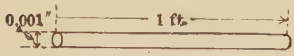


FIG. 49.—The circular-mil-foot.

Example.—What is the resistance of a 750,000-C.M. copper cable, 2,500 ft. long?

If the cable had a cross-section of 1 cir. mil it would have a resistance of $2{,}500 \times 10.37 = 25{,}900$ ohms. However, the cross-section is actually 750,000 cir. mils, therefore,

$$R = \frac{25{,}900}{750{,}000} = 0.0346 \text{ ohm} \quad \textit{Ans.}$$

or Eq. (3), page 35, may be used directly.

$$R = 10.37\frac{2{,}500}{750{,}000} = 0.0346 \text{ ohm.}$$

When applying Eq. (3), L must be expressed in *feet* and A in *cir. mils.*

43. The Circular-mil-inch.—The circular-mil-inch is also a very convenient unit of resistivity, particularly in connection with coils and windings. Such windings usually operate at a temperature which gives a resistivity of approximately 12 ohms per cir.-mil-foot or one ohm per cir.-mil-inch. (A temperature of 60° C. gives this resistivity.) It follows that the resistance of a copper conductor,

$$R = 1\frac{l}{\text{C.M.}}$$

where l is in inches.

That is the resistance is equal to the length in inches divided by the cross-section in circular mils.

Example.—A certain circular magnet coil having an inner diameter of 6 in., an outer diameter of 9 in., and a length of 4 in. is wound with 1,000 turns of No. 14 d.c.c. magnet wire, which has a cross-section of 4,110 C.M. Assuming that the resistance of 1 cir.-mil-inch of the wire is one ohm, find the resistance of the coil.

The mean length of turn is $\pi\frac{9+6}{2} = \pi 7.5 = 23.6$ in.

The total length of copper, $1{,}000 \times 23.6 = 23{,}600$ in.

The resistance, $R = \frac{23{,}600}{4{,}110} = 5.74\,\Omega$ *Ans.*

44. Resistor Materials and Alloys.—Resistor materials are used where it is desired purposely to introduce resistance into a circuit. Frequently the resistor materials must operate at high temperature, as in heating devices, such as electric furnaces, ranges, toasters, etc. The material must have high resistivity, high melting point, and must be able to resist oxidation at high temperatures. With field rheostats and controllers, long life and overload capacity are highly desirable. Hence they are not normally operated at the high temperatures of many of the heating devices. This reduces the effects of corrosion and disintegration due to high temperatures. Resistor materials are also used in connection with instruments, as for ammeter shunts, series resistances for voltmeters, etc. Although the temperature may not be high when used for these purposes, it is highly desirable that the temperature coefficient of resistance be small.

One of the earliest resistor alloys is *german silver* or nickel silver, which is composed of copper, nickel, and zinc, in varying proportions. The alloy is usually listed on the percentage of nickel, 18 per cent. alloy containing 18 per cent. nickel. The 18 per cent. alloy has a resistivity about eighteen times that of copper, and the 30 per cent. about twenty-eight times. German silver has a high temperature coefficient of resistance. Manganin is a copper-manganese alloy of about 65 per cent. copper, 30 per cent. ferro-manganin, 5 per cent. nickel. It has a very low temperature coefficient, and hence is used extensively with instruments for multipliers, shunts, etc. Iron-nickel alloys have high resistivity but are far inferior to chromium-nickel alloys in resistance to corrosion. Copper-nickel alloys have resistivities of from ten to thirty times the resistivity of copper and are used commonly for resistor materials. The chromium-nickel alloys, such as "Nichrome," have resistivities from sixty to seventy times that for copper, and are used for resistor materials where high resistivity is essential. Iron wire and cast iron are also used as resistors, cast-iron grids being commonly used for starting and controller resistances. Table 45 gives the properties of the more common metals and resistor materials.

45. Electrical Properties of the Metals and Alloys

Metals	Resistivity 20° C. Cm. cube (microhms)	Resistivity 20° C. Cir.-mil.-ft. (ohms)	Temperature coefficient of resistance at 20° C.	Approximate maximum working temperature, ° C.	Melting point, ° C.
Aluminum	2.828	17.02	0.0039	....	659
Antimony	41.7	251.0	0.0036	....	630
Bismuth	110.0	663.0	0.004	....	271
Carbon:					
Amorphous	3,800 to 4,100	......	(−)	....	>3500
Retort (graphite)	†720 to 812	......	(−)		
Copper (drawn)	1.724	10.37	0.00393	260	1083
Gold	2.44	14.7	0.0034	....	1063
Iron:					
Electrolytic	9.96	59.9	....	....	1530
Cast	74.4 to 97.8	448 to 588			
Lead	*20.4	*123	0.00387	....	327
Mercury	*94.07	*566	0.00072	....	−38.9
Nickel	*6.93	*41.7	0.0062	600	1452
Platinum	*10.96	*66.0	0.003	1,500	1754
Silver	1.629	9.8	0.0038	....	961
Steel:					
Soft	15.9	95.8	0.0016	....	1430
Glass hard	45.7	275			
Silicon 4 per cent	51.15	308			
Transformer	11.09	66.8			
Tungsten	5.51	33.2	0.005	....	3400
Zinc	*5.75	*34.6	0.0037	....	419
Alloys					
Brass	7	42.1	0.002	....	940
Constantan (Cu 60, Ni 40)	49	295	0.00005	....	1290
German silver Ni 18	33	198.5	0.0004		
IaIa	49	295	0.000005	370	1230
Ideal	50	300	0.000018	350	1090
Manganin (Cu 84, Mn 12, Ni 4)	44	265	0.000006	100	
Monel metal (Ni-Cu)	42	294	0.0020	450	1360
Nichrome (FeNiCr)	109	670	0.00019	950	
Phosphor-bronze: Cu, Sn 2–6, Ph 0.005 to 0.13	3.95	23.7			

* 0° C.

† Furnace electrodes, 3000° C.

46. Temperature Coefficient of Resistance.—The resistance of copper and other non-alloyed metals increases very appreciably with the temperature. As the temperature of the windings of electric machinery is necessarily much higher than that of the surrounding air, it is important to know the relation between

temperature and resistance. The relation may be expressed as follows:

$$R_t = R_0(1 + \alpha t) \tag{13}$$

where R_t is the resistance at the temperature t, R_0 the resistance at 0° C., and α is the *temperature coefficient of resistance* at 0°. For copper α is 0.00427 and for most of the unalloyed metals is sensibly of this value. The above is equivalent to saying that the resistance increases 0.427 of 1 per cent. for each degree Centigrade increase of temperature above 0°. For example, assume that a coil has a resistance of 100 ohms at 0° C. For every degree increase of temperature the resistance will increase

$$100 \times 0.00427 \text{ ohm or } 0.427 \text{ ohm.}$$

At 40° C. the increase of resistance will be $40 \times 0.427 = 17.08$ ohms, and the resistance at 40° will be $100 + 17.08 = 117.08$ ohms.

If the resistance at some definite temperature other than 0° C. is known, ordinarily the resistance at 0° C. must first be found before the resistance at other temperatures can be determined.

For this purpose Eq. (13) may be put in the form

$$R_0 = \frac{R_t}{1 + \alpha t} \tag{14}$$

Example.—The resistance of an electromagnet winding of copper wire at 20° C. is 30 ohms. What is its resistance at 80° C.?

The resistance at 0° C.

$$R_0 = \frac{30}{1 + 0.00427 \times 20} = \frac{30}{1.085} = 27.65 \text{ ohms}$$

$$R_{80} = 27.65(1 + 0.00427 \times 80) = 37.11 \text{ ohms.} \quad \textit{Ans.}$$

This process of working back to 0° is a little inconvenient, although it is fundamental and easy to remember. It is possible, however, to determine the temperature coefficient for *any* initial temperature.

Let R_1 and R_2 be the resistances at temperatures t_1 and t_2 respectively. Then

$$R_1 = R_0(1 + \alpha t_1) \tag{I}$$

$$R_2 = R_0(1 + \alpha t_2) \tag{II}$$

Dividing Eq. (II) by Eq. (I) and solving for R_2

$$R_2 = R_1\frac{(1 + \alpha t_2)}{(1 + \alpha t_1)} = R_1[1 + \alpha_1(t_2 - t_1)] \tag{III}$$

Where α_1 is the temperature coefficient at the initial temperature t_1.

Solving Eq. (III) for α_1

$$\alpha_1 = \frac{\alpha(t_2 - t_1)}{(\alpha t_1 + 1)(t_2 - t_1)} = \frac{\alpha}{\alpha t_1 + 1} = \frac{1}{t_1 + \frac{1}{\alpha}}$$

If α (for copper) equals 0.00427, $\frac{1}{\alpha} = 234.5.$

Then $\alpha_1 = \frac{1}{t_1 + 234.5}$, which is easy to remember.

For example, $\alpha_{40} = \frac{1}{274.5} = 0.00364$ (see Par. 47).

Paragraph 47 gives the temperature coefficients of copper for different initial temperatures. By using this table, temperature coefficient problems are simplified, as shown by solving the example of Par. 46.

Example.—The temperature coefficient of copper at 20° initial temperature from Par. 47 is 0.00393. The rise in temperature = 80° − 20° = 60°.
Then the resistance at 80° C.

$$R_{80} = 30(1 + 0.00393 \times 60) = 37.07 \text{ ohms.} \quad \textit{Ans.}$$

47. Temperature Coefficients of Copper at Different Initial Temperatures
(From formula $1/(234.5 + t)$)

Initial temperatures	Increase in resistance per 1° C.
0	0.00427
5	0.00418
10	0.00409
15	0.00401
20	0.00393
25	0.00385
30	0.00378
35	0.00371
40	0.00364
45	0.00358
50	0.00352

48. Inferred Zero Resistance.—If the resistance of copper at ordinary temperatures be plotted as ordinates and temperature as

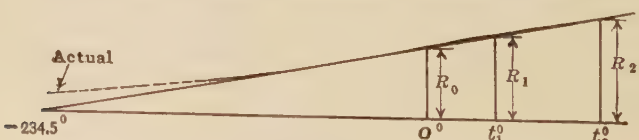


Fig. 50.—Variation of resistance with temperature.

abscissas the graph is practically a straight line (Fig. 50). If this line be extended, it will intersect the zero resistance line at −234.5° C. (an easy number to remember), as shown in Fig. 50. This is equivalent to saying that between ordinary limits of temperature, copper behaves as if it had zero resistance at −234.5° C.

(Actually the curve bends at these extremely low temperatures, as shown by the dotted line (Fig. 50).) This gives a convenient method for determining temperature-resistance relations.

By the law of similar triangles (Fig. 50),

$$\frac{R_0}{234.5^\circ} = \frac{R_1}{234.5^\circ + t_1} \tag{15}$$

$$\frac{R_1}{234.5^\circ + t_1} = \frac{R_2}{234.5^\circ + t_2} \tag{16}$$

Applying this equation to the example, of Par. 46.

$$\frac{30}{234.5^\circ + 20^\circ} = \frac{R_{80}}{234.5^\circ + 80^\circ}.$$

$$R_{80} = 30\,\frac{234.5^\circ + 80^\circ}{234.5^\circ + 20^\circ} = 30\,\frac{314.5^\circ}{254.5^\circ} = 37.1 \text{ ohms.} \quad \textit{Ans.}$$

49. The American Wire Gage (A.W.G.).—The A.W.G. (formerly Brown & Sharpe Gage) is based on a constant ratio between diameters of successive gage numbers; that is, the diameters taken in order form a geometrical progression. The diameter of 0000 is defined as 0.4600 in. (1.168 cm.) and the diameter of No. 36 as 0.0050 in. (0.0127 cm.). Between No. 0000 and No. 36 are 38 gage numbers. Hence the ratio of any diameter to the diameter of the next greater gage number must be

$$\sqrt[39]{\frac{0.4600}{0.0050}} = \sqrt[39]{92} = 1.123.$$

The square of this ratio $(1.123)^2$ is 1.2610. Since the cross-section varies as the square of the diameters, the ratio of any cross-section to the cross-section of the next greater number is 1.261. The sixth power of 1.123 is 2.0050. Hence the ratio of any diameter to the diameter of a number which is six greater is practically 2. It follows that the cross-section either doubles or halves for every three gage numbers. Since $(1.261)^2$ is 1.590, the ratio of any cross-section to the cross-section of a number greater by 2 is 1.60, practically. The ratio of any cross-section to one of a smaller number differing by 10 is $(92^{10/39})^2 = 10$ (practically).

50. Approximations with the A.W.G.—The diameter of No. 10 wire is 0.102 in. (0.259 cm.) and has a cross-section of 10,400 cir. mils. As an approximation, the diameter may be considered as being 0.1 in. (0.254 cm.) and the cross-section 10,000 cir. mils.

From the foregoing and the fact that the weight of 1,000 ft. of No. 10 wire is 31.4 (10π) lb., the following approximate rules may be given:

(1) No. 10 wire has a diameter of 0.1 in. and a resistance of 1 ohm per 1,000 ft. (2) The resistance of the wire doubles with every increase of 3 gage numbers. (3) Therefore the resistance increases $\sqrt[3]{2} = 1.26$ ($1\frac{1}{4}$) times for each successive gage number and $(1.26)^2 = 1.6$ times for every two numbers. (4) The resistance is multipled or divided by 10 for every difference of 10 gage numbers. (5) The weight of 1,000 ft. of No. 10 wire is 31.4 lb. and the weight of 1,000 ft. of No. 2 wire is 200 lbs. These rules make it a comparatively simple matter to determine the weight or resistance of any gage number without reference to the table.

Example.—What are the resistance and weight of 1,000 ft. of 0000 wire?

The resistances will decrease as follows:

Gage No.	10	7	4	1	000	
Resistance	1	0.5	0.25	0.125	0.0625	(rules 1 and 2)

Resistance of 0000 = 0.0625/1.25 = 0.050 ohm (rules 3). *Ans.*

Weight of 1,000 ft. No. 2 = 200 lb.

Weight of 1,000 ft. 00 = 400 lb.

Weight of 1,000 ft. 0000 = 400 × 1.6 = 640 lb. (rules 5, 2 and 3). *Ans.*

The example might have been worked more quickly by rule 4.

Resistance of 1,000 ft. of No. 10 = 1 ohm.

Resistance of 1,000 ft. of 0 = 0.1 ohm (rule 4).

Resistance of 1,000 ft. of 0000 = 0.050 ohm (rule 2).

51. Working Table, Standard Annealed Copper Wire, Solid[1]
American Wire Gage (B. & S.). English Units

Gage No.	Diameter in mils	Cross-section		Ohms per 1000 ft.		Ohms per mile	Pounds per 1,000 ft.
		Circular mils	Square inches	25° C. (=77° F.)	65° C. (=149° F.)	25° C. (=77° F.)	
0000	460.0	212,000.0	0.166	0.0500	0.0577	0.264	641.0
000	410.0	168,000.0	0.132	0.0630	0.0727	0.333	508.0
00	365.0	133,000.0	0.105	0.0795	0.0917	0.420	403.0
0	325.0	106,000.0	0.0829	0.100	0.116	0.528	319.0
1	289.0	83,700.0	0.0657	0.126	0.146	0.665	253.0
2	258.0	66,400.0	0.0521	0.159	0.184	0.839	201.0
3	229.0	52,600.0	0.0413	0.201	0.232	1.061	159.0
4	204.0	41,700.0	0.0328	0.253	0.292	1.335	126.0
5	182.0	33,100.0	0.0260	0.319	0.369	1.685	100.0
6	162.0	26,300.0	0.0206	0.403	0.465	2.13	79.5
7	144.0	20,800.0	0.0164	0.508	0.586	2.68	63.0
8	128.0	16,500.0	0.0130	0.641	0.739	3.38	50.0
9	114.0	13,100.0	0.0103	0.808	0.932	4.27	39.6
10	102.0	10,400.0	0.00815	1.02	1.18	5.38	31.4
11	91.0	8,230.0	0.00647	1.28	1.48	6.75	24.9
12	81.0	6,530.0	0.00513	1.62	1.87	8.55	19.8
13	72.0	5,180.0	0.00407	2.04	2.36	10.77	15.7
14	64.0	4,110.0	0.00323	2.58	2.97	13.62	12.4
15	57.0	3,260.0	0.00256	3.25	3.75	17.16	9.86
16	51.0	2,580.0	0.00203	4.09	4.73	21.6	7.82
17	45.0	2,050.0	0.00161	5.16	5.96	27.2	6.20
18	40.0	1,620.0	0.00128	6.51	7.51	34.4	4.92
19	36.0	1,290.0	0.00101	8.21	9.48	43.3	3.90
20	32.0	1,020.0	0.000802	10.4	11.9	54.9	3.09
21	28.5	810.0	0.000636	13.1	15.1	69.1	2.45
22	25.3	642.0	0.000505	16.5	19.0	87.1	1.94
23	22.6	509.0	0.000400	20.8	24.0	109.8	1.54
24	20.1	404.0	0.000317	26.2	30.2	138.3	1.22
25	17.9	320.0	0.000252	33.0	38.1	174.1	0.970
26	15.9	254.0	0.000200	41.6	48.0	220.0	0.769
27	14.2	202.0	0.000158	52.5	60.6	277.0	0.610
28	12.6	160.0	0.000126	66.2	76.4	350.0	0.484
29	11.3	127.0	0.0000995	83.4	96.3	440.0	0.384
30	10.0	101.0	0.0000789	105.0	121.0	554.0	0.304
31	8.9	79.7	0.0000626	133.0	153.0	702.0	0.241
32	8.0	63.2	0.0000496	167.0	193.0	882.0	0.191
33	7.1	50.1	0.0000394	211.0	243.0	1,114.0	0.152
34	6.3	39.8	0.0000312	266.0	307.0	1,404.0	0.120
35	5.6	31.5	0.0000248	335.0	387.0	1,769.0	0.0954
36	5.0	25.0	0.0000196	423.0	488.0	2,230.0	0.0757
37	4.5	19.8	0.0000156	533.0	616.0	2,810.0	0.0600
38	4.0	15.7	0.0000123	673.0	776.0	3,550.0	0.0476
39	3.5	12.5	0.0000098	848.0	979.0	4,480.0	0.0377
40	3.1	9.9	0.0000078	1,070.0	1,230.0	5,650.0	0.0299

NOTE 1.—The *fundamental resistivity* used in calculating the tables is the International Annealed Copper Standard viz., 0.15328 ohm (meter, gram) at 20° C. The *temperature coefficient* for this particular resistivity is $\alpha_{20} = 0.00393$, or $\alpha_0 = 0.00427$. The *density* is 8.89 grams per cubic centimeter.

NOTE 2.—The values given in the table are only for annealed copper of the standard resistivity. The user of the table must apply the proper correction for copper of any other resistivity. Hard-drawn copper may be taken as about 2.7 per cent. higher resistivity than annealed copper.

NOTE 3.—Pounds per mile may be obtained by multiplying the respective values above by 5.28.

[1] From Circular of the Bureau of Standards, No. 31.

52. **Bare Concentric Lay Cables of Standard Annealed Copper**

English Units

A. W. G. No.	Circular mils	Ohms per 1,000 ft. 25° C. (=77° F.)	Ohms per 1,000 ft. 65° C. (=149° F.)	Pounds per 1,000 ft.	Standard concentric stranding: Number of wires	Standard concentric stranding: Diameter of wires, in mils	Standard concentric stranding: Outside diameter, in mils
	2,000,000	0.00539	0.00622	6,180	127	125.5	1,631
	1,700,000	0.00634	0.00732	5,250	127	115.7	1,504
	1,500,000	0.00719	0.00830	4,630	91	128.4	1,412
	1,200,000	0.00899	0.0104	3,710	91	114.8	1,263
	1,000,000	0.0108	0.0124	3,090	61	128.0	1,152
	900,000	0.0120	0.0138	2,780	61	121.5	1,093
	850,000	0.0127	0.0146	2,620	61	118.0	1,062
	750,000	0.0144	0.0166	2,320	61	110.9	998
	650,000	0.0166	0.0192	2,010	61	103.2	929
	600,000	0.0180	0.0207	1,850	61	99.2	893
	550,000	0.0196	0.0226	1,700	61	95.0	855
	500,000	0.0216	0.0249	1,540	37	116.2	814
	450,000	0.0240	0.0277	1,390	37	110.3	772
	400,000	0.0270	0.0311	1,240	37	104.0	728
	350,000	0.0308	0.0356	1,080	37	97.3	681
	300,000	0.0360	0.0415	926	37	90.0	630
	250,000	0.0431	0.0498	772	37	82.2	575
0000	212,000	0.0509	0.0587	653	19	105.5	528
000	168,000	0.0642	0.0741	518	19	94.0	470
00	133,000	0.0811	0.0936	411	19	83.7	418
0	106,000	0.102	0.117	326	19	74.5	373
1	83,700	0.129	0.149	258	19	66.4	332
2	66,400	0.162	0.187	205	7	97.4	292
3	52,600	0.205	0.237	163	7	86.7	260
4	41,700	0.259	0.299	129	7	77.2	232

53. Conductors.—Although silver is a better conductor than copper, its use as a conductor is very limited because of its cost. In a few instances it is used where a delicate but highly conducting material is necessary, such as in the brushes and occasionally in the commutator of watthour meters. Copper, because of its high conductivity and moderate cost, is used more extensively

as a conductor than any other material. It has many good qualities such as ductility, high tensile strength, not easily abraided, not corroded by the atmosphere, and it can be readily soldered.

Aluminum has only 61 per cent. of the conductivity of copper, but for the same length and weight, it has about twice the conductance of copper. It is softer than copper, its tensile strength is much less, and it cannot be readily soldered. It is not affected by exposure to the atmosphere. The large diameter for a given conductance prohibits its use where an insulating covering is required. Aluminum is used extensively as a conductor for high-voltage transmission lines, where its lightness and large diameter are an advantage. It is used to some extent for low-voltage bus-bars as it offers much greater radiating surface than copper of the same conductance.

Iron and steel have about nine times the resistance of copper for the same cross-section and length. The large cross-section for a given conductance prohibits their use where an insulating covering is necessary and the increased weight prevents their use in most cases where the conductors must be placed on poles. They are most commonly used as resistors in connection with rheostats and for third rails of electric railways. Iron and steel ordinarily must be protected from oxidation by galvanizing or other protective covering. Copper-clad steel ("Copper-weld") consists of a steel wire coated or covered with a layer of copper, fused or welded to the steel. The advantages claimed for it are that it possesses the high tensile strength of steel, combined with the high conductivity of copper. Further, the copper protects the steel from corrosion. Its field is the transmission line conductor, where long spans make high tensile strength necessary. It is also used as an overhead ground wire on transmission lines.

CHAPTER IV

OHM'S LAW AND THE ELECTRIC CIRCUIT

The exact nature of electricity is not known, but recent investigations indicate that it consists of infinitesimal charges called electrons.[1] When these electrons are forced to travel in the same direction an electric current results. The flow of electricity through a circuit resembles in many ways the flow of water through pipes, for it acts as an incompressible fluid would act, undergoes pressure drop, etc., as will be shown later.

54. Absolute Systems of Electrical Units.—There are two basic or fundamental systems of electrical units, the *electrostatic* system and the *electromagnetic* system.

The units in the *electrostatic system* are based on Coulomb's experimental law (see p. 239) which states that the force acting between two electrostatic charges Q_1 and Q_2, placed r centimeters apart in air, is $\frac{Q_1Q_2}{r^2}$ dynes. All the other electrical units such as those of current, potential, etc. may be derived from this simple relationship. The electrostatic system is used to calculate the capacitance of condensers from their geometry (see Eq. 104, p. 254), and is also used to a large extent in the calculation of electron phenomena, since electrons themselves are minute electric charges. The units of the electrostatic system ordinarily have the prefix, *stat*, for example, statvolt, statfarad, etc.

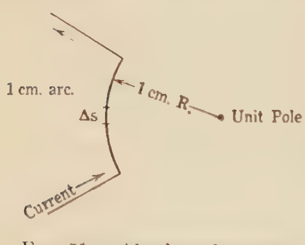


FIG. 51.—Absolute determination of current

The units in the c.g.s. (cm.-gm.-sec.) *electromagnetic* or *absolute* system are based fundamentally on the law which gives the force between magnetic poles, Eq. 1, p. 6. The unit of current is

[1] See Vol. II, Chap. XIII. [All references to volume II are to the second edition (revised).]

derived by means of the Biot-Savart law, which gives the relation existing between the current in a conductor and the force which it exerts on a unit magnetic pole. If a conductor carrying a current be bent into an arc of a circle 1 cm. in length and 1 cm. radius (Fig. 51) and the unit pole be placed at the center of the *circle*, the force acting on the unit pole due to the current in *any* small length of arc Δs is proportional to the current and the length of arc Δs. (It follows that any two arcs of equal length exert the same force on the unit pole. Also note that the leading-in connections to the arc (Fig. 51) are radial so that the current in them exerts no force on the unit pole.) If the current is adjusted until the force on the unit pole is *one dyne*, its value is one c.g.s. or *absolute ampere* (*absampere*). All other units in the system may then be derived from the current whose value is thus determined. For example, the quantity Q is equal to the product of current and time, etc. The units of this c.g.s. electromagnetic system are commonly used in magnetic calculation (see Chap. VIII). They also serve as the basis of the units of the practical system (Par. 55). Absolute units ordinarily have the prefix *ab* or *abs*, for example, abvolt, absampere, etc.

55. The Practical System of Electrical Units.—Most of the units of the absolute electromagnetic system are many times smaller than the quantities which are used in practice. For example, the absolute volt or abvolt has 1/100,000,000 (10^{-8}) the magnitude of the practical volt. In the practical system, which is derived from the absolute system, the units are of the order of magnitude of the quantities which are ordinarily met in practice. These units of the practical system differ from the units of the absolute system only by definite numerical ratios. For example, the practical ampere has one-tenth (10^{-1}) the magnitude of the absampere (Par. 54), the practical ohm has one billion (10^{9}) times the magnitude of the absohm, the henry has one billion (10^{9}) times the magnitude of the abhenry, etc.

Current.—Since it is difficult to determine experimentally the value of current by measuring the force which it exerts on a unit pole (Fig. 51) it is necessary to use methods which are more practicable but which give values corresponding to the fundamental definition. Hence the ampere is defined by an act of Congress, 1894, as follows:

The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the centimeter-gram-second system of electromagnetic units and is the practical equivalent of the unvarying current, which when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of one thousand one hundred and eighteen millionths (0.001118) of a gram per second.

It should be kept in mind that the ampere is the *rate* of flow of electricity. It corresponds in hydraulics to the rate of flow of water, which is expressed as cubic feet per second, gallons per minute, etc.

Quantity.—The unit of quantity is the *coulomb.* This is equal to the quantity of electricity conveyed by one ampere in one second. The coulomb is analagous to the unit quantity of water in hydraulics, such as the cubic foot, the gallon, etc.

From this definition it is evident that an electric current may be expressed in *coulombs per second* rather than in *amperes.*

Difference of Potential and Electromotive Force (e.m.f.).—The difference of potential in volts between two points is defined fundamentally as the work in joules expended in moving a unit quantity of electricity, the coulomb, from one point to the other. Difference of potential and electromotive force tend to cause a flow of electricity. The unit of potential difference or of e.m.f. is the *volt,* and is that potential difference which when impressed across the terminals of a resistance of one ohm will cause a current of one ampere to flow. The *international volt* is now more specifically defined as 1/1.01830 of the voltage of a normal Weston cell (see Par. 96).

The mechanical analogy of potential difference is pressure. The difference in hydraulic pressure between the ends of a pipe causes or tends to cause the flow of water. The pressure of water behind the dam tends to cause water to flow through the penstock or through any leaks. The pressure in a boiler tends to cause steam to flow through the pipes, valves, etc. Likewise electric pressure or difference of potential tends to cause current to flow.

Resistance.—The ohm, the unit of resistance, has already been defined in Chap. III as that resistance which will allow one ampere to flow if one volt is impressed across its terminals. The *international ohm* is specifically defined as the resistance of a column of mercury at the temperature of melting ice (0° C.),

14.4521 g. in mass, of a constant cross-sectional area and of a length of 106.300 cm.

56. Absolute Potential.—The absolute potential of a body in c.g.s. units is defined as the work in ergs necessary to bring a unit charge of electricity up to it from infinity. It is practically impossible to determine the absolute potential of a body. For example, the absolute potential of the earth is not even approximately known, but for convenience the earth is assumed to be at zero potential and the potentials of bodies are usually given with reference to earth potential.

It is only occasionally that absolute potential is of interest. Usually it is desired to know *difference of potential.* The difference of potential (in c.g.s. units) between any two points is defined fundamentally as the work in ergs necessary to move a unit quantity of electricity from one point to the other. In the practical system the difference in potential would be given by the work in joules (or 10^7 ergs) necessary to move 1 coulomb from one point to the other (see page 54). Practically, difference of potential is measured by means of some measuring device, such as a voltmeter (see Par. 59).

57. Nature of the Flow of Electricity.—The flow of electricity through a circuit resembles in many ways the flow of water through a closed system of pipes. For example, in Fig. 52 water enters the mechanically-driven centrifugal pump P at a pressure h_1 (represented by the length of a column of mercury) above the point of zero pressure shown by the line h_0. In virtue of the action of the pump blades, its pressure through the pump is increased from h_1 to h_2, representing a net increase of pressure H_1. The water then flows out along pipe F_1 to the hydraulic motor W. Because of the friction loss in the pipe F_1, the pressure at the motor terminals h_3 is slightly less than h_2. In other words, a pressure of $h_2 - h_3$ is required to overcome the frictional resistance of the pipe F_1. The line ab gives the pressure at each point along the pipe F_1. The pressure decreases uniformly in F_1, as shown by line ab.

In Fig. 53 the mechanically-driven electrical generator G raises the potential of the current entering its negative terminal, from v_1 and v_2 where v_1 and v_2 are measured from the earth whose potential is ordinarily assumed as zero. (The various voltages

are measured with voltmeters v'_1, v'_2, etc.) The generator, in raising the potential of this portion of the circuit from v_1 to v_2, produces a net increase in pressure $v_2 - v_1 = V_1$. The current now flows out through the line L_1 to the + terminal of the motor M. Because of the line resistance, the potential drops from v_2 at

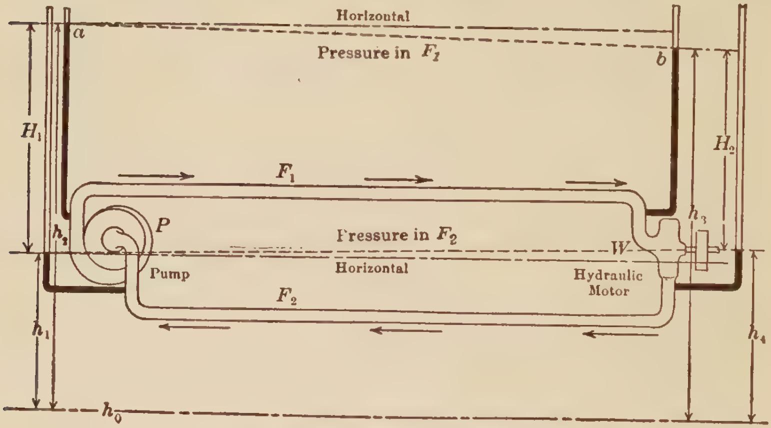


FIG. 52.—Flow of water through a hydraulic motor and pipe system.

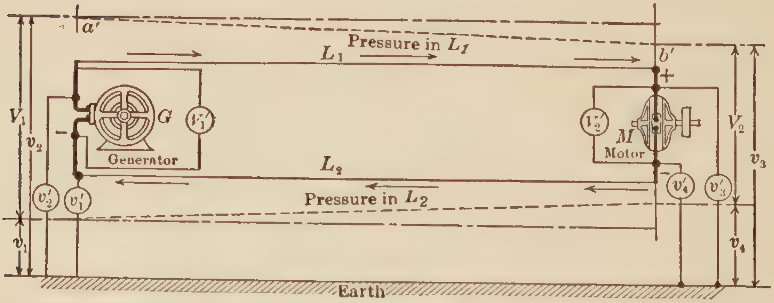


FIG. 53.—Flow of an electric current through an electric motor and the connecting feeder system.

the generator to v_3 at the motor in practically the same manner that the water pressure drops in pipe F_1 (Fig. 52). A voltage $v_2 - v_3$ is necessary to force the current through the line L_1. The line $a'b'$ shows the actual voltage at each point along the wire, the distance of $a'b'$ from the ground line being proportional to the voltage at each point. The voltage drop is uniform.

Referring to Fig. 52, the water enters the hydraulic motor W and in overcoming the back pressure of the revolving blades its pressure drops from h_3 to h_4, representing a net drop in pressure H_2. Pressure h_4 must necessarily be greater than h_1 in order that the water may flow back through the pipe F_2. The pressure $h_4 - h_1$ is necessary to overcome the friction loss in the pipe F_2. It is to be noted that H_2, the net pressure at the motor terminals, is less than the pressure H_1 at the pump, by the sum of the pressures necessary to overcome the friction in the two pipes F_1 and F_2.

In a similar manner, the pressure of the electric current in passing through the motor M, drops from v_3 to v_4, representing a net drop in pressure V_2. A large percentage of this voltage V_2 is necessary to overcome the back electromotive force of the motor. v_4 is necessarily greater than v_1, or the current could not flow along L_2 back to the negative terminal of the generator. It is to be noted that, as in the case of Fig. 52, the net potential difference V_2 at the motor M is less than the potential difference V_1 at the generator, by the drop in potential due to the resistance of both the *outgoing* and the *return* wire.

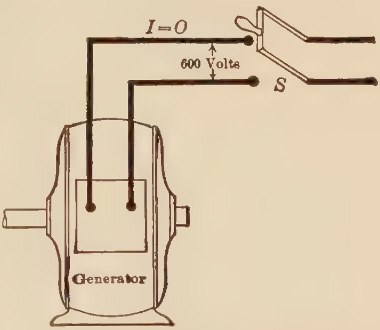


FIG. 54.—Illustrating the existence of potential difference without current.

Difference of potential is therefore the equivalent of pressure and *tends* to send current through a circuit; current is quantity of electricity per second. Potential difference may exist with no current flow, in the same manner that a boiler may have a very high steam pressure with no steam flow, due to all the valves being closed. Likewise a generator (Fig. 54) may have a very high potential difference at its terminals, yet, because the switch S is open, no current flows.

58. Difference of Potential.—In order that current may flow between two points, there must be a *difference of potential* between the two points as shown in Fig. 53. This is further illustrated in Fig. 55. A large reservoir and a small tank are connected by a pipe P. The water level in the tank and in the reservoir is the

same, that is, there is pressure in each but there is no *difference in pressure* between them. When the valve V is opened, no water flows from the reservoir to the tank. However, if the valve V' is opened, allowing the water level in the tank to fall, a difference of pressure between the two tanks results and water flows from the reservoir to the tank.

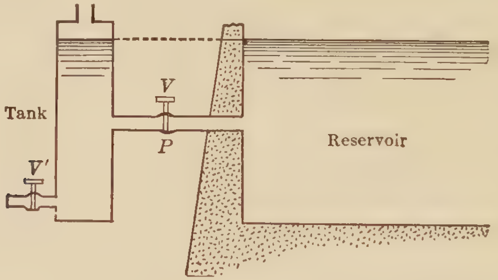


FIG. 55.—Tank and reservoir at the same pressure.

Figure 56 shows two batteries A_1 and A_2 each having an e.m.f. of 2 volts. The positive terminal of A_1 has a potential of $+2$ volts above its negative terminal; likewise the $+$ terminal of A_2 has a potential of $+2$ volts above its negative terminal. The negative terminals of both batteries are at the same potential

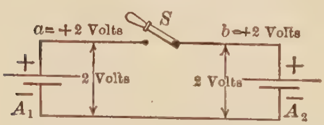


FIG. 56.—Two batteries having equal electromotive forces.

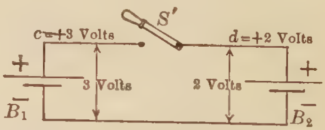


FIG. 57.—Two batteries having unequal electromotive forces.

because they are connected by a copper wire through which no current flows, and consequently there can be no potential difference between the ends of the copper wire. Therefore points a and b must each be at the same potential of $+2$ volts. If now the switch S be closed, *no* current will flow from a to b, because there is no *difference* of *potential* between a and b.

In Fig. 57 the e.m.f. of battery B_1 is 3 volts and therefore the potential of its positive terminal is 3 volts above that of its negative terminal. The e.m.f. of battery B_2 is 2 volts and therefore the potential of its positive terminal is 2 volts above that of its

negative terminal. The negative terminals are at the same potential. If this potential be assumed as zero, the point c is at a potential of $+3$ volts whereas the potential of d is $+2$ volts. Therefore the point c is at a potential of $3 - 2$ or 1 volt higher than d. When switch S' is closed, a current will flow from c to d, in virtue of c being at a higher potential than d.

59. Measurement of Voltage and Current.—Voltage or potential difference is ordinarily measured with a voltmeter. It is only occasionally that *absolute potential* is of interest. Ordinarily *difference of potential* is the quantity desired. The voltmeter therefore should be connected *across* or *between* the wires whose difference of potential is to be measured, as shown in Fig. 58.

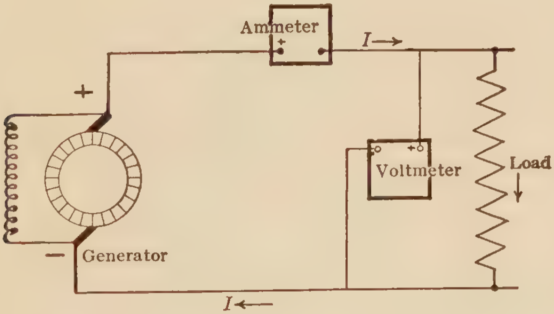


FIG. 58.—Proper method of connecting a voltmeter and an ammeter.

Current is ordinarily measured with an ammeter. As current is the *quantity* of *electricity* per second passing in the wire, the ammeter must be connected so that only the current to be measured passes through it. This is accomplished by opening one of the wires of the circuit and inserting the ammeter, just as a water meter is inserted in a pipe when it is desired to measure the flow of water in the pipe. When the ammeter is so connected, the current passing through to the load is measured by the ammeter (Fig. 58). *Never connect an ammeter across the line.*

60. Ohm's Law.—Ohm's law states that for a steady current the current in a circuit is *directly* proportional to the *total* electromotive force acting in the circuit and is *inversely* proportional to the total resistance of the circuit.

The law may be expressed by the following equation if the current I is in *amperes*, the e.m.f. E is in *volts*, and the resistance R is in *ohms*.

$$I = \frac{E}{R} \tag{17}$$

That is, the current in amperes in a circuit is equal to the e.m.f. of the circuit in volts divided by the resistance of the circuit in ohms. Potential difference may be represented by either the letter "V" or "E," V usually signifying terminal voltage and E electromotive force or induced voltage.

Example.—The resistance of the field winding of a shunt motor is 30 ohms. What current will flow through the winding when it is connected across 115-volt mains?

$$I = \frac{E}{R} = \frac{115}{30} = 3.83 \text{ amp.} \quad \textit{Ans.}$$

Example.—Figure 59 shows a simple series circuit in which two batteries are connected whose e.m.fs. are 6 volts and 10 volts, respectively, and two resistances of 2 and 6 ohms. Neglecting the resistance of the batteries, find the current in the circuit.

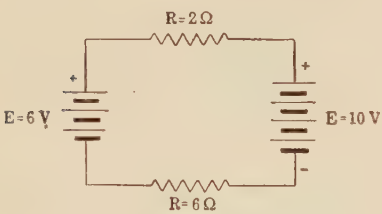


FIG. 59.—Current with two e.m.fs. and two resistances.

It is obvious from a study of Fig. 59 that the 6-volt battery tends to send current in a clockwise direction, and the 10-volt battery tends to send current in a counter-clockwise direction. Hence they are in *opposition* and the net e.m.f. is 10 − 6 or 4 volts. The total resistance is 2 + 6 or 8 ohms. Hence, by Ohm's law,

$$I = \frac{10 - 6}{2 + 6} = \frac{4}{8} = 0.5 \text{ amp.} \quad \textit{Ans.}$$

By transformation, Eq. (17) becomes

$$E = IR \tag{18}$$

That is, the voltage across any part of a circuit is equal to the product of the current in amperes and the resistance in ohms, provided the current is steady and there are no sources of e.m.f. within this part of the circuit.

Example.—The resistance of the field winding of a shunt generator is 48 ohms and the resistance of its rheostat is 22 ohms (see Fig. 60). If the field current is 3.2 amp., what is the voltage across the field winding terminals,

the voltage across the rheostat, and the voltage across the generator terminals?

$$E_1 = IR_1 = 3.2 \times 22 = 70.4 \text{ volts across rheostat.}$$
$$E_2 = IR_2 = 3.2 \times 48 = \underline{153.6} \text{ volts across field winding.}$$
$$\text{Total } 224.0 \text{ volts at terminals.}$$

Also

$$E = I(R_1 + R_2) = 3.2(22 + 48) = 224.0 \text{ volts (check).} \quad \textit{Ans.}$$

Again, if Eq. (17) be solved for the resistance the result is

$$R = \frac{E}{I} \tag{19}$$

That is, the resistance of a circuit is equal to the voltage divided by the current, provided the current is steady and there are no sources of e.m.f. within the circuit. This formula is very useful in making resistance measurements (see Par. 127).

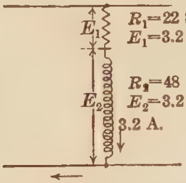


FIG. 60.—Voltage drops across a generator field and its rheostat.

Example.—The voltage across the terminals of a generator field is 220 volts and the field current is 4 amp. What is the resistance of the field circuit?

$$R = \frac{E}{I} = \frac{220}{4} = 55 \text{ ohms.} \quad \textit{Ans.}$$

61. The Series Circuit.—As was stated in Par. 40, if several resistances are connected in series, the total resistance is the sum of the individual resistances. That is,

$$R = r_1 + r_2 + r_3, \text{ etc.} \tag{20}$$

and the current

$$I = \frac{E}{R} = \frac{E}{r_1 + r_2 + r_3 + \cdots} \tag{21}$$

Example.—A 50-ohm relay is connected in series with a resistance tube of 30 ohms and with a small pilot lamp having a resistance of 5 ohms. The operating voltage is 155 volts. What current flows in this relay circuit?

$$I = \frac{115}{50 + 30 + 5} = \frac{115}{85} = 1.35 \text{ amp.} \quad \textit{Ans.}$$

62. The Parallel Circuit.—In Par. 40, the relation of total resistance to the component resistances in a parallel circuit was proved by transforming conductances into resistances. This equation may be proved by Ohm's law as follows: Consider the circuit of Fig. 61, consisting of resistances r_1, r_2, and r_3 in parallel across the voltage E. Let I_1 = the current in resistance r_1, I_2 = current in r_2, and I_3 = the current in r_3.

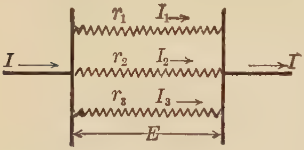


FIG. 61.—A parallel circuit.

Then

$$\left.\begin{aligned} I_1 &= \frac{E}{r_1} \\ I_2 &= \frac{E}{r_2} \\ I_3 &= \frac{E}{r_3} \end{aligned}\right\} \quad \text{(Eq. 17)}$$

Adding these together:

$$I_1 + I_2 + I_3 = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

Let the total current be $I = I_1 + I_2 + I_3$.

Let the equivalent resistance be R

$$I = \frac{E}{R}$$

Substituting I for $I_1 + I_2 + I_3$

$$I = \frac{E}{R} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

or

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \tag{22}$$

That is, *the reciprocal of the equivalent resistance of a parallel circuit is the sum of the reciprocals of the individual resistances.*

If but two resistances are involved,

$$R = \frac{r_1 r_2}{r_1 + r_2} \tag{23}$$

If three resistances are involved,

$$R = \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1} \tag{24}$$

Example.—Two rheostats, connected in parallel, are in series with the armature of an electric motor (Fig. 62). One rheostat is adjusted until its resistance is 3.2 ohms and the other until its resistance is 1.8 ohms. (*a*) What resistance is connected in series with the armature? (*b*) If the armature current is 25 amp., what is the voltage drop in the two resistances?

(*a*) Using Eq. (22)

$$\frac{1}{R} = \frac{1}{3.2} + \frac{1}{1.8} = 0.312 + 0.556 = 0.868 \text{ mho}$$

$$R = \frac{1}{0.868} = 1.152 \text{ ohms.} \quad \textit{Ans.}$$

Using Eq. (23) $R = \dfrac{3.2 \times 1.8}{3.2 + 1.8} =$ 1.152 ohms (check).

(*b*) $E = 25 \times 1.152 = 28.8$ volts. *Ans.*

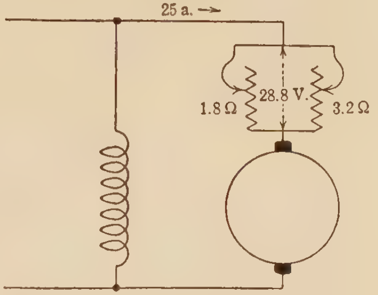


FIG. 62.—Parallel-connected rheostats in series with a motor armature.

(The voltage drop in each rheostat must be the same, since they are in parallel.)

Example.—Determine the total current in a circuit consisting of 4 resistances of 4, 6, 8, and 10 ohms respectively, connected in parallel across a 10-volt source.

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} = 0.25 + 0.167 + 0.125 + 0.10$$

$$= 0.642 \text{ mho}$$

$$R = \frac{1}{0.642} = 1.56 \text{ ohms.}$$

$$I = \frac{10}{1.56} = 6.42 \text{ amp.} \quad \textit{Ans.}$$

63. Division of Current in a Parallel Circuit.—In Fig. 63, two resistances, R_1 and R_2, are connected in parallel across the voltage E. Then:

$$I_1 = \frac{E}{R_1}$$

$$I_2 = \frac{E}{R_2}$$

$$\frac{I_1}{I_2} = \frac{E/R_1}{E/R_2} = \frac{R_2}{R_1}. \tag{25}$$

That is, in a parallel circuit of two branches, the currents are inversely as the resistances. (This relationship does not apply when there is a source of e.m.f. in either branch. For example it does not give the division of current through the field and

armature of a shunt motor when the motor is running since the motor armature generates an electromotive force.)

Example.—A current of 12 amp. divides between two paths in parallel, part passing through a branch having a resistance of 8 ohms, the other branch having a resistance of 12 ohms. How much current passes through each branch?

Let I_1 be the current in the 8-ohm branch and I_2 be the current in the 12-ohm branch.

$$\frac{I_1}{I_2} = \frac{12}{8} \text{ (Eq. 25).} \tag{I}$$

Also

$$I_1 + I_2 = 12.$$

$$I_1 = I_2\frac{12}{8} = I_2\frac{3}{2} \text{ from Eq. (I).} \tag{II}$$

substituting in Eq. (II)

$$I_2\frac{3}{2} + I_2 = 12.$$

$$\frac{5I_2}{2} = 12 \qquad I_2 = 4.8 \text{ amp. } \textit{Ans}$$

$$I_1 = 4.8\frac{3}{2} = 7.2 \text{ amp. } \textit{Ans.}$$

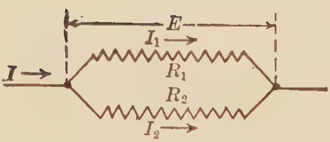


FIG. 63.—Division of current in a two-branch parallel circuit.

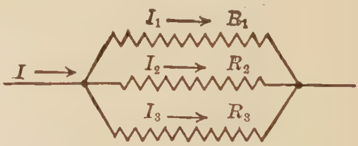


FIG. 64.—Division of current in a three-branch parallel circuit.

If the circuit consists of three branches (Fig. 64)

$$R_1, R_2, \text{ and } R_3,$$

the respective currents may be found as follows:

Let I be the total current $= I_1 + I_2 + I_3$.

It can be shown that the current in each branch is given by:

$$I_1 = I\left(\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{26}$$

$$I_2 = I\left(\frac{R_3R_1}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{27}$$

$$I_3 = I\left(\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{28}$$

(Note the cyclic order of the subscripts.)

Example.—A current of 25 amp. passes through a parallel circuit of three resistances of 2.5, 4, and 6 ohms, respectively. How does the current divide?

Current in 2.5 ohms.

$$I_1 = 25\frac{4.0 \times 6.0}{(2.5 \times 4.0) + (4.0 \times 6.0) + (6.0 \times 2.5)}$$

$$= 25\frac{24}{10 + 24 + 15} = 12.25 \text{ amp.}$$

$$I_2 = 25\frac{6.0 \times 2.5}{49} = 7.65 \text{ amp.}$$

$$I_3 = 25\frac{2.5 \times 4.0}{49} = \underline{5.10} \text{ amp.}$$

Total 25.00 amp (check).

64. The Series-parallel Circuit.—A circuit may consist of groups of parallel resistances in series with other resistances or groups of resistances as shown in Fig. 65. When such is the case, each group of parallel resistances is first combined into its equivalent single resistance by Eq. (22) and the whole is then treated as a series circuit.

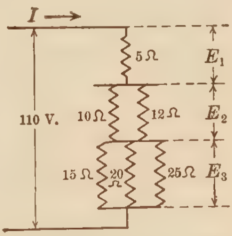


Fig. 65.—Series-parallel circuit.

Example.—Determine the total current in the circuit shown in Fig. 65; determine the voltage across each portion of the circuit; determine the current in each resistance.

Combine first the 10- and 12-ohm resistances into a resistance R_1

$$\frac{1}{R_1} = \frac{1}{10} + \frac{1}{12} = 0.10 + 0.0833 = 0.1833 \text{ mho}$$

$$R_1 = 5.45 \text{ ohms}$$

Likewise combining the group of three resistances into R_2

$$\frac{1}{R_2} = \frac{1}{15} + \frac{1}{20} + \frac{1}{25} = 0.0667 + 0.050 + 0.040$$

$$= 0.1567 \text{ mho.}$$

$$R_2 = \frac{1}{0.1567} = 6.38 \text{ ohms.}$$

$$I = \frac{110}{5 + 5.45 + 6.38} = \frac{110}{16.83} = 6.54 \text{ amp.}$$

$$E_1 = 6.54 \times 5.0 = 32.7 \text{ volts.}$$

$$E_2 = 6.54 \times 5.45 = 35.6 \text{ volts.}$$

$$E_3 = 6.54 \times 6.39 = \underline{41.7} \text{ volts.}$$

Total 110.0 volts (check).

$$\text{Current in } 10\ \Omega = \frac{35.6}{10} = 3.56 \text{ amp.}$$

$$\text{Current in } 12\ \Omega = \frac{35.6}{12} = \underline{2.97} \text{ amp.}$$

Total 6.53 amp. (check).

$$\text{Current in } 15\ \Omega = \frac{41.7}{15} = 2.78 \text{ amp.}$$

$$\text{Current in } 20\ \Omega = \frac{41.7}{20} = 2.09 \text{ amp.}$$

$$\text{Current in } 25\ \Omega = \frac{41.7}{25} = 1.67 \text{ amp.}$$

Total 6.54 (check).

65. Electrical Power.—The unit of electrical power is the *watt* which is defined fundamentally as 10^7 ergs, or one joule, per second. One joule of work is done when one coulomb is carried through a potential difference of one volt (see par. 55). If 1 coulomb per second is carried through a potential difference of 1 volt the *power* must be equal to 1 joule per second or 1 *watt.* One coulomb per second gives a current of 1 *amp.* Hence the watt may be defined as the power developed by 1 amp. in falling through a potential difference of 1 volt. Watts are therefore equal to the product of the volts and the amperes. Thus the power

$$P = EI \text{ watts.} \tag{29}$$

Since $E = IR$ in a circuit containing resistance only, (Eq. 18), Eq. (29) may be written

$$P = (IR)I = I^2R. \tag{30}$$

Substituting for I its value $\left(I = \frac{E}{R}\right)$ in Eq. (29)

$$P = \frac{E^2}{R} \tag{31}$$

Equation (29) is useful when the volts and the amperes are known; Eq. (30) is useful when the current and the resistance are known; and Eq. (31) is useful when the voltage and the resistance are known.

Example.—The resistance of a 150-scale voltmeter is 12,000 ohms. What power is consumed by this voltmeter when it is connected across a 125-volt circuit?

Since the voltage and the resistance are known, Eq. (31) is most convenient:

$$P = \frac{(125)^2}{12{,}000} = 1.3 \text{ watts.} \quad \textit{Ans.}$$

This may be checked by Eq. (29)

$$I = \frac{125}{12{,}000} = 0.0104 \text{ amp.}$$

$$P = 125 \times 0.0104 = 1.3 \text{ watts (check).}$$

Example.—A field rheostat is adjusted until the field current is 4.7 amp. The resistance of the rheostat is found to be 12.8 ohms. How much power is lost in the rheostat?

Since the current and the resistance are given, Eq. (30) is most convenient.

$$P = (4.7)^2 \times 12.8 = 283 \text{ watts.} \quad \textit{Ans.}$$

The watt is often too small a unit for commercial use and the *kilowatt* (equal to 1,000 watts) is used when large amounts of power are being considered. It is often necessary to transform from mechanical horsepower to electrical power and conversely, and a knowledge of the relation of the two is therefore useful:

$$746 \text{ watts} = 1 \text{ hp.} \tag{32}$$

$$0.746 \text{ kw.} = 1 \text{ hp.} \tag{33}$$

and

$$1 \text{ hp.} = \frac{3}{4} \text{ kw. very nearly} \tag{34}$$

$$1 \text{ kw.} = \frac{4}{3} \text{ hp. very nearly} \tag{35}$$

Example.—An electric motor takes 28 amp. at 550 volts and has an efficiency of 89 per cent. What horsepower does it deliver?

$$\text{Input} = 28 \times 550 = 15{,}400 \text{ watts.}$$

$$\text{Output} = 15{,}400 \times 0.89 = 13{,}700 \text{ watts.}$$

$$\frac{13{,}700}{746} = 18.35 \text{ hp. at the pulley.} \quad \textit{Ans.}$$

66. Electrical Energy.—Power is the *rate of doing work*, or is the *rate of expenditure of energy*. Therefore electrical energy is equal to the product of electrical power and time (see Par. 65).

The fundamental c.g.s. unit of work or energy is the *erg* or *dyne-centimeter* which is the work done when a force of 1 dyne is exerted through a distance of 1 centimeter in the direction of the force. This unit, however, is too small for practical purposes, hence the *watt-second* or *joule*, equal to 10,000,000 (10^7) ergs is the unit of energy in the practical electrical system.

It follows that electrical energy

$$W = EIt \text{ watt-seconds or joules,}$$

where t is in seconds, E is in volts, and I is in amperes.

The watt-second even is ordinarily too small a unit for commercial purposes, so the larger unit, the *kilowatt-hour* (kw.-hr.) is commonly used. 1 kilowatt-hour = $1{,}000 \times 60 \times 60 = 3{,}600{,}$-000 joules or watt-seconds.

The distinction between power and energy (or work) should be clearly kept in mind. Power is *rate* of doing work, just as velocity is rate of motion. On the other hand, energy is the total work done and is equal to the power multiplied by the time during which it acts just as distance covered is the velocity or rate of motion multiplied by the time. To speak of a train traveling at a rate of 40 mi. per hour gives no information as to the total distance which the train travels. Likewise, to speak of 50 kw. does not state the amount of energy that is involved. The statement "electricity is sold for so many cents per kilowatt" is incorrect. The correct expression is "electrical *energy* is sold for so many cents per kilowatt-*hour*." To illustrate:

Example.—If energy is sold for 10-cts. per kilowatt-hour (kw.-hr.,) how many kilowatts may be purchased for 20 cts.? This question as it stands cannot be answered, since the *time* is not given. If, however, it is assumed that the power is to be used for 1 hr.:

$$\frac{20 \text{ cents.}}{10 \text{ cents.}} = 2 \text{ kw.-hr. available.}$$

$$\frac{2 \text{ kw.-hr.}}{1 \text{ hr.}} = 2 \text{ kw. are available.} \qquad \textit{Ans.}$$

$$\text{If used in 0.5 hr., } \frac{2 \text{ kw.-hr.}}{0.5 \text{ hr.}} = 4 \text{ kw.} \qquad \textit{Ans.}$$

$$\text{If used in 0.001 hr., } \frac{2 \text{ kw.-hr.}}{0.001 \text{ hr.}} = 2{,}000 \text{ kw.} \qquad \textit{Ans.}$$

so that the 20-cts. could purchase *any number* of *kilowatts*, depending on the time during which the power is supplied.

In a similar way, horsepower is *rate of doing work* and is equivalent to 33,000 ft.-lb. *per minute* and *not* to 33,000 ft.-lb. A motor developing ⅛ hp. could do 33,000 ft.-lb. of work if allowed 8 min. in which to do it. When speaking of *work* in connection with horsepower, the *horsepower-hour* is the unit ordinarily used.

Example.—How many watt-seconds are supplied by a motor developing 2 hp. for 5 hr.?

$$2 \times 5 = 10 \text{ hp.-hr.}$$

$$10 \text{ hp.-hr.} \times 746 = 7{,}460 \text{ watt-hr.}$$

$$7{,}460 \times 3{,}600 = 2.68 \times 10^7 \text{ watt-sec.} \qquad \textit{Ans.}$$

67. Heat and Energy.—It is well known that heat may be converted into mechanical and electrical energy, and, conversely, that electrical and mechanical energy may be converted into heat. The complete cycle of energy transformation is well illustrated by a steam power plant. The energy is brought to the plant in the coal, as *chemical energy*. The ingredients of the coal combine

with the oxygen of the air, thus converting the chemical energy into *heat energy*. A certain percentage of this heat energy is transferred to the boiler and produces steam. The expansion of the steam in the engine cylinders, or through the buckets and blades of the turbine, converts the heat energy of the steam into *mechanical energy*. This mechanical energy drives the generator, which converts a large percentage of this energy into *electrical energy*. A portion of this electrical energy is transformed into heat in the wires, bus-bars, etc. Finally, the remainder is used to supply lamps, propel electric cars, operate motors, and some may be used for chemical processes. Ultimately all the energy appears again as heat or else is converted into chemical or other forms of energy.

The following table shows approximately what becomes of each 100 heat units existing initially in the coal in the most efficient modern power plants, using superheaters, condensers, and large units.

EFFICIENCY OF ENERGY CONVERSION

	Form of energy	Efficiency (per cent.)	Heat units converted
Coal	Chemical	..	100.0
Boiler	Heat	80	80.0
Turbine	Mechanical	25	20.0
Generator	Electrical	95	19.0
Distribution system (to point of utilization)	Electrical	85	16.2
Small motors	Mechanical	65 (av.)	10.5
Lamps	Light	2	0.32

Figure 66[1] (by R. A. Philip) shows graphically the flow of power from the boiler to the point of utilization. It is apparent that even in the most modern power plants the over-all efficiency is very low.

68. Thermal Units.—The unit of heat in the English system is the B.t.u. (British thermal unit) and is equal to the amount of heat required to raise 1 pound of water 1° F. It is equal to 778 ft.-lb. (called the Mechanical Equivalent of Heat).

[1] A. I. E. E. *Trans.*, Vol. XXXIV, p. 779 (1915).

In the c.g.s. system the heat unit is the gram-calorie and is equal to the amount of heat required to rise 1 g. of water 1° C.[1] A gram-calorie is equal to 4.2 watt-sec. or joules.

By Joule's law the heat developed in a circuit is:

$$W = \frac{1}{4.2} I^2Rt = 0.24\ I^2Rt \text{ calories} \tag{36}$$

where t is in seconds, I in amperes and R in ohms.

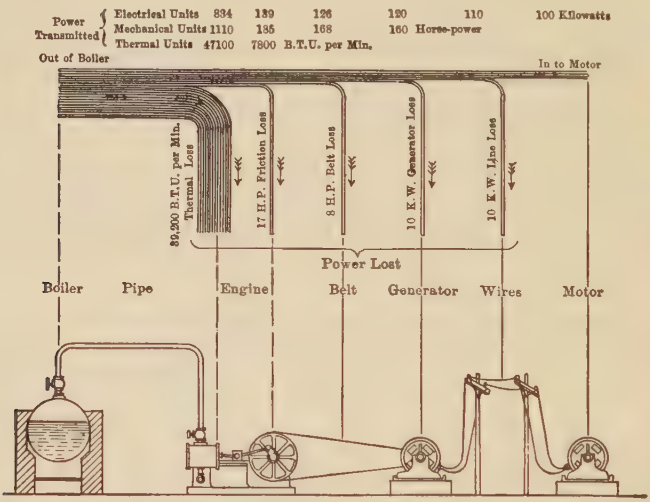


FIG. 66.—Energy flow—thermal, mechanical and electrical transmission.

Example.—Ten horsepower is delivered by a pump circulating 400 gal. of water per minute through a certain cooling system. How many degrees F. is the temperature of the water raised by the action of the pump?

$$10 \text{ hp.} = 10 \times 33{,}000 = 330{,}000 \text{ ft.-lb. per min.}$$

$$\frac{330{,}000}{778} = 424 \text{ B.t.u. per min.}$$

$$400 \text{ gal.} = 400 \times 8.34 = 3{,}336 \text{ lb.}$$

$$\frac{424}{3{,}336} = 0.13° \text{ F.} \quad \textit{Ans.}$$

Example.—An incandescent lamp taking 0.5 amp. from 110-volt mains is immersed in a small tank, containing 2,000 c.c. of water. Neglecting radiation, by how many degrees per minute is the temperature of the water raised?

$$W = 0.24 \times 0.5 \times 110 \times 60 = 792 \text{ calories per minute.}$$

$$\frac{792}{2{,}000} = 0.396° \text{ C.} \quad \textit{Ans.}$$

[1] See Appendix A, page 473.

69. Potential Drop in a Feeder Supplying One Concentrated Load.—Figure 67 shows a feeder (consisting of a positive and a negative wire) supplying a motor load. The feeder is connected to bus-bars having a constant potential difference of 230 volts. The feeder is 1,000 ft. long and consists of two 250,000-C.M. conductors. The maximum load on the feeder is 250 amp. It is required to determine the voltage at the motor terminals and the efficiency of transmission.

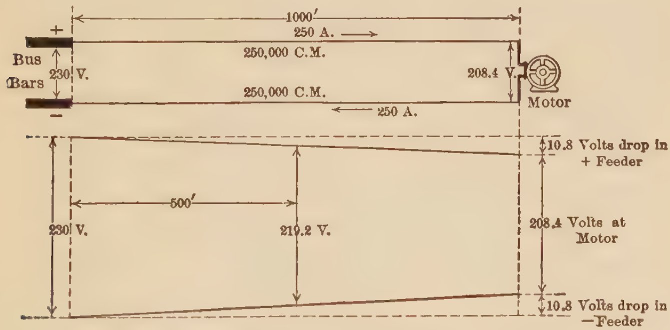


FIG. 67.—Voltage-drop in a feeder due to a single load.

As was stated in Par. 57, the voltage at the motor must be less than that at the bus-bars because of the voltage lost in supplying the resistance-drop in the feeder.

From Table 52, the resistance of 1,000 ft. of 250,000-C.M. cable is 0.0431 ohm. As was shown in Par. 57, the net voltage at the receiving end of the line is less than the voltage at the sending end by the voltage loss in both the *outgoing* and the *return* wire. Therefore the drop in 2,000 ft. of cable must be taken, the total resistance being 0.0862 ohm.

The current is 250 amp.

By Eq. (18) the voltage-drop in the line:

$$E' = 250 \times 0.0862 = 21.55 \text{ volts.}$$

Therefore, the voltage at the motor terminals is

$$230 - 21.6 = 208.4 \text{ volts.} \quad \textit{Ans.}$$

In Fig. 67 the voltage along the line is shown graphically. The voltage at the sending end of the line in 230 volts, and there is a uniform drop in each wire, this drop increasing uniformly to 10.8 volts, making a total voltage loss of 21.6 volts. The

potential difference between the two wires 500 ft. from the sending end will be $230 - 10.8 = 219.2$ volts as shown.

The power delivered to the motor $= 208.4 \times 250$ watts.

The power delivered to the line $= 230 \times 250$ watts.

$$\text{The efficiency of the line} = \frac{\text{output}}{\text{input}} = \frac{208.4 \times 250}{230 \times 250}$$

$$= \frac{208.4}{230} \text{ or } 90.6 \text{ per cent.}$$

With one concentrated load the efficiency of transmission is given by the voltage at the load divided by the voltage at the sending end of the line.

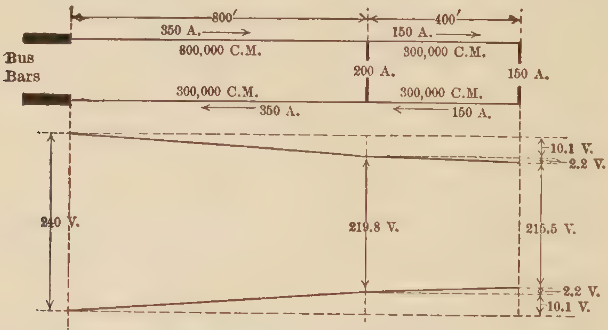


FIG. 68.—Voltage-drops in a feeder supplying two loads.

70. Potential-drop in a Feeder Supplying Two Concentrated Loads at Different Points.—In Fig. 68 a 300,000-C.M. feeder supplies 200 amperes to a load 800 ft. from the bus-bars, and 150 amp. to a load 400 ft. farther on. If the bus-bar voltage is maintained constant at 240 volts, determine the voltage at each load, the total line loss and the efficiency of transmission.

From Table 52, the resistance of 1,000 ft. of 300,000-C.M. cable is 0.0360 ohm. The resistance of 800 ft. $= 800/1{,}000 \times 0.0360 = 0.0288$ ohm.

Voltage drop to the 200-amp. load

$$E' = 350(2 \times 0.0288) = 20.16 \text{ volts.}$$

Voltage at 200-amp. load

$$E_1 = 240 - 20.2 = 219.8 \text{ volts.} \quad \textit{Ans.}$$

Resistance of one cable from the 200-amp. load to the 150-amp. load $= 400/1{,}000 \times 0.0360 = 0.0144$ ohm.

Voltage drop from 200-amp. load to 150-amp. load

$$E'' = 150(2 \times 0.0144) = 4.32 \text{ volts.}$$

Voltage at 150-amp. load

$$E_2 = 219.8 - 4.3 = 215.5 \text{ volts.} \quad \textit{Ans.}$$

The voltage distribution along this line is shown graphically in Fig. 68.

To determine the efficiency:

Line loss to 200-amp. load

$$P_1 = (350)^2(2 \times 0.0288) = 7{,}060 \text{ watts (Eq. 30 p. 66).}$$

Line loss from 200-amp. load to 150-amp. load

$$P_2 = (150)^2(2 \times 0.0144) = 649 \text{ watts (Eq. 30).}$$

Total line loss

$$P_1 + P_2 = 7{,}060 + 649 = 7{,}709 \text{ watts or } 7.709 \text{ kw.}$$

Efficiency =

$$\frac{\text{input} - \text{losses}}{\text{input}} = \frac{(240 \times 350) - 7{,}709}{240 \times 350} = \frac{76{,}290}{84{,}000} = 90.8 \text{ per cent.}$$

71. Estimation of Feeders.—It was stated in Par. 42 that a cir.-mil-foot of copper has a resistance of 10.37 ohms. In many cases it is sufficiently exact to assume a value of 10 ohms. Assume the current density in a feeder to be 1,000 cir. mils per ampere, or 0.001 amp. per cir. mil. Call this the *normal* current density. (Bus-bars and large feeders operate at a density very nearly equal to this.)

The voltage-drop through a cir.-mil-foot carrying 0.001 amp. is:

$$E = IR = 0.001 \times 10 = 0.01 \text{ volt.}$$

Another cir.-mil-foot, carrying 0.001 amp., will also have a drop of 0.01 volt between its ends. If these be placed side by side, the drop across the two will still be 0.01 volt. With any number of wires, each having one circular mil cross-section, a length of 1 ft. and a current of 0.001 amp., the drop across the ends of each wire will be 0.01 volt. The wires may be separated or they may be made into a cable.

In Fig. 69 (a) are shown four separate conductors, each of 1 cir.-mil-foot and each carrying 0.001 amp. The voltage across each must be 0.01 volt. In Fig. 69 (b) these same four conductors are grouped together and as each carries 0.001 amp., the total current must be 0.004 amp. The voltage-drop across the group is still 0.01 volt. If any number of cir.-mil-ft. conductors

each carrying 0.001 amp. are added in parallel to the group of Fig. 69 (*b*), the drop remains 0.01 volt.

From the foregoing the following rule may be deduced:

The voltage-drop per foot of copper conductor is always 0.01 *volt provided that the current density is* 0.001 *amp. per circular mil. Further, if the density is other than* 0.001 *amp. per circular mil, the voltage-drop will be in direct proportion to the current density.* This last follows from Eq. (18), page 60.

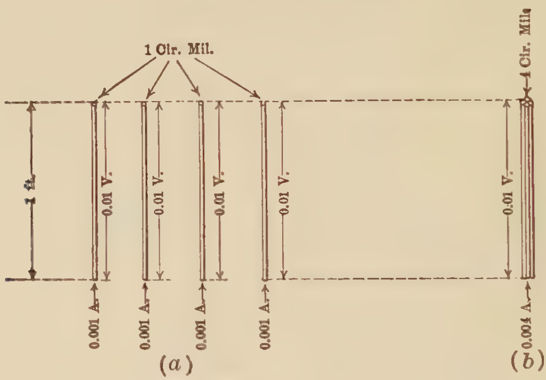


FIG. 69.—Voltage-drop in a cir.-mil-foot.

Example.—A motor 800 ft. from the power house is to take 500 amp. from 230-volt bus-bars. What size cable is necessary in order that the voltage drop shall not exceed 20 volts?

A cable to operate at the normal density must have

$$500 \times 1{,}000 = 500{,}000 \text{ C.M.}$$

The total voltage drop then becomes

$$0.01 \times 800 \times 2 = 16 \text{ volts.}$$

The allowable drop is 20 volts, so a smaller cable may be used.

$$500{,}000 \times \frac{16}{20} = 400{,}000 \text{ C.M.} \quad \textit{Ans.}$$

This makes the actual current density

$$\frac{500}{400} = 1.25 \text{ amp. per } 1{,}000 \text{ cir. mils.}$$

72. Power Loss in a Feeder.—The method of Par. 71 may be used to estimate the power loss in a copper conductor. At the normal density

$$P' = I^2R = (0.001)^2 10 = 0.00001 \text{ (or } 10^{-5}\text{) watt per cir.-mil-ft.}$$

The total power loss at the *normal density*

$$P_0 = 0.00001 \times \text{C.M.} \times l$$

where C.M. is the conductor cross-section in circular mils and l its length in feet.

The *actual* power loss is proportional to the *square* of the ratio of the actual to the normal density.

That is,

$$P = P_0D^2$$

where P is the actual power loss, P_0 the power loss at the normal density and D is the actual current density in amperes per 1,000 cir. mils.

Example.—Determine the power loss in the example of Par. 71.

$P_0 = 0.00001 \times 400{,}000 \times 800 \times 2 = 6{,}400$ watts at the normal density.

The actual power loss

$$P = 6{,}400 \times (1.25)^2 = 10{,}000 \text{ watts} = 10 \text{ kw.} \quad \textit{Ans.}$$

Also if the voltage-drop and the current are known, the power loss is readily determined. For example, in the foregoing problem the voltage-drop is 20 volts and the current is 500 amp. Hence the power loss

$$P = 20 \times 500 = 10{,}000 \text{ watts} = 10 \text{ kw. (check).}$$

The foregoing give an easy and rapid method of solution of many problems. It is sufficiently exact for most practical purposes.

CHAPTER V

BATTERY ELECTROMOTIVE FORCES—KIRCHHOFF'S LAWS

73. Battery Electromotive Force and Resistance.—If a voltmeter be connected across the terminals of a battery (Fig. 70), the switch S being open, the instrument will indicate a certain voltage E. If now the switch S be closed, allowing the current I to flow, the instrument will indicate a voltage V which is less than E.

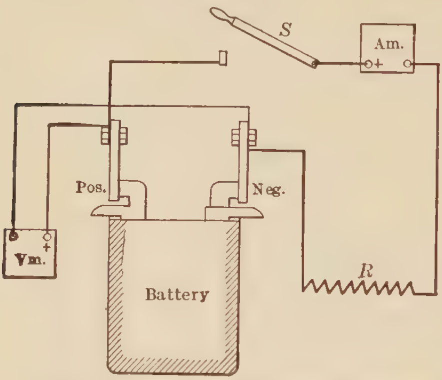


Fig. 70.—Connections for measuring battery resistance.

The voltage E, measured when the battery delivers no current, is the *internal voltage* or the *electromotive force* of the battery; the voltage V, measured when a current I flows, is known as the *terminal voltage* of the battery for that particular current value.

The difference between the open-circuit voltage E and the voltage V, measured when current is being taken from the battery, is the *voltage drop* in the battery due to the passage of current through the battery resistance. Every cell has a certain resistance, lying for the most part in the electrolyte, but partly in the battery plates and terminals. When the external circuit is closed so that current can flow, a certain voltage is required to send this current through the battery resistance, just as voltage is required to send current through an external resistance.

If the voltage E, measured at the battery terminals when the circuit is open, drops to V when the circuit is closed, the voltage $e = (E - V)$ is the voltage drop through the cell due to the

passage of the current I. Let the cell resistance be r. Then, by Ohm's law,

$$E - V = e = Ir \quad \text{(by Eq. 18)}$$

or

$$r = \frac{e}{I} = \frac{E - V}{I} \text{ (by Eq. 19)} \tag{37}$$

$$E = V + Ir \tag{38}$$

That is, the internal resistance of the battery is equal to the difference of the open-circuit and closed-circuit terminal voltages divided by the current.

Also the electromotive force of the battery is equal to the closed-circuit terminal voltage *plus* the resistance-drop in the battery.

Example.—The open-circuit voltage of a storage cell is 2.20 volts. The terminal voltage measured when a current of 12 amp. flows is found to be 1.98 volts. What is the internal resistance of the cell?

The voltage drop through the cell

$$E - V = 2.20 - 1.98 = 0.22 \text{ volt.}$$

Then $$r = \frac{0.22}{12} = 0.0183 \text{ ohm.} \quad \textit{Ans.}$$

In making a measurement of this character, it must be remembered that under open-circuit conditions even the ordinary voltmeter takes some current. If the cell capacity is small (as in the case of a Weston cell) the voltmeter current alone may reduce the terminal voltage to a value one-half, or even less, of the open-circuit voltage. Under these conditions the ordinary voltmeter cannot be used to measure the electromotive force of the cell.

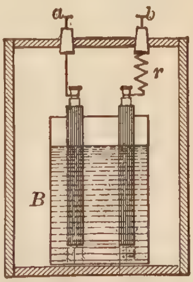


FIG. 71.—The internal resistance of a cell.

Moreover, it is impossible to measure directly the internal voltage of the battery when the battery delivers current, for the voltage drop occurs *within* the cell itself. Figure 71 represents these conditions so far as their effect on the external circuit is concerned. A battery cell B is enclosed in a sealed box. Its resistance r is considered as removed from the cell itself and connected external to the cell, but within the sealed box. The cell

then may be considered as having no resistance, its resistance having been replaced by r. The connections are brought through bushings in the box to terminals a and b. When no current is being delivered by the cell, if a voltmeter be connected across the two terminals a and b, the instrument will measure the e.m.f., E. If, however, a current I flows, the terminal voltage will drop from E to V, due to the voltage drop in the resistance r. Under these conditions it is impossible to measure E when the current is flowing, since the voltmeter can only be connected outside the resistance, through which the voltage drop occurs.

The voltage E and the resistance r are seldom constants but are more or less dependent upon the current. They are also affected by temperature, change in specific gravity of the electrolyte, polarization, etc.

74. Battery Resistance and Current.—As was shown in Par. 73, the resistance within the battery tends to reduce the flow of current. If, in Fig. 70, the switch be closed, the cell electromotive force E will be acting upon a circuit consisting of the internal resistance of the cell r and the resistance of the external circuit R. The resistances r and R being in series, the total resistance in the circuit is their sum. The current is therefore

$$I = \frac{E}{r + R} \tag{39}$$

The power lost in the battery is

$$P = I^2 r$$

If the cell is short-circuited, R becomes zero and $I = \frac{E}{r}$. Under these conditions all the electrical energy developed by the cell is converted into heat within the cell itself.

Example.—A battery-cell having an electromotive force of 2.2 volts and an internal resistance of 0.03 ohm is connected to an external resistance of 0.10 ohm. What current flows and what is the efficiency of the battery as used?

$$I = \frac{2.2}{0.03 + 0.10} = \frac{2.2}{0.13} = 16.9 \text{ amp.} \quad \textit{Ans.}$$

Power lost in the battery

$$P = (16.9)^2 \times 0.03 = 8.57 \text{ watts.}$$

The useful power

$$P' = (16.9)^2 \times 0.10 = 28.6 \text{ watts.}$$

P' is equal to the total power developed by the battery minus the battery loss.

$$2.2 \times 16.9 = 37.2 \text{ watts.}$$
$$P' = 37.2 - 8.6 = 28.6.$$
$$\text{Eff.} = \frac{28.6}{28.6 + 8.6} \text{ or } 76.9 \text{ per cent.} \quad \textit{Ans.}$$

From the foregoing, the following rule may be deduced: *The current in a circuit is equal to the total electromotive force acting in the circuit divided by the total resistance of the circuit* (see p. 59).

75. Maximum Power Delivered by a Battery.—Assume that the electromotive force E and the internal resistance r of the battery (Fig. 70) are constant. If the battery delivers a current i, the power p delivered to the external circuit is vi where v is the terminal voltage of the battery. Since the terminal voltage $v = E - ir$, v will *decrease* as the current i increases, as was demonstrated in Par. 74. Hence, the power p delivered to the external circuit is the product of two variables, one of which (v) decreases as the other (i) increases. The external power will be a maximum when the product vi is a maximum. It can be shown that this maximum occurs when $v = E/2$ (also, see p. 459, Par. 284). Under these conditions the voltage-drop ir within the battery is equal to the terminal voltage v of the battery. If the power is delivered to an external resistance R,

$$ir = v = iR.$$

Or

$$r = R.$$

That is, the resistance of the external circuit is equal to the internal resistance of the battery.

The *total* power which the battery is developing equals the product of its internal e.m.f. E and the current i. That is, this total power,

$$P' = Ei.$$

The power lost within the battery is i^2r and the power given to the external circuit is i^2R. Since $r = R$, it follows that $i^2r = i^2R$.

That is, *when a battery is delivering its maximum power, half the total power is lost within the battery itself.* The efficiency of the battery under these conditions is *50 per cent.*

As has already been shown, the maximum current occurs when the external resistance R is zero. Under these conditions, the

terminal voltage v is zero and the power delivered to the external circuit is zero. The total power, Ei, developed by the battery is lost in heating the battery itself.

Usually, it is not economical to operate a battery so that it delivers maximum power, because of the low efficiency and the fact that the battery heats unduly. Occasionally, however, batteries are so operated for short periods. For example, automobile-starting batteries, during the cranking period, frequently operate at such a current that their terminal voltage is one-half their electromotive force, and therefore the battery delivers maximum power.

Example.—A battery cell has an electromotive force of 1.2 volts and an internal resistance of 0.8 ohm. (*a*) For what value of current will the power delivered by the cell be a maximum? (*b*) What is the power under these conditions? (*c*) What will be the value of the external resistance under these conditions? (*d*) Determine the current and the power when the external resistance is 1.0 ohm. (*e*) When it is 0.6 ohm. (*f*) What is the maximum current which this cell can deliver? (*g*) What is the external power under these conditions?

(*a*) The power is a maximum when the terminal voltage $v = \frac{E}{2} =$ 0.60 volt. The current, $I = \frac{1.2 - 0.60}{0.8} = \frac{0.6}{0.8} = 0.75$ amp. *Ans.*

(*b*) $p = 0.60 \times 0.75 = 0.45$ watt. *Ans.*

(*c*) $R = \frac{v}{i} = \frac{0.60}{0.75} = 0.8$ ohm. *Ans.*

(*d*) $i = \frac{1.2}{0.8 + 1.0} = \frac{1.2}{1.8} = 0.667$ amp. *Ans.*

$$v = 1.2 - 0.667 \times 0.8 = 1.2 - 0.533 = 0.667 \text{ volt}$$
$$p = vi = 0.667 \times 0.667 = 0.444 \text{ watt.} \quad \textit{Ans.}$$

(*e*) $i = \frac{1.2}{0.8 + 0.6} = \frac{1.2}{1.4} = 0.857$ amp. *Ans.*

$$v = 1.2 - 0.857 \times 0.8 = 1.2 - 0.686 = 0.514 \text{ volt.}$$
$$p = vi = 0.514 \times 0.857 = 0.440 \text{ watt.} \quad \textit{Ans.}$$

(*f*) $i = \frac{1.2}{0.8} = 1.5$ amp. *Ans.*

(*g*) Since $v = 0$, $p = 0 \times 1.5 = 0$. *Ans.*

It will be noted that the product of vi is a maximum when $v = E/2$. In (*d*), v is greater than in (*a*), but i is so much less that the product vi is less. In (*e*) i is greater than in (*a*) but v is so much less that the product vi is less. (*d*) and (*e*) may be solved more readily by using $p = i^2R$, but this equation does not so clearly illustrate the fact of the product being a maximum.

76. Batteries Receiving Energy.—If a resistance load be connected across a battery, current will immediately flow from the positive terminal of the battery and will return to the battery though its negative terminal. As has already been pointed out, the battery terminal voltage will be less than its open-circuit value, due to the current flowing through the internal resistance of the battery. Under these conditions, the battery is a source of energy and is acting as a generator, that is, it *delivers* energy.

If current is forced to *enter* at the positive terminal of the battery, the battery will no longer be supplying energy but will be receiving energy. This energy must be supplied from some other source, as from another battery, or, as is more common, from a generator. The cell shown in Fig. 72 has an electromotive force of 2 volts, and a voltmeter V, connected across its terminals,

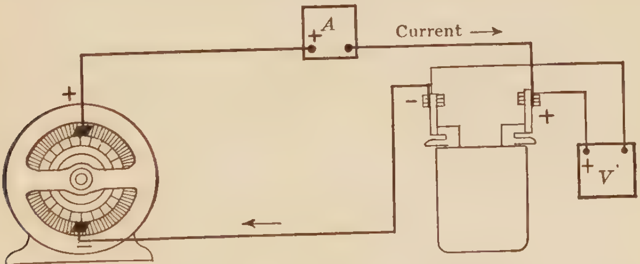


Fig. 72.—Generator charging a battery.

indicates 2 volts when no current flows. If another source of electrical energy, such as a direct-current generator, supply a potential difference of just 2 volts and its + terminal be connected to the + terminal of the battery and its − terminal connected to the − terminal of the battery, as shown in the figure, the voltmeter V will still read 2 volts and the ammeter A will read zero. That is, the battery neither delivers nor receives energy and no effect is noted other than those observed when the battery stood open-circuited. Under these conditions the battery is said to be "floating." If, however, the voltage of the generator be raised slightly, the ammeter A will indicate a current flowing from the + terminal of the generator *into* the + terminal of the battery, a direction just opposite to that which the current has when the battery *supplies* energy. The voltmeter will no

longer read 2 volts, but will indicate a potential difference somewhat in excess of 2 volts.

What actually happens may be illustrated by a mechanical analogy. Figure 73 shows a car standing on the track. A force of 400 lb. is necessary to overcome the standing friction of the car on the track. At one end of the car a force F is applied. Before the force F can move the car its value must at least equal 400 lb. When F is exactly 400 lb. the car will not move, just as no current flows into the battery when the generator voltage is just equal to that of the battery. When the force F exceeds 400 lb., however, the car will move, the force effective in producing this motion being the amount by which F exceeds 400 lb. Thus, if F = 450 lb., 400 lb. of this is utilized in overcoming the 400 lb. opposing force due to friction and 50 lb. is effective in moving the car.

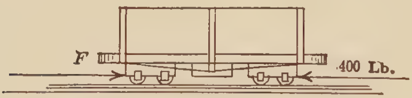


FIG. 73.—Force necessary to start a car.

In the case of the battery no current will flow until voltage in excess of the 2 volts is produced by the generator. Thus, if the generator voltage be raised to 2.4 volts, 2.0 volts of this is utilized to "buck" the 2.0 volts of the cell and 0.4 volt is effective in sending current into the cell. Thus, if the cell resistance be 0.1 ohm, the current will be

$$I = \frac{0.4}{0.1} = 4.0 \text{ amp.}$$

This assumes that the resistance of the leads is negligible.

Therefore, if E is the electromotive force of a battery, r its resistance and V the terminal voltage when current flows *in* at its positive terminal,

$$I = \frac{V - E}{r}, \tag{40}$$

and

$$E = V - Ir. \tag{41}$$

That is, the electromotive force of the cell is less than the terminal voltage by the amount of the resistance drop in the cell itself. These equations should be compared with Eqs. (37) and (38), respectively.

Under the foregoing conditions, the cell is *receiving* electric energy, as is the case when a storage battery is being charged.

77. Battery Cells in Series.—Strictly speaking, a battery consists of more than one unit or cell. However, the term battery has come also to mean a single cell, when this cell is not acting in conjunction with others.

When cells are connected in series, their electromotive forces are added together to obtain the total electromotive force of the battery, and their resistances are added together to obtain the total resistance of the battery.

Thus, if several cells, having electromotive forces, E_1, E_2, E_3, E_4, etc., and resistances r_1, r_2, r_3, r_4, etc., are connected in series, the total electromotive force of the combination is

$$E = E_1 + E_2 + E_3 + E_4, \text{ etc.} \tag{42}$$

and the total resistance is

$$r = r_1 + r_2 + r_3 + r_4, \text{ etc.} \tag{43}$$

Equation (42) assumes that the cells are all connected + to − so that their electromotive forces are additive. If any cell be connected so that its electromotive force opposes the others, its voltage in Eq. (42) must be preceded by a minus sign.

If an external resistance R is connected across these cells in series, then by Eq. (39) the current is

$$I = \frac{E}{r + R} = \frac{E_1 + E_2 + E_3 + E_4, \text{ etc.}}{r_1 + r_2 + r_3 + r_4, \text{ etc.}, + R} \tag{44}$$

Example.—Four dry cells having electromotive forces of 1.30, 1.30, 1.35, and 1.40 volts and resistances of 0.3, 0.4, 0.2, and 0.1 ohm, respectively, are connected in series to operate a relay having a resistance of 10 ohms. What current flows in the relay?

$$I = \frac{1.30 + 1.30 + 1.35 + 1.40}{0.3 + 0.4 + 0.2 + 0.1 + 10} = \frac{5.35}{11.0} = 0.486 \text{ amp.} \quad \textit{Ans.}$$

A battery consisting of n equal cells in series has an e.m.f. n times that of one cell, but has the current capacity of one cell only.

78. Equal Batteries in Parallel.—To operate satisfactorily in parallel all the batteries should have the same electromotive force. The behavior of batteries having unequal electromotive forces can be treated as special problems (see Par. 81).

Figure 74 shows a battery of six cells, each having an electromotive force of 2.0 volts and a resistance of 0.2 ohm. It is clear that the e.m.f. of the entire battery is no greater than the e.m.f. of any one cell. The current, however, has 6 paths through which

to flow. Therefore, for a fixed external current, the voltage drop in each cell is one-sixth that occurring if all the current passed through one cell. If the internal resistance of one cell is 0.2 ohm, the resistance of the battery as a whole must be $0.2/6 = 0.033$ ohm.

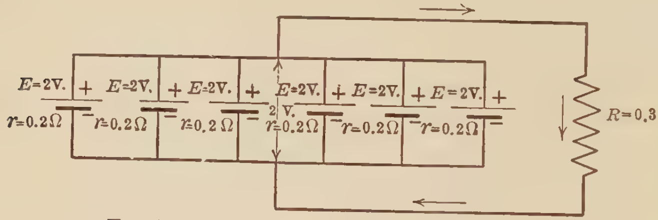


FIG. 74.—Parallel arrangement of equal cells.

Example.—If the external resistance connected across the terminals of the battery in Fig. 74 is 0.3 ohm, what current flows?

Resistance of battery $= 0.2/6 = 0.033$ ohm.

$$I = \frac{2.0}{0.033 + 0.3} = \frac{2.0}{0.333} = 6 \text{ amp.} \quad \text{(Eq. 39)}. \quad \textit{Ans.}$$

If the e.m.fs. are equal but the resistances of the cells are not all equal, but are r_1, r_2, r_3, r_4, etc., the battery resistance r is found by considering these resistances as being in parallel (Eq. (9), Chap. III, p. 39).

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \text{etc.} \tag{45}$$

Example.—A battery consists of 4 cells connected in parallel, each having an electromotive force of 2.0 volts, but resistances of 0.30, 0.25, 0.22, and 0.20 ohm, respectively. If a resistance of 0.5 ohm is connected across the terminals of the battery, what current flows, and how much current does each cell supply? What is the voltage across the battery terminals?

$$\frac{1}{r} = \frac{1}{0.30} + \frac{1}{0.25} + \frac{1}{0.22} + \frac{1}{0.20} = 16.87 \text{ mhos.}$$

$$r = \frac{1}{16.87} = 0.0593 \text{ ohm.}$$

$$I = \frac{2.0}{0.0593 + 0.50} = \frac{2.0}{0.5593} = 3.58 \text{ amp.} \quad \textit{Ans.}$$

The terminal voltage

$$E_1 = IR = 3.58 \times 0.5 = 1.79 \text{ volts.} \quad \textit{Ans.}$$

The current in each cell may be found by means of Eq. (37), page 77.

$$\frac{2.0 - 1.79}{I_1} = 0.30.$$

Solving $$I_1 = \frac{2.0 - 1.79}{0.30} = \frac{0.21}{0.30} = 0.70 \text{ amp.}$$

Likewise $$I_2 = \frac{2.0 - 1.79}{0.25} = \frac{0.21}{0.25} = 0.84 \text{ amp.}$$

$$I_3 = \frac{2.0 - 1.79}{0.22} = \frac{0.21}{0.22} = 0.95 \text{ amp.}$$

$$I_4 = \frac{2.0 - 1.79}{0.20} = \frac{0.21}{0.20} = 1.05 \text{ amp.}$$

Total current 3.54 (check). *Ans.*

That is, *the current in any cell is equal to the voltage drop in the cell divided by the resistance of the cell.*

It will be found that the products of the current and the resistance of each cell are all equal.

$$0.7 \times 0.3 = 0.84 \times 0.25 = 0.95 \times 0.22 = 1.05 \times 0.20$$

Cells connected in parallel *must all have the same terminal voltage* since all the positive terminals are connected together and all the negative terminals are connected together. If the e.m.fs. of the cells are all equal, the total battery e.m.f. is equal to the e.m.f. of but one cell. The total battery resistance may be found by the equation for resistances in parallel. The current in each cell is inversely proportional to the resistance of the cell *if the electromotive forces are all equal.* The current capacity of the battery is the sum of the current capacities of the individual cells.

79. Series-parallel Grouping of Cells.—Rows of series-connected cells may be so connected that the rows themselves are grouped in parallel. Figure 75 shows a row of 4 cells in series,

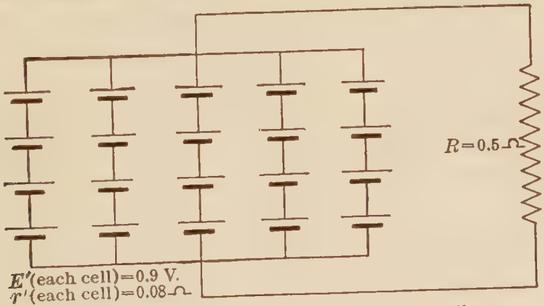


FIG. 75.—Series-parallel grouping of cells.

and five of these rows in parallel. If there are m equal cells in series in each row, then the e.m.f. of each row must be

$$E = mE' \qquad \text{(by Eq. 42)}$$

where E' is the e.m.f. of one cell.

The resistance of each row must be

$$r_1 = mr' \text{ (by Eq. 43)}$$

where r' is the resistance of one cell.

Since there are n rows in parallel, the resistance of the whole combination must be

$$r = \frac{r_1}{n} = \frac{m}{n} r' \tag{46}$$

If an external resistance R is connected to the battery, the current is

$$I = \frac{mE'}{\frac{m}{n}r' + R} \tag{47}$$

Example.—Let each of the cells of Fig. 75 have an e.m.f. of 0.9 volt and an internal resistance of 0.08 ohm. If the external resistance R is 0.5 ohm, what current flows?

$$I = \frac{4 \times 0.9}{\frac{4}{5} 0.08 + 0.5} = \frac{3.6}{0.564} = 6.4 \text{ amp.} \quad \textit{Ans.}$$

80. Grouping of Cells.—(*a*) *To obtain the best economy,* group the cells so that the battery resistance is as low as possible. This usually means a large number of parallel connections. Under these conditions the life of the battery will be prolonged but the initial cost is excessive.

(*b*) *To obtain the maximum current with fixed external resistance* make the internal resistance $\left(\frac{m}{n}r'\right)$ of the battery equal to the external resistance. This is not economical, since only one-half of the energy developed by the battery is available in the external circuit; the other half is lost in the cells themselves. Under these conditions the battery delivers the maximum power (see Par. 75).

(*c*) *To obtain quick action* for the intermittent operation of relays, bells, etc., group the cells in series if possible.

Example.—In the example of Par. 79, how should the cells be arranged to obtain the maximum current?

The total battery resistance $\frac{m}{n} 0.08$ must be equal to the external resistance.

That is, $$\frac{m}{n} 0.08 = 0.5.$$

Also $$m \times n = 20 \quad n = \frac{20}{m}.$$

Solving
$$\frac{m}{\left(\frac{20}{m}\right)}0.08 = 0.5.$$
$$m^2 = \frac{20}{0.08}0.5 = 125.$$
$$m = 11+. \quad \textit{Ans.}$$

The best arrangement is ten cells in series, and two rows in parallel (Eleven cells in series would not operate satisfactorily if connected in parallel with the remaining nine cells in series.)

81. Division of Current among Unequal Batteries in Parallel.—A simple method of determing the division of current among batteries in parallel, having unequal e.m.fs. and unequal internal resistances, is illustrated by the following example:

Example.—Two batteries A and B (Fig. 76), having e.m.fs. of 12 and 10 volts and internal resistances of 0.5 and 0.3 ohm are connected in parallel. An external resistance R connected across the batteries takes 8 amp. Determine: (a) the current delivered by each battery; (b) the voltage E_{ab} across their terminals; (c) the value of the external resistance; (d) the total power developed by each battery; (e) the power delivered by each battery.

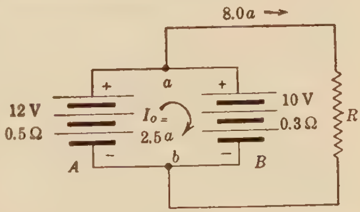


Fig. 76.—Unequal batteries in parallel.

(a) Assume that the external load is entirely disconnected. The net voltage acting around the local circuit of the two batteries is 12 − 10 or 2 volts, and the total resistance is 0.5 + 0.3 = 0.8 ohm. Hence a circulatory current, $I_0 = \frac{12 - 10}{0.5 + 0.3} = 2.5$ amp. flows in a clockwise direction, as shown. That is, battery A delivers +2.5 amp. and battery B delivers −2.5 amp. This current brings the batteries to equality of terminal voltage. That is, $12 - 2.5 \times 0.5 = 10 + 2.5 \times 0.3 = 10.75$ volts. After this equality of terminal voltage is obtained, any additional current drawn from the system divides between the two batteries *inversely* as their resistances.

Now apply the 8-amp. load. This current must be divided between the batteries inversely as their resistances (see Par. 78). That is, of this 8 amp., battery A delivers $\frac{0.3}{0.8} \times 8 = 3.0$ amp. and battery B delivers $\frac{0.5}{0.8} \times 8 = 5.0$ amp.

But battery A already delivers +2.5 amp., so that its *total* current now is $3.0 + 2.5 = +5.5$ amp. Battery B already delivers −2.5 amp., so that its *total* current is now $5.0 - 2.5 = +2.5$ amp. *Ans.*

(b) $E_{ab} = 12 - (5.5 \times 0.5) = 10 - (2.5 \times 0.3) = 9.25$ volts. *Ans.* (check)

(c) $R = \frac{9.25}{8} = 1.156$ ohms. *Ans.*

(d) $P_A' = 12 \times 5.5 = 66$ watts.

$P_B' = 10 \times 2.5 = 25$ watts. *Ans.*

(e) $P_A = 9.25 \times 5.5 = 50.88$ watts.

$P_B = 9.25 \times 2.5 = 23.13$ watts. *Ans.*

In (a) after the circulatory current has brought about equality of terminal voltage for the cells, the two cells behave as a single cell, having an e.m.f. of 10.75 volts and an internal resistance equal to the equivalent parallel resistance of the two cell resistances in parallel or $\frac{0.3 \times 0.5}{0.3 + 0.5} = 0.1875$ ohm. Hence, the external current is $\frac{10.75}{0.1875 + 1.156} = 8.0$ amp. (check).

82. Kirchhoff's Laws.—By means of Kirchhoff's Laws it is possible to solve many circuit networks that would otherwise be difficult of solution.

1. *In any branching network of wires, the algebraic sum of the currents in all the wires that meet at a point is zero.*

2. *The sum of all the electromotive forces acting around a complete circuit is equal to the sum of the resistances of its separate parts multiplied each into the strength of the current that flows through it,* or *the total change of potential around any closed circuit is zero.*

The first law is obvious. It states that the total current *leaving* a junction is equal to the total current *entering* the junction. If this were not so electricity would accumulate at the junction.

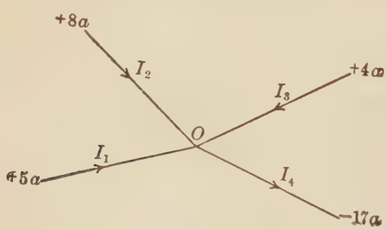


Fig. 77.—Illustrating Kirchhoff's first law.

The law is illustrated by Fig. 77. Four currents, I_1, I_2, I_3, and I_4 meet at the junction O. The first three currents flow *toward* the junction so have plus signs as they *add* to the quantity at the point O. The last current I_4 flows away from the junction, so has a minus sign as it subtracts from the quantity at the point O. Then

$$I_1 + I_2 + I_3 - I_4 = O \qquad (48)$$

Assume that $I_1 = 5$ amp., $I_2 = 8$ amp. and $I_4 = 17$ amp. Then

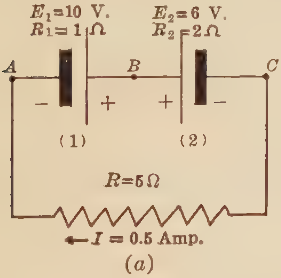
$$5 + 8 + I_3 - 17 = O$$

and $I_3 = +4$ amp., the plus sign indicating that the current flows toward the junction.

The second law is but another application of Ohm's law (Eq. 18). The basis of the law is obvious; if one starts at a certain point in a circuit, and follows continuously around the paths of the circuit until the starting point is again reached, he must again be at the same potential with which he started. Therefore the sources of electromotive force encountered in this passage must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign.

This second law is illustrated by the following example.

Two batteries (Fig. 78), having electromotive forces of 10 and 6 volts and internal resistances of 1 and 2 ohms respectively, are connected in series opposing (their + terminals connected



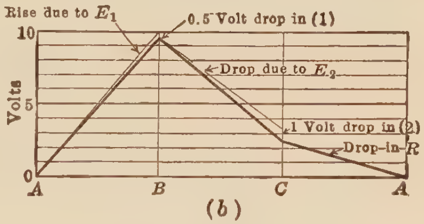


FIG. 78.—Voltage relations in an electric circuit.

together) and in series with an external resistance of 5 ohms. Determine the current and the voltage at each part of the circuit.

Since the two batteries act in opposition, the net electromotive force of the two batteries is 10 − 6 = 4 volts.

The current is,

$$I = \frac{10 - 6}{1 + 2 + 5} = \frac{4}{8} = 0.5 \text{ amp.}$$

Consider the point A as being at reference potential. In passing from A to B there is a 10-volt *rise* in potential due to the electromotive force of battery No. 1, but around the circuit in the direction of the current flow there occurs a simultaneous 0.5-volt drop of potential due to the current flowing through the 1-ohm resistance of cell No. 1. Therefore the net potential at B is but 9.5 volts greater than that at A, as is shown in Fig. 78 (b). In passing from B to C there is a drop of 6 volts due to passing from the + to the − terminal of battery No. 2, and there is

also a further drop of 1 volt due to the current of 0.5 amp. flowing through the 2-ohm resistance of battery No. 2. This makes the net potential at $C = 9.5 - 6 - 1 = +2.5$ volts. In passing from C to A there is a drop in potential of 2.5 volts due to the current of 0.5 amp. flowing through the 5-ohm resistance. When point A is reached the potential has dropped to zero.

Therefore the sum of all the electromotive forces in the circuit, taken with their proper signs, is equal to the sum of the Ir drops. This is illustrated as follows:

Electromotive forces		Ir drops	
Cell No. 1 =	+10 volts	Cell No. 1 = -0.5×1 =	-0.5 volt
Cell No. 2 =	$-$ 6 volts	Cell No. 2 = -0.5×2 =	-1.0 volt
Total	+ 4 volts	5-ohm res. = -0.5×5 =	-2.5 volts
		Total	-4.0 volts

$$+4 + (-4) = 0.$$

The current also may be found by applying Kirchhoff's second law directly. Starting at A,

$$+10 - I(1) - 6 - I(2) - I(5) = 0$$

$$8I = +4 \qquad I = +0.5 \text{ amp.}$$

83. Applications of Kirchhoff's Laws.—In the application of Kirchhoff's second law to specific problems the question of algebraic signs may be troublesome and is a frequent source of error. If, however, the following rules are kept in mind no difficulties should occur.

A rise in potential should be preceded by a + sign.

A drop in potential should be preceded by a − sign.

For example, in passing *through a battery* from the − to the + terminal, the potential *rises* so that this voltage should be preceded by a + sign. On the other hand, when passing from the + terminal to the − terminal, the potential *drops*, so that a − sign should precede this voltage. These points are illustrated in Par. 82 and by Fig. 78.

When going through a resistance in the *same* direction as the current flow, the voltage drops in the same manner that the level of a stream of water decreases when one goes in the direction of stream flow. Hence, a voltage taken through a resistance in the direction of current flow, whether in a battery resistance or

in an external resistance, should be preceded by a − sign. When going through a resistance in the direction *opposite* to the current flow, the voltage rises in the same manner that the level of a stream of water rises when one goes in the direction opposite to the stream flow. Hence, a voltage taken through a resistance in a direction opposed to that of the current flow, whether in a battery resistance or in an external resistance, should be preceded by a + sign.

This is further illustrated by the electric circuit shown in Fig. 79. Three batteries having e.m.fs. E_1, E_2, and E_3 are connected as shown in different parts of the network of resistances, R_1, R_2, R_3, R_4. The *assumed* directions for the various currents are indicated by the arrows. The battery resistances are assumed negligible as compared with the other circuit resistances.

Starting at the point a, and applying Kirchhoff's second law to the path $abcda$, an equation may be written

$$+E_1 - I_1R_1 - I_2R_2 + E_2 - I_1R_4 = 0 \qquad \text{(I)}$$

Starting at f and passing along the path $febcdf$:

$$-E_3 + I_3R_3 - I_2R_2 + E_2 = 0 \qquad \text{(II)}$$

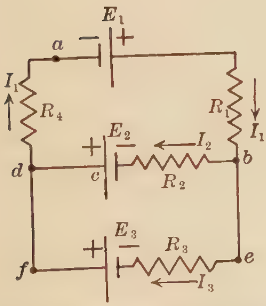


FIG. 79.—Application of Kirchhoff's laws.

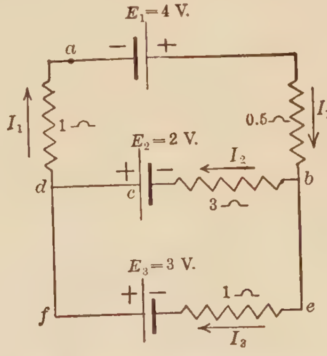


FIG. 80.—Application of Kirchhoff's laws.

This gives but two equations for the determination of three unknown currents. Three equations are necessary. A third equation may be obtained by applying Kirchhoff's second law to a third path *febadf*. This equation, however, when combined with either Eq. (I) or Eq. (II) would give either Eq. (II) or Eq. (I), showing that another condition must be placed on the net-

work in order that a solution may be found. This third condition must be obtained by applying Kirchhoff's first law to some junction as b.

$$+I_1 - I_2 - I_3 = 0$$

since I_1 is assumed to flow *toward* the junction and I_2 and I_3 *away* from the junction.

With these three equations it is possible to determine the three currents.

Example.—Figure 80 shows a network identical with that shown in Fig. 79, except that numerical values are used. The battery resistances are assumed to be small compared with the circuit resistances, and are neglected.

Considering path $abcda$,

$$+4 - (I_1 0.5) - (I_2 3) + 2 - (I_1 1) = 0$$

or

$$1.5I_1 + 3I_2 = 6 \tag{A}$$

Similarly, path $febcdf$, starting at f,

$$-3 + (I_3 1) - 3I_2 + 2 = 0$$

or

$$3I_2 - I_3 = -1 \tag{B}$$

and at the junction b,

$$+I_1 - I_2 - I_3 = 0$$

or

$$I_1 = I_2 + I_3. \tag{C}$$

Substituting I_1 (C) in (A)

$$1.5(I_2 + I_3) + 3I_2 = 6$$
$$4.5I_2 + 1.5I_3 = 6,$$

and combining with (B)

$$9I_2 - 3I_3 = -3. \qquad (B)$$
$$9I_2 + 3I_3 = 12.$$
$$-6I_3 = -15.$$
$$I_3 = 2.5 \text{ amp.} \quad Ans.$$

Substituting this value in (B)

$$3I_2 - 2.5 = -1.$$
$$3I_2 = 1.5.$$
$$I_2 = 0.5 \text{ amp.}$$
$$I_1 = I_2 + I_3 = 3.0 \text{ amp.} \quad (C) \quad Ans.$$

84. Assumed Direction of Current.—In the solution of this type of problem, the question of assuming the proper direction of

current often arises. The current may be *assumed* to flow in either direction. If the assumed direction of the current is not the actual direction, this current will be found to have a minus sign when the equations are solved.

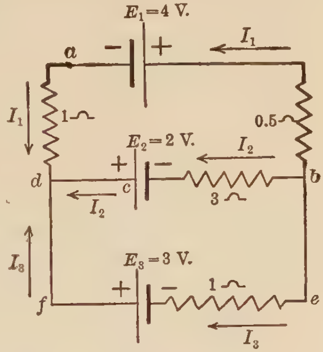


FIG. 81.—Application of Kirchhoff's laws.

Example.—This is illustrated by assuming that the three currents of Par. 83 have such a direction that they all meet at point d as is shown in Fig. 81. This condition is of course impossible.

Considering circuit *abcda*, starting at a,

$$+4 + 0.5I_1 - 3I_2 + 2 + I_1 = 0.$$

$$1.5I_1 - 3I_2 + 6 = 0.$$

Similarly circuit *febcdf*, starting at f,

$$-3 + I_3 - 3I_2 + 2 = 0.$$

$$I_3 - 3I_2 - 1 = 0.$$

The three currents I_1, I_2, I_3 all flow toward junction d, therefore

$$I_1 + I_2 + I_3 = 0.$$

Substituting and solving

$$I_1 = -3 \text{ amp.} \quad \textit{Ans.}$$

$$I_2 = 0.5 \text{ amp.} \quad \textit{Ans.}$$

$$I_3 = 2.5 \text{ amp.} \quad \textit{Ans.}$$

The minus sign preceding I_1 signifies that this current flows in the opposite direction to that assumed and indicated by the arrow, Fig. 81. The + signs before I_2 and I_3 indicate that the assumed directions for these two currents were the actual directions of flow.

85. Further Applications of Kirchhoff's Laws.—A study of the equations and their algebraic solutions in Pars. 83 and 84 indicates that in order to solve any network, Kirchhoff's first law must be applied to a sufficient number of junctions to include every current at least once. Also Kirchhoff's second law must be applied a sufficient number of times to include every path at least once. By the proper application of these two rules, the total number of equations can be made equal to the number of unknowns.

It is possible, however, to reduce the number of unknown currents, and hence the number of equations, by combining the currents, in accordance with Kirchhoff's first law, directly on the diagram. This is illustrated by the following example, in which the resistance of the batteries is not negligible as was assumed in Pars. 83 and 84.

Example.—Determine all the currents in the network of Fig. 82 and also the voltages between points *ed* and *dc*. There are six unknown currents, but by combining at the junctions *d* and *e*, as shown in the diagram, the number of unknown currents is reduced to three. Applying Kirchhoff's second law to path *abcdea*,

$$-12 + 0.5I_1 + 6(I_1 + I_2 + I_3) + 3(I_1 + I_2) + (1)I_1 = 0.$$

or

$$10.5I_1 + 9I_2 + 6I_3 = 12. \qquad \text{(I)}$$

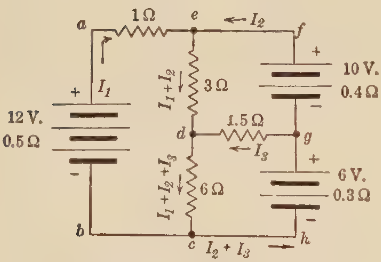


FIG. 82.—Electric network.

Path *efgde*

$$-10 + 0.4I_2 - 1.5I_3 + 3(I_1 + I_2) = 0.$$

or

$$3I_1 + 3.4I_2 - 1.5I_3 = 10. \qquad \text{(II)}$$

Path *cdghc*

$$+6(I_1 + I_2 + I_3) + 1.5I_3 - 6 + 0.3(I_2 + I_3) = 0.$$

$$6I_1 + 6.3I_2 + 7.8I_3 = 6 \qquad \text{(III)}$$

(I), (II), and (III) give three equations and since there are but three unknowns, the currents I_1 I_2, and I_3, these can be obtained by the simultaneous solution of (I), (II), and (III).

For example, if I_3 is eliminated between (I) and (II) and then between (II) and (III), there results

$$22.5I_1 + 22.6I_2 = 52.$$

and

$$32.4I_1 + 35.97I_2 = 87,$$

from which

$$I_1 = -1.244 \text{ amp.} \quad \textit{Ans.}$$

$$I_2 = +3.540 \text{ amp.} \quad \textit{Ans.}$$

By substituting these values in (II)

$$I_3 = -1.126 \text{ amp.} \quad \textit{Ans.}$$

Hence, I_1 and I_3 flow in directions opposite to those assumed.

The current from *e* to *d* $= I_1 + I_2 = +2.296$ amp. *Ans.*

The current from *d* to *c* $= I_1 + I_2 + I_3 = +1.17$ amp. *Ans.*

The current from *c* to *h* to *g* $= I_2 + I_3 = +2.414$ amp. *Ans.*

The voltage across *ed* $= +2.296 \times 3 = 6.89$ volts. *Ans.*

The voltage across *dc* $= +1.17 \times 6 = 7.02$ volts. *Ans.*

The voltage across *ce* $= 12 - (-1.244)(1.5) = 13.9$ volts (check).

86. Applications of Kirchhoff's Laws to Power Systems.—Kirchhoff's laws might be applied to problems involving distribution systems, electric railways, etc.,where power is fed to the loads through different feeders and from different substations. In practice, however, Kirchhoff's laws are rarely applied directly

to electric railway systems, since the widely fluctuating loads which are constantly shifting their location make it impossible to formulate a definite problem. Only occasionally is it necessary to apply these laws to power and lighting systems, since the feeder layout in such systems is usually determined by various operating considerations.

The following problem illustrates the possible application of these laws:

Example.—In Fig. 83, a 240-volt sub-station at A supplies two distributing centers B and C, by a ring system of feeders. Between A and B, a distance of 800 ft., two 1,000,000-C.M. feeders are paralleled; between A and C, a distance of 1,200 ft., three 1,000,000-C.M. feeders are paralleled; between

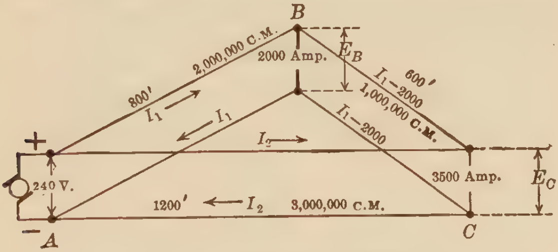


FIG. 83.—Ring-feeder system.

B and C, a distance of 600 ft., a 1,000,000-C.M. tie line is connected. Determine the current in each feeder and the voltage at each distributing center, when the load at B is 2,000 amp. and that at C is 3,500 amp.

Assuming 10 ohms per cir.-mil-foot:

$$\text{Resistance per wire } A \text{ to } B = \frac{800 \times 10}{2,000,000} = 0.004 \text{ ohm.}$$

$$\text{Resistance per wire } B \text{ to } C = \frac{600 \times 10}{1,000,000} = 0.006 \text{ ohm.}$$

$$\text{Resistance per wire } A \text{ to } C = \frac{1,200 \times 10}{3,000,000} = 0.004 \text{ ohm.}$$

Going from A to B to C, out on the positive and back on the negative conductor,

$$240 - I_1(0.004) - (I_1 - 2,000)0.006 - E_C - (I_1 - 2,000)0.006 - I_1(0.004) = 0$$

$$240 - I_1(0.02) + 24 = E_C. \qquad (1)$$

Likewise going direct from A to C

$$240 - I_2(0.004) - E_C - I_2(0.004) = 0.$$

$$240 - I_2(0.008) = E_C. \qquad (2)$$

Equating Eqs. (1) and (2)

$$240 - I_1(0.02) + 24 = 240 - I_2(0.008).$$

$$0.02I_1 - 0.008I_2 = 24. \qquad (3)$$

At the junction at C

$$I_1 - 2{,}000 + I_2 = 3{,}500.$$

$$I_1 + I_2 = 5{,}500. \tag{4}$$

Substituting in Eq. (3) for $I_1 = 5{,}500 - I_2$.

$$0.02(5{,}500 - I_2) - 0.008I_2 = 24.$$

$$110 - 0.02I_2 - 0.008I_2 = 24.$$

$$0.028I_2 = 86.$$

$$I_2 = 3{,}070 \text{ amp.} \quad \textit{Ans.}$$

$$I_1 = 2{,}430 \text{ amp.} \quad \textit{Ans.}$$

Voltage at C Eq. (2)

$$E_C = 240 - 3{,}070(0.008) = 215.44 \text{ volts.} \quad \textit{Ans.}$$

$$E_B = 240 - 2{,}430(0.008) = 220.56 \text{ volts.} \quad \textit{Ans.}$$

The problem may be solved, however, without involving the voltages at A, B, or C. Taking a path from the + bus at A and going along the positive wires of the two feeders to B, and C and then returning to A along the positive wire of the 3,000,000-C.M. feeder, the following equation is obtained:

$$-0.004I_1 - 0.006(I_1 - 2{,}000) + 0.004I_2 = 0.$$

or

$$0.01I_1 - 0.004I_2 = 12.$$

which is equal to Eq. (3) divided by 2;
and at C,

$$I_1 - 2{,}000 + I_2 = 3{,}500,$$

or

$$I_1 + I_2 = 5{,}500 \text{ amp.}$$

which is the same as Eq. (4).

CHAPTER VI

PRIMARY AND SECONDARY BATTERIES

87. Principle of Electric Batteries.—If two copper strips or plates be immersed in a dilute sulphuric acid solution (Fig. 84 (*a*)) and be connected to the terminals of a voltmeter, no appreciable deflection of the voltmeter will be observed. This shows that no appreciable difference of potential exists between the copper strips. If, however, one of the copper strips (Fig. 84

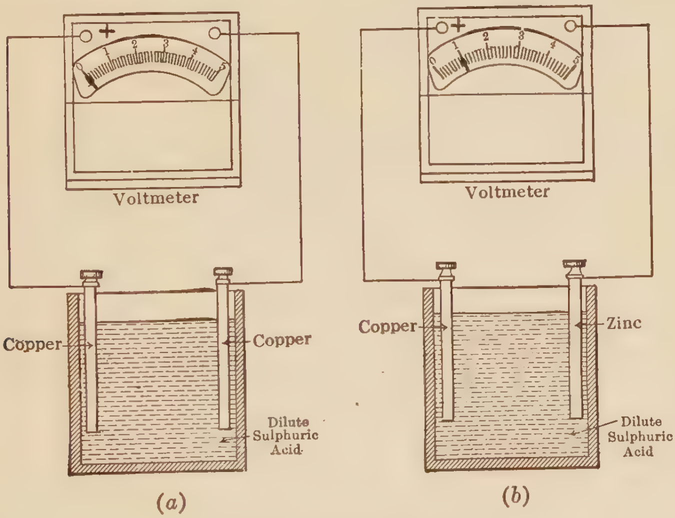


FIG. 84.—Simple primary cell.

(*b*)) be replaced by a zinc strip, the voltmeter needle will deflect and will indicate approximately one volt, showing that a potential difference now exists. It will be necessary to connect the copper to the + terminal of the voltmeter and the zinc to the − terminal in order that the voltmeter may read up scale. This shows that so far as the *external* circuit is concerned, the copper is positive to the zinc.

The above experiment may be repeated with various metals. For example, carbon or lead may be substituted for the copper and a potential difference will be found to exist between each of these and the zinc, although it will not be of the same value as it was for the copper-zinc combination. Likewise other metals may be substituted for the zinc, and potential differences will be found to exist.

Furthermore, it is not necessary that sulphuric acid be used for the solution. Other acids such as hydrochloric, chromic, etc., may be substituted for the sulphuric; or even salt solutions such as common salt (sodium chloride), ammonium chloride (sal ammoniac), copper sulphate, zinc sulphate, etc., may be used.

In order to obtain a difference of potential between the two metal plates, but two conditions are necessary.

1. The plates must be of different metals.

2. They must be immersed in some electrolytic solution, such as an acid, alkali, or salt.

Again, if current be taken from the cell shown in Fig. 84 (*b*) by connecting a resistance across its terminals (Fig. 85), current will flow from the copper through the resistance AB and into the cell through the zinc. Inside the cell, however, the current will flow *from* the *zinc* through the solution *to* the *copper* as shown in Fig. 85. Since current flows *from zinc to copper within the cell*, zinc is said to be electrochemically positive to copper. Therefore, when considering such an electrochemical cell, the copper is positive to the zinc when the external circuit is considered, but the zinc is electro-positive to the copper when the plates and the solution alone are considered.

88. Definitions.—The metal strips or plates of a cell are called *electrodes*. The electrode at which current enters the solution (as the zinc, Fig. 85) is the *anode*, and the electrode at which current leaves the solution (as the copper, Fig. 85) is the *cathode*.

The solution used in a cell is called the *electrolyte*.

If current be taken from the cell under proper conditions and for a considerable time, the zinc plate will diminish in weight. This is true not only for this particular cell, but in practically all cells the flow of current is accompanied by a loss in weight of at least one of the plates. Energy is stored in the cell

chemically, and the electrical energy is delivered at the expense of the plate which goes into solution. That is, one plate is either oxidized or converted into another chemical compound, this change being accompanied by a decrease in the available chemical energy of the system. Therefore *chemical energy* is converted into *electrical energy*, when the cell delivers a current.

Hence:

An electric cell or battery is a device for transforming chemical energy into electrical energy.

Such cells or batteries are divided into two classes: *primary cells* and *secondary cells*.

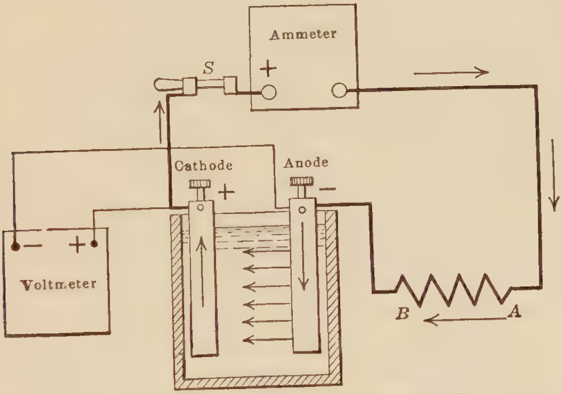


Fig. 85.—Current-flow in a single cell.

In a *primary cell* it is necessary from time to time to renew the electrolyte and the electrode which goes into solution by fresh solution and new plates, respectively.

In a *secondary cell* the electrolyte and the electrodes which undergo change during the process of supplying current are restored electrochemically by sending a current through the cell in the reverse direction.

89. Primary Cells.—Although it was stated in Par. 87 that there are many combinations of metals and solutions capable of generating an electromotive force and so forming a cell, only a limited number of such combinations are commercially practicable. The general requirements of a good cell are as follows:

(*a*) There must be little or no wastage of the materials when the cell is not delivering current.

(*b*) The electromotive force must be of such a magnitude as to enable the cell to deliver a reasonable amount of energy with a moderate current flowing.

(*c*) Frequent replacement of materials must not be necessary and such materials must not be expensive.

(*d*) The internal resistance and the polarization effects must not be excessive, otherwise the battery cannot supply even moderate values of current, at least for any appreciable time.

As an illustration, the cell shown in Fig. 84 (*b*) would not be practicable, because both the copper and the zinc would waste away even were the battery delivering no current. Polarization (see Par. 91) would be excessive, and hence the battery would be capable of delivering only a comparatively small current.

90. Internal Resistance.—As was pointed out in Chap. V, every cell or battery has an internal resistance, which tends to reduce the magnitude of the current and causes the terminal voltage to drop when current is taken from the cell. Such resistance lies in the electrodes, in the contact surface between the electrodes and the electrolyte, and in the electrolyte itself. This resistance may be reduced by changing the dimensions of the cell in the same way as would be done for any electric conductor. The cross-section of the path through which the current flows inside the cell should be made as large as is practicable. This means large area of electrodes in contact with the electrolyte. Also the cross-section of the plates must be large enough to carry the current to the cell terminals without excessive drop in voltage. Little difficulty is experienced in making this voltage drop negligible. It will be appreciated that larger electrodes mean a larger cell, with a greater current capacity. In addition to increasing the area of the electrodes, the resistance of the cell may be diminished by decreasing the distance between the plates. This reduces the length of the path through which the current flows within the cell and correspondingly reduces the cell resistance.

Increasing the size of the cell *does not increase its electromotive force*. This electromotive force depends solely on the material of the two electrodes, and the electrolyte. Thus, Fig. 86 shows two gravity cells, made up of the same materials, but differing materially in size. The cells are in opposition, that

is, their + terminals are joined and their − terminals are joined. A galvanometer G connected in one of the leads reads zero, indicating that no current flows from the larger to the smaller cell. Hence their electromotive forces must be equal.

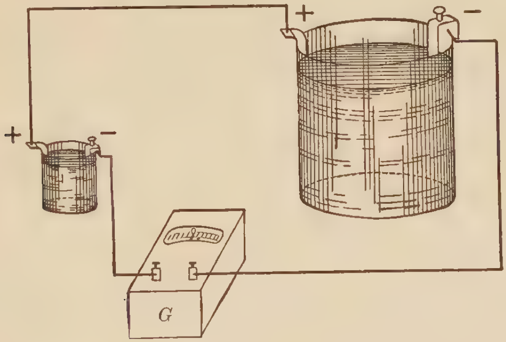


FIG. 86.—Equality of electromotive forces in cells of unequal sizes.

91. Polarization.—If a test be made to determine the fall of terminal voltage as current is taken from a cell, by connecting a voltmeter, ammeter, and an external resistance as in Fig. 85, the results will be somewhat as follows:

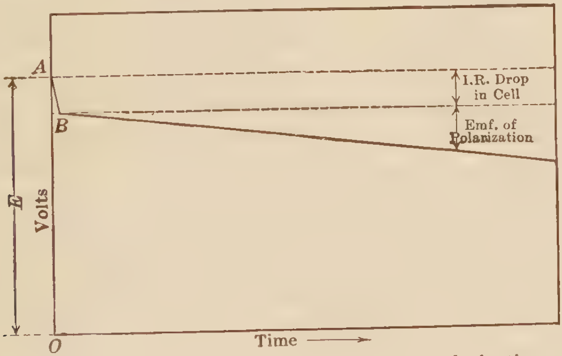


FIG. 87.—Drop of voltage in a cell due to polarization.

When the cell is on open-circuit the voltmeter will indicate the cell electromotive force E, represented by the distance OA, Fig. 87. When the switch S is closed, current will flow and the voltage will drop immediately from OA to OB. The distance AB represents the voltage drop due to the internal resistance

of the cell and this has been considered earlier, in some detail. As time elapses the terminal voltage will be observed to drop still further, even though the current be maintained constant. This further drop of voltage is due to *polarization.*

When the cell delivers current, small bubbles of hydrogen appear upon the positive plate or cathode, practically covering it. These bubbles have two effects:

They cause a substantial increase in the resistance at the contact surface between the cathode and the electrolyte.

Hydrogen acting in conjunction with the cathode or positive plate sets up an electromotive force which opposes that of the cell.

These two effects explain the reduction in the current capacity of many types of cells after they have delivered current for some time.

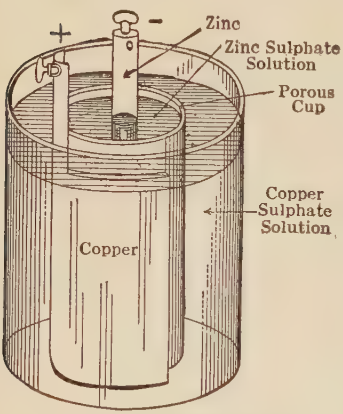


Fig. 88.—Daniell cell.

Remedies for Polarization.—These hydrogen bubbles may be removed mechanically by brushing them off or by agitating the electrolyte. This is impracticable under commercial conditions. If the plate be roughened, the bubbles form at the projections and come to the surface more readily.

The more practicable method is to remove the hydrogen bubbles chemically by bringing oxidizing agents, such as chromic acid or manganese peroxide, into intimate contact with the cathode. The hydrogen readily combines with the oxygen of these compounds to form water (H_2O). This method is used in the bichromate cell, in the Le Clanché cell and in dry cells.

92. Daniell Cell.—This cell, Fig. 88, is a two-fluid cell having copper and zinc as electrodes. It consists of a glass jar, inside of which is a porous cup containing zinc sulphate solution or a solution of zinc sulphate and sulphuric acid. The anode or negative electrode is immersed in this electrolyte. The porous cup is placed in a solution of copper sulphate with copper sulphate

crystals in the bottom of the jar. The copper plate, which is cathode, surrounds the porous cup. The porous cup keeps the two solutions separated. As the copper is in a copper sulphate solution, there is no polarization. This cell is designed for use in a circuit which is kept continually closed. If left idle the electrodes waste away. When the cell is taken out of service for some time, the electrodes should be removed and the porous cup should be thoroughly washed. The electromotive force of this cell is about 1.1 volts.

93. Gravity Cell.—The gravity cell is similar to the Daniell cell, except that gravity, rather than a porous cup, is depended upon to keep the electrolytes separated. This cell is shown in Fig. 89. The cathode, which is of copper, is made of thin strips riveted together and placed in the bottom of the cell together with copper sulphate crystals. A solution of copper sulphate is then poured to within a few inches of the top of the jar. The connection to the copper is usually an insulated copper wire fastened to the copper and carried out through the solution to the top of the jar. There should always be copper sulphate crystals at the bottom of the cell.

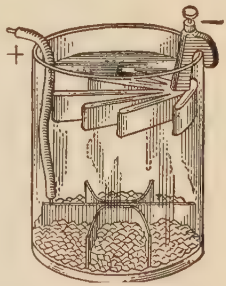


FIG. 89.—Gravity cell.

The anode is zinc, is usually rather massive and is cast in the form of a crow's foot and hung on the top of the jar. This is surrounded by a zinc sulphate solution. The solutions are kept separated by gravity. The copper sulphate is the heavier of the two solutions and therefore tends to remain at the bottom. The solutions should be poured in carefully for if the copper sulphate solution comes in contact with the zinc, copper will be deposited. This copper should be removed, if by chance it becomes deposited in any way. In the operation of the cell the zinc goes into solution as zinc sulphate, and metallic copper comes out of the copper sulphate solution and is deposited upon the copper electrode. The cathode will therefore gain in weight, whereas the anode will lose in weight. This is the reason for having the zinc electrode massive, and the copper electrode of very thin sheet copper, when the cell is set up initially.

Due to capillary action the electrolyte tends to creep up over the top of the jar forming a crystalline deposit. To prevent this creeping, the top of the jar should be paraffined. To prevent evaporation, the upper surface of the electrolyte may be covered with oil. When the cell is replenished, metallic zinc and copper sulphate are supplied and metallic copper and zinc sulphate are removed.

The gravity cell is a *closed circuit* battery, and the circuit should therefore be kept closed for the best results. Otherwise the copper sulphate will gradually mix with the zinc sulphate. The cell has been found very useful in connection with railway signals, fire alarm systems, and telephone exchanges, all closed circuit work, although the storage battery has replaced it in many instances. The electromotive force of the cell is practically that of the Daniell cell, being about 1.09 volts, but varies slightly with the concentration of the solutions.

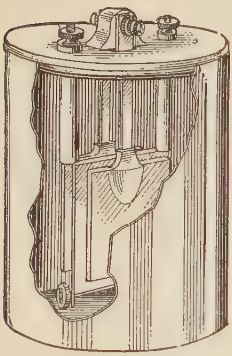


FIG. 90.—Edison-Lalande cell or Edison primary battery.

94. Edison-Lalande Cell.—The Edison-Lalande cell (Fig. 90) is still used to some extent. The cathode is of copper oxide and is suspended between two zinc plates which form the anode. All the plates are fastened to a porcelain cover by means of bolts which serve as binding posts as well as supports for the plates. The electrolyte is caustic soda (NaOH), one part by weight of soda to three of water. To prevent the soda being acted upon by the air, the electrolyte is covered with a layer of mineral oil. The copper oxide of the cathode gives up its oxygen very readily to the hydrogen which forms on it, thus preventing any substantial polarization. These cells are capable of delivering a very heavy current. The electromotive force is about 0.95 volt, and when delivering current the terminal voltage drops to 0.75 volt. There is little or no local action in this cell and it can therefore be used to advantage on both open-circuit and closed-circuit work. Its chief disadvantage is its low electromotive force.

95. Le Clanché Cell.—The Le Clanché cell is perhaps the most familiar type of primary battery, because of its wide applica-

tion. The cathode is molded carbon and the anode is amalgamated zinc. The electrolyte is sal ammoniac or ammonium chloride. This type of cell is suited only for open-circuit work because of the rapidity with which it polarizes. The electromotive force is 1.4 volts, but because of the drop due to its internal resistance and that due to polarization, not over 1 volt per cell should be allowed in planning an installation. The most common method of reducing polarization is to bring manganese dioxide into intimate contact with the carbon. This gives up oxygen readily which unites with the hydrogen bubbles to form water.

In one type of Le Clanché cell a pencil zinc is suspended in the center of a hollow cylinder of carbon and manganese dioxide. An improved type, the porous cup cell, is shown in Fig. 91. In this form a hollow carbon cylinder is filled with manganese dioxide, and the zinc, bent into cylindrical form, surrounds the carbon cylinder, being separated therefrom by rubber rings.

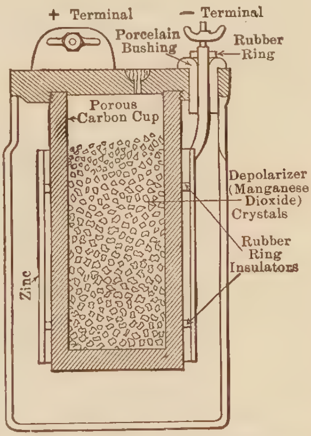


Fig. 91.—Porous cup Le Clanché cell.

The solution should consist of 3 oz. of sal ammoniac to 1 pt. of water. A more concentrated solution produces zinc chloride crystals on the zinc and carbon. To prevent the solution "creeping," the top of the cell is dipped in paraffin and the top of the carbon is covered with wax.

This cell owes its wide use to its simplicity, to the small amount of attention that it requires, and to the fact that it contains no injurious acids or alkalis. Its uses are for intermittent work, such as ringing door-bells, telephone work, and open-circuit telegraph work. Because of its more convenient form, the dry cell has to a large extent replaced this type of cell.

96. Weston Standard Cell.—It is essential in practical work to be able to reproduce accurately standards of current, voltage, and resistance. Obviously if two of the above quantities are known, the third is readily obtainable by Ohm's law. It is a matter

of no great difficulty to make and reproduce resistance standards, as such standards are nothing more than metals in strips and in other forms, carefully mounted and calibrated. Such standards are very permanent and their resistance remains constant indefinitely.

A standard of either current or voltage is much more difficult to reproduce and maintain than is the standard of resistance. Of the two, it has been found more practicable to produce and maintain a voltage standard rather than a current standard. This voltage standard is obtained in a *standard cell.* The electromotive force of a cell depends upon its materials and their impurities, the concentration of the electrolyte, the temperature, the polarization effects, etc. It is difficult, therefore, to select such materials for a cell as will enable it to be reproduced at different times and at various places with a high degree of accuracy. The Clark cell was the first of the standard cells to prove commercially successful. This had a cathode of mercury, an anode of zinc, and an electrolyte of mercurous sulphate and zinc sulphate. The objections to this cell were that the electromotive force changed very appreciably with the temperature and that this change lagged behind the change in temperature.

In the Weston cell, cadmium is substituted for the zinc of the Clark cell. An unmounted Weston cell is shown in Fig. 92. The cathode is mercury, located at the bottom of one leg of an H-tube. Above this is mercurous sulphate paste. These materials are held in position by means of a porcelain tube, expanded at the bottom and packed with cotton. This tube extends to the top of the cell and acts as a vent for any gases that are formed. In the bottom of the other leg of the H-tube is the anode, of cadmium amalgam. This is held in place by another porcelain tube packed with cotton. The electrolyte is cadmium sulphate. The leads from the cathode and the anode are sealed into the tubes at the bottom. The top of the cell is sealed with cork, paraffin, and wax. The entire cell is mounted in a wood and metal case with binding posts at the top.

The cell is made in two forms, the *normal cell* and the unsaturated or *secondary* cell. In the normal cell, cadmium sulphate crystals are left in the bottom of the solution so that it is always saturated. Its electromotive force is affected slightly by tem-

perature, but corrections can be accurately made. It is possible to reproduce such cells with electromotive forces differing by only a few parts in 100,000.

In the unsaturated cell, the solution is saturated at 4° C. and as no crystals are left in the solution, its concentration is substantially constant at other temperatures. Such cells have practically no temperature coefficient. They are not as accurately reproducible as is the normal cell. A certificate should accompany each one giving its electromotive force, which usually is about 1.0186 volts. The unsaturated type of cell rather than the normal cell is used almost entirely in practical work.

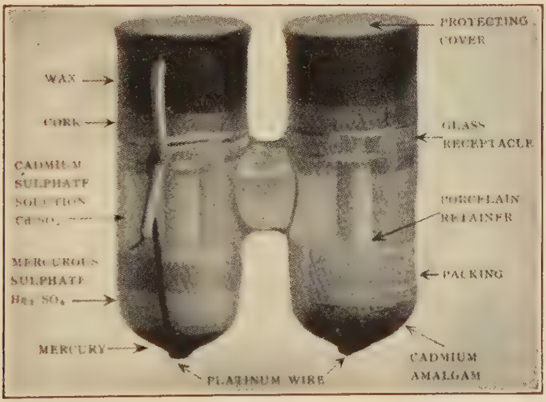


FIG. 92.—Unmounted Weston unsaturated cell.

The terminal voltage of any cell differs from its electromotive force by the IR drop due to the cell resistance. As the resistance of a Weston cell is about 200 ohms, it is evident that if any appreciable current be taken from the cell its terminal voltage will be quite different from its electromotive force. The cell must be used, therefore, in such a manner that it delivers no appreciable current. By means of the so-called Poggendorff method, described in Par. 136, the cell is used without delivering current. Not more than 0.0001 amp. should be taken from the cell at any time. If appreciable current is taken, the electromotive force drops, but when the circuit is again opened the electromotive force slowly recovers its initial value.

97. Dry Cells.—Dry cells are a modification of the Le Clanché cell and as they are very light, portable, and convenient, they are rapidly replacing other types of cells. The name "dry cell" is really a misnomer, for no cell that is dry will deliver any appreciable current. In fact the chief cause of dry cells becoming exhausted is their actually becoming dry.

A cross-section of a typical dry cell is shown in Fig. 93. The anode is sheet zinc, made in the form of a cylinder with an open top, and acts as the container of the cell. The binding post is soldered to the top of the zinc. The zinc is lined with some non-conducting material such as blotting paper or plaster of paris. The anode consists of a carbon rod, with the mixture of coke, carbon, etc., which surrounds this rod. The rod itself varies in shape among various manufacturers. It is located axially in the zinc container and the binding post is secured to the top of it. The depolarizing agent, powdered manganese dioxide, is mixed with finely crushed coke and pressed solidly into the container between the carbon and the non-conducting material which lines the zinc. It fills the cell to within about an inch of the top. Sal ammoniac, with perhaps a little zinc sulphate, is added and the cell then sealed with wax or some tar compound. The outside of the zinc is frequently lacquered, and the cells are always set in close-fitting cardboard containers.

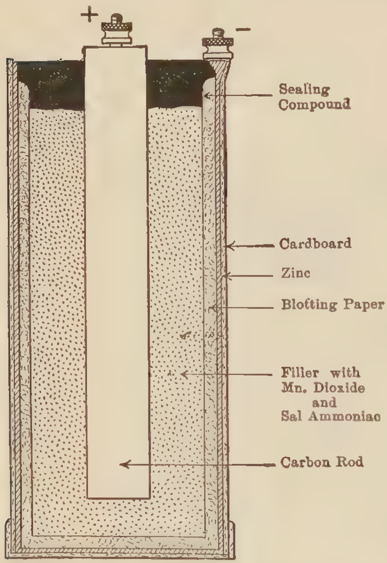


FIG. 93.—Sectional view—dry cell.

The electromotive force of a dry cell is about 1.5 volts when new but this drops to about 1.4 volts with time, even though the cell remains idle. A cell is practically useless after a year to 18 months, even if not used at all. The internal resistance of the cell is about 0.1 ohm when new and increases to several times

this value with time. The polarization effect is large as compared with the internal resistance so that a low value of internal resistance is not important except as an indication of the condition of the cell. A method for testing the condition of a cell is to short-circuit it through an ammeter, when it should deliver an instantaneous value of 1.5/0.1 or 15 amp., if in good condition. When new, the current under these conditions may reach even 25 amp. When delivering appreciable current the terminal voltage is very nearly 1 volt.

One of the chief causes of a cell's becoming useless is the using up of the zinc as a result of electrochemical actions in the cell. This allows the solution to leak out and to dry up and the cell then becomes worthless. The life of a cell may be prolonged temporarily by introducing fresh solution, but the results are usually far from satisfactory.

As is well known, dry cells have many applications. Their field is limited to supplying moderate currents intermittently, but they are capable of supplying very small currents of the magnitude of 0.1 amp. continuously. They are used extensively for door bells, electric bells, buzzers, telephones, telegraph instruments, gas-engine ignition, flash lamps, and for many other purposes. Their use as "A," "B," and "C" batteries in radio receiving sets is now common.

STORAGE BATTERIES

98. Storage Batteries.—A storage or secondary cell (sometimes called an accumulator) involves the same principles as a primary cell, but the two differ from each other in the manner in which they are renewed. The materials of a primary cell which are used up in the process of delivering current are replaced by new materials, whereas, in the storage cell, the cell materials are restored to their initial condition by sending a current through the cell in a reverse direction. For this reason the electrochemical products resulting from the discharge of such a cell must remain within the cell. Therefore if a cell in its operation gives off material, usually in the form of gases, so that it cannot be brought back to its original condition with a reverse current, it is not suitable for a storage cell. For example, the Le Clanchè cell gives off free ammonia gas and therefore cannot be used as a

storage cell. The Daniell and gravity cells are both reversible and hence are theoretically capable of being used as storage cells; but as the active materials go into solution and do not all return during the reverse cycle, the life of such a cell would be limited. There are but two types of storage cells in common use, the *lead-lead-acid* type and the *nickel-iron-alkali* type. In both of these cells the active materials do not leave the electrodes.

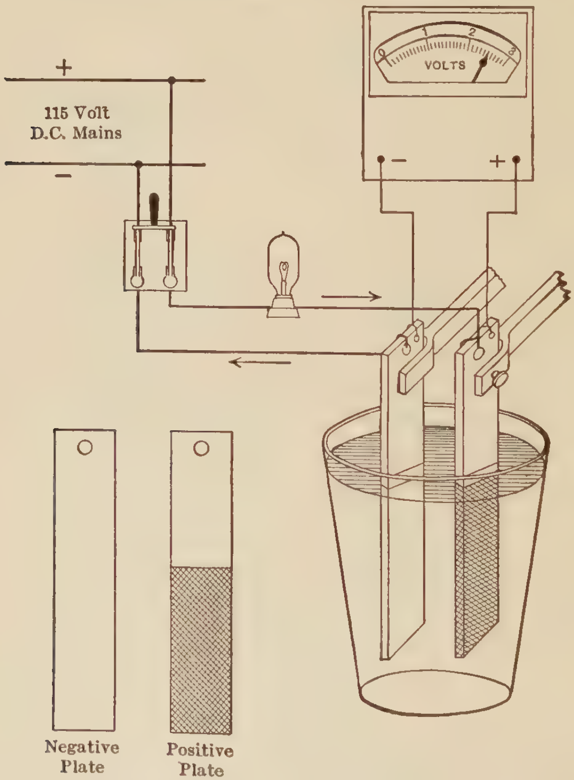


FIG. 94.—Forming the plates of an elementary lead storage cell.

99. The Lead Cell.—The principle underlying the lead cell may be illustrated by the following simple experiment. Two plain lead strips (Fig. 94) are immersed in a glass of dilute sulphuric acid (sp. gr. = 1.200 approximately). These are connected in series with an incandescent lamp supplied from 115-volt direct-current mains, or from a battery. When current flows

through this cell bubbles of gas will be given off from each plate, but it will be found that a much greater number come from one plate than from the other. After a short time one plate will be observed to have changed to a dark chocolate color, and the other apparently will not have changed its appearance. A careful examination, however, will show that the metallic lead at the surface of the latter plate has started to change from solid metallic lead to spongy lead.

When the current is flowing as shown in Fig. 94 the voltmeter connected across the cell will indicate about 2.5 volts. If the current be interrupted by pulling the switch the voltmeter reading will fall to about 2.05 volts, and the cell will now be found to be capable of delivering a small current. This current is of sufficient magnitude to operate a small buzzer for a very short period, but the amount of energy that such a cell can deliver is very limited; even the small current taken by the voltmeter is sufficient to exhaust the cell in a very short time. As the cell discharges the voltage drops off slowly to about 1.75 volts, after which it drops more rapidly until it becomes zero and the cell is apparently exhausted. The color of the dark brown plate will now have become lighter and will more nearly resemble its initial lead color. After a short rest the cell will recover slightly and will again deliver current for a very brief period.

The plate which is a dark chocolate color in the above experiment is the positive plate or cathode and the one which is partially converted to spongy lead is the negative plate or anode. The bubbles which were noted come mostly from the negative plate and are free hydrogen gas. When the current is passed through such a cell the metallic lead of the positive plate becomes converted into lead peroxide, whereas the negative plate is not changed chemically, but is converted from solid lead into the spongy form which is softer and more porous than ordinary metallic lead. When the cell is discharged the lead peroxide of the positive plate is changed to lead sulphate and the spongy lead of the negative plate becomes lead sulphate so that they both tend to become electrochemically equivalent.

The principle of the cell is the same as that of the primary cell. When the two lead plates are the same electrochemically, that is, when both are lead sulphate, no potential difference

exists between them. When the positive is converted to the peroxide and the negative to spongy lead by the action of an electric current, the two plates become dissimilar and an electromotive force now exists between them. This electromotive force is about 2.05 volts, the excess of 0.45 volt observed in charging the cell being necessary to overcome the internal resistance and polarization effects. This simple experiment illustrates the principle underlying the operation of lead storage cells.

The chemical reactions which take place in a storage cell are as follows:

Battery discharged					Battery charged				
(+ plate)		(− plate)			(+ plate)		(− plate)		
$PbSO_4$	+	$PbSO_4$	+ $2H_2O$	$\leftrightarrows$	PbO_2	+	Pb	+	$2H_2SO_4$
Lead sulphate	+	lead sulphate	+ water	is changed to	Lead peroxide	+	lead	+	sulphuric acid

The above equation shows the changes that occur when the battery is charged. The reverse takes place on discharge. It will be noted that when the battery is being charged the only change that takes place in the electrolyte is that water is converted into sulphuric acid. This accounts for the rise of specific gravity on charge. On discharge the sulphuric acid is dissociated, and reacts with the lead peroxide to form water. Therefore the specific gravity of the electrolyte decreases when the cell is discharging. When charging, free hydrogen is given off at the negative plate and oxygen at the positive plate. Because of the explosive nature of hydrogen, *no flame should be allowed to come in proximity to a storage battery, while the battery is charging.*

100. Planté Plates.—It would not be practicable to construct storage cells of plain lead sheets such as were used in this experiment. The current capacity of the cell would be so small that the cell could not deliver currents of commercial value for any length of time, unless the cell were made prohibitively large in order to secure the necessary plate area.

If the charging of the elementary cell (Fig. 94) were carried further, the dark lead peroxide of the positive plate would be observed to fall off in flakes and drop to the bottom of the tumbler. Therefore in a commercial cell provision must be made to minimize this flaking of the active material.

It was recognized very early that in order to make the storage cell commercial, a large plate area must be exposed to the action

of the acid and a large amount of the lead must be converted into the peroxide and so become active material. There are two methods of obtaining this result, the Planté process and the Faure process. In the Planté process the active material on the plates is formed from the metallic lead by passing a current through the cell first in one direction and then in the reverse direction, which procedure works the lead on the surface of the plates into active material. This process is slow but may be accelerated by adding certain acids to the sulphuric acid during the forming process. The Gould plate shown in Fig. 95 is made by this process. The plate is first passed under revolving steel wheels which convert its surface into ridges and furrows, increasing the surface area of the plate. As this process weakens the plate mechanically, certain portions of it are not acted upon by the wheels. These portions act as ribs which give support and mechanical strength to the plate and tend to prevent buckling. The active material is then formed electrically by the Planté process. The negative plate is made from the positive by reducing the peroxide to spongy lead by an electric current.

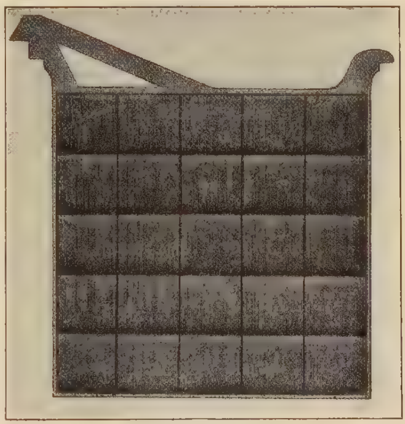
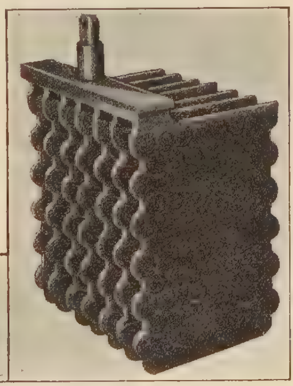


Fig. 95.—Gould ploughed plate, Planté process.



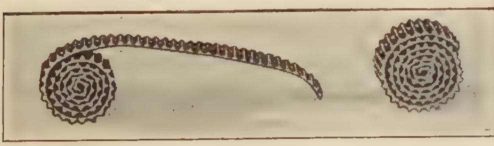


Fig. 96.—Planté (Manchester) positive group and button.

Another type of Planté plate, the Manchester type, is shown in Fig. 96. A grid made of lead and antimony is perforated. The active material consists of a corrugated lead ribbon, which is coiled into spirals and pressed into the perforations of the grid. The peroxide has a greater volume than the lead from which it is derived. Therefore when the cell is charged, these spirals expand and become more firmly embedded in the plate. The grid itself is not acted upon to any great extent, but serves as a mechanical support. The advantage of this type of plate is its rigidity and mechanical strength. Since the grid acts merely as the support for the lead buttons, the plate can be used until all the lead is converted into active material, without the plate as a whole disintegrating.

Planté positive plates should be designed to give from 1,800 to 2,400 cycles of complete charge and discharge. With Planté plates, the charging and discharging during the use of the battery converts more and more of the plate into active material. Hence, the positive plates must gradually shed some of the active material to make room for this new active material. Therefore, sufficient space between the bottom of the jars and the bottom of the plates must be allowed in order to prevent the accumulation of lead peroxide from short-circuiting the plates.

Planté negative plates should give between 2,500 and 3,000 complete cycles of charge and discharge before their capacity falls to 80 per cent. of its initial value. They fail from a loss of capacity due to the lead losing its spongy form, rather than from mechanical disintegration of the plate. Only one or two companies use Planté negative plates, the Faure plate being almost always used, even with Planté positives.

101. Faure or Pasted Plate.—This type of plate consists of a lead-antimony lattice work or skeleton into which lead oxide is applied in the form of a paste. The battery is then charged. The paste on the positive grid is converted into peroxide and that on the negative grid into spongy lead. Two types of pasted plates are shown in Fig. 97.

The chief advantage of the pasted plate is its high overload capacity, especially for short periods, together with its lesser size, cost, and weight for a given discharge rate. It is therefore very useful where lightness and compactness are necessary, such

as in electrical vehicle batteries, ignition and starting batteries for gasoline cars, etc.

Flat-pasted positive plates also lose active material by shedding. A pasted plate should give from 300 to 400 complete cycles of charge and discharge before its capacity falls to 80 per cent. of its initial value. Pasted positive plates gain in capacity during the early part of their life, increasing in capacity to about 120 per cent. of the initial value. This extra capacity is available during the greater part of their life, since new active material

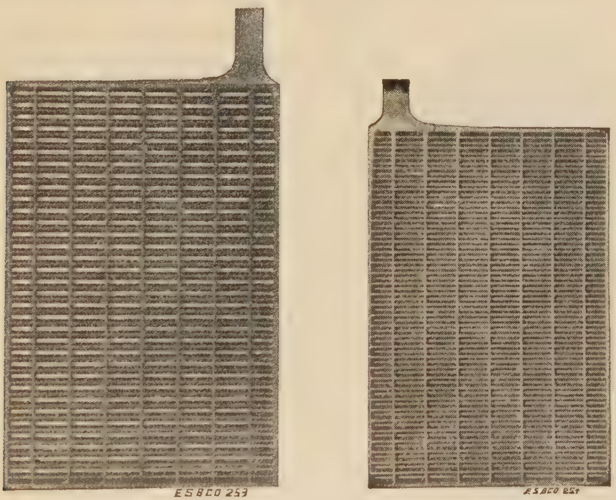


FIG. 97.—Pasted positive and negative plates.

forms as rapidly as it is shed. However, as soon as all the new active material has been utilized, further erosion results in a considerable loss in capacity. The erosion and the rate at which the plates lose capacity then become so rapid that the useful life of the plate is practically ended.

Pasted plates have a much greater ampere capacity per unit area than Planté plates. On the other hand, the Planté plate is more rugged and has a much longer life.

In all batteries there is one more negative than positive plate. This allows all the positives to be worked on *both* sides. Were any of the positives to be worked on one side only, the expansion of the active material, which occurs when it is converted to the

peroxide on charge, would be unequal on the two sides of the plate and buckling would result.

"Exide Ironclad."—For the propulsion of vehicles of various kinds and for many purposes where it is desirable to combine the characteristics of the pasted plate with much of the ruggedness and long life of the Planté plate, the Exide Ironclad is the lead storage battery most generally used. Its positive consists of a lead-antimony frame which supports a number of slotted hard-rubber tubes. An irregular lead-antimony core passes through the center of each tube and serves as a collecting device for the current. The peroxide or active material is pressed into the tubes, filling the space between the core and the inner wall of the tube. The perforations are so small that the peroxide does not drop out readily and erosion of the positive plate is practically eliminated. An ordinary pasted plate, somewhat thickened, is used for the negative plate of this cell. Although slightly more expensive, this type of cell has a life of from two to three times that of the usual flat-pasted plate cell. This type of battery can also stand considerable rough usage. A view of an Exide Ironclad, cut away to show the assembly, is given in Fig. 98.

FIG. 98.—Cut-away of an Exide Ironclad cell.

Storage batteries are divided into two general classes, *stationary batteries* and *portable batteries*.

102. Stationary Batteries.—The plates of this type of battery may be either of the Planté type or of the pasted type, depending on the nature of the service. For merely regulating duty, involving only moderate, though continual, charging and discharging, the Planté plate is preferable. Where a battery is installed for emergency service, to carry an enormous overload for a very short period during a temporary shut-down of the generating apparatus, the Faure or pasted plate is preferable. For a given floor

area the pasted plate can discharge at the 1-hr. rate, twice the current that the Planté plate can and at less than the 1-hr. rate this ratio becomes greater. This is a very important factor in congested city districts where such batteries are usually located and where floor area is very valuable.

103. Tanks.—The containing tanks are of two types: glass and lead-lined wooden tanks. Glass jars are used only for cells of small capacity, as they are expensive and have not the requisite mechanical strength in the larger sizes.

When glass jars are used, the cells may be of either the open or sealed type (see Fig. 105). In the open type, the plates are suspended by projecting lugs which rest on the edges of the jar (Fig. 105 (*a*)). In the sealed type, the plates are usually hung by the connecting strap and post from the cover which is sealed with compound to the top edge of the jar (Fig. 105 (*b*)).

The wooden tanks must be strong and well made. They are lined with sheet lead. The seams of the lead lining must be sealed by burning the lead with a non-oxidizing flame. Solder should never be used. The wood should be painted with an acid-resisting paint, such as asphaltum. An occasional application of linseed oil will prevent decomposition due to the acid.

In the lead-lined tanks, the plates are also suspended by projecting lugs resting on two glass slabs, 3/8 in. thick, which in turn rest on the bottom of the tank (see Fig. 99).

The plates of like polarity are burned to a heavy lead strip or bus-bar to which the current-carrying lead is either burned or bolted. There should always be a liberal space between the plates and the bottom of the tank to allow the red lead-peroxide to accumulate without short-circuiting the plates. All types of stationary batteries should have a glass cover to reduce evaporation and to intercept the fine acid spray which occurs during the charging periods.

104. Separators.—To prevent the positive and negative plates from coming in contact with one another, several types of separators have been tried. Very thin perforated hard rubber usually in combination with a wood diaphragm is still in use for small cells, but this is unsuitable for larger cells as the limited area of the perforations offers too much resistance to the passage of the

current to the active material. Glass rods have been suspended between the plates, but these are unsatisfactory because there is still opportunity for bits of peroxide dropping from the positive plate to lodge between the plates and cause a short-circuit. Moreover, the rods are not a complete barrier between plates so that the expansion of the active material on either the positive or the negative plate may cause a short-circuit. The most satisfactory separators are made of wood. These are very thin and are grooved vertically to permit the circulation of the electrolyte.

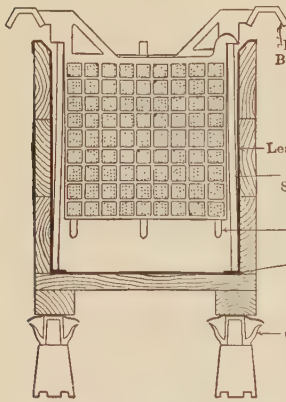


FIG. 99.—Lead-lined wooden tank storage cell.

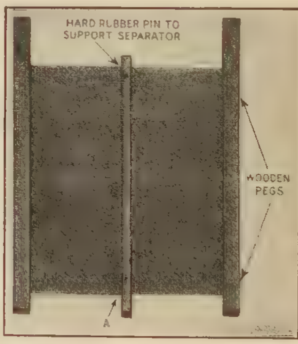


FIG. 100.—Assembly of a wooden separator.

They are specially treated to remove ingredients that would be detrimental to the electrolyte. The wood, after being treated, is not attacked by the acid. These separators should never be allowed to become dry, as they then decompose very readily. After being received, they should be kept wet until installed. In larger sizes of batteries the separators are held in place by dowel pins (Fig. 100). In the better types of starting batteries, a hard rubber separator, with a large perforated area, is used to reinforce the wooden separator. The hard rubber is placed adjacent to the positive plate so that the oxygen liberated from the positive plate during discharge does not act on the wood.

105. Electrolyte.—The electrolyte should be chemically pure sulphuric acid. When fully charged the specific gravity should be 1.210 for Planté plates and not higher than 1.300 for pasted

plates. This solution may be made from concentrated acid (oil of vitriol sp. gr. 1.84) by *pouring the acid into water* in the following ratios:

PARTS WATER TO ONE PART ACID

Specific gravity	Volume	Weight
1.200	4.3	2.4
1.210	4.0	2.2
1.240	3.4	1.9
1.280	2.75	1.5

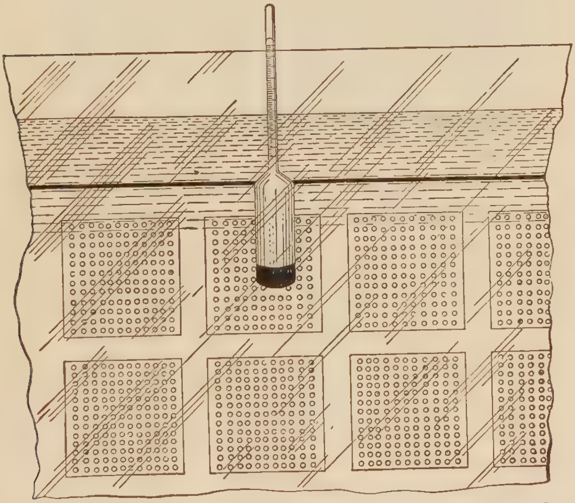


FIG. 101.—Measurement of specific gravity in a stationary battery.

A large amount of heat is evolved when acid and water are mixed. This results in a large amount of steam being generated if the water is added to the acid. This should be avoided as it may scatter the acid, break the container and may cause personal injury.

The specific gravity of a solution may be determined directly by the use of a hydrometer. This consists of a weighted bulb and a graduated tube which floats in the liquid as shown in Fig. 101. The bulb floats in the liquid whose specific gravity is to be measured, and the specific gravity is read at the point where the surface of the liquid intercepts the tube. Such a tube may be left floating permanently in stationary batteries of the open type in a representative cell called a *pilot cell* (Fig. 101).

The small amount of liquid and the inaccessibility of vehicle and starting batteries as well as stationary batteries of the sealed type make the use of such a hydrometer impossible. To determine the specific gravity with such batteries, the syringe hydrometer shown in Fig. 102 is used. The syringe contains a small hydrometer and when sufficient liquid is drawn into the syringe tube, the small hydrometer floats and may be read directly.

Fig. 102.—Syringe hydrometer.

Fig. 103 shows the change in specific gravity during charge and discharge at constant normal current rate. This relation is very important, as the specific gravity of the electrolyte is an accurate indication of the condition of charge of the battery.

106. Specific Gravity.—When the battery is charged, oxygen (O_2) is given to the positive plate to convert it into the peroxide, and sulphate ions (SO_4) are given off at the negative plate, leaving

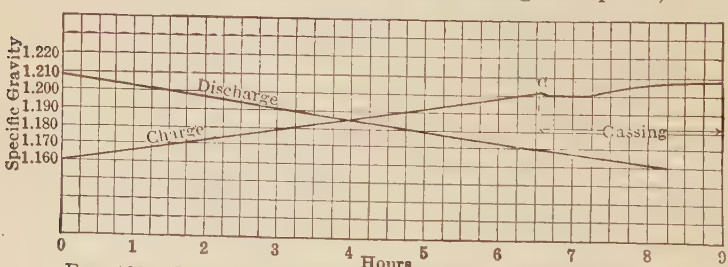


Fig. 103.—Variation of specific gravity in a stationary battery.

spongy lead. The oxygen given to the positive plate is obtained from the oxygen ion of the water, leaving the hydrogen ion of the water to combine with the SO_4 ion, given up by the negative plate, to form H_2SO_4, or sulphuric acid (see chemical equation, p. 112). Hence, the electrolyte gives up water and sulphuric acid is formed, which means that during charge the solution becomes more and more concentrated. The specific gravity will rise from the complete discharge value of 1.160 to 1.210 when

fully charged as shown in Fig. 103. Point *a* is called the gassing point because it is the point at which all the hydrogen is not absorbed in the chemical reactions and hence is given off rapidly as a gas. Here the specific gravity drops off slightly due to the presence of the hydrogen bubbles in the electrolyte. After the charging has ceased the specific gravity continues to rise for some time. This is due to the very concentrated acid in the pores of the active material diffusing out into the solution and also to the fact that the hydrogen bubbles have escaped from the solution. The discharge curve shown in Fig. 103 is very similar to the charge curve. The specific gravity continues to decrease even after the battery has ceased to deliver current. This is now due to the dilute acid in the pores of the active material diffusing into the solution. The specific gravity is such a good indicator of the state of charge of the battery that the hydrometer reading is generally used to determine how nearly charged or discharged the battery may be.

As the hydrogen and the oxygen gas which escape from the battery during the charging and the discharging periods are only dissociated water, the battery loses nothing but the equivalent of water. Ordinarily, therefore, nothing but water need be added to replace the electrolyte. A small amount of the acid is carried away as a spray by the gas bubbles, but this loss is rarely of appreciable magnitude. Acid need only be added when an actual loss of electrolyte takes place, such as occurs with a leaky tank. Distilled water is used, as a rule, to replace the evaporation of the electrolyte. If any doubt exists as to the suitability of local water, the battery companies upon receipt of a sample will analyze the water and report upon the matter.

107. Temperature.—Below is given the relation between the freezing point of the electrolyte and its specific gravity. It will be noted that the freezing point is very considerably reduced with increasing values of the specific gravity, so that if a battery is well charged there is no danger of freezing in the temperate zone.

Specific gravity	Freezing temperature, Fahrenheit
1.180	− 6°
1.200	−16°
1.240	−51°
1.280	−75°

At the higher temperatures the rate of diffusion of the acid throughout the pores of the active material is increased so that the rating of a battery increases very appreciably with increasing temperature. Above 70° this increase is of the order of from 0.5 to 1.0 per cent. per degree Fahrenheit.

108. Installing and Removing from Service.—The plates, tanks, electrolyte, and containers of stationary batteries are packed

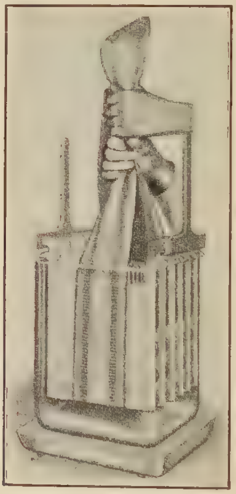
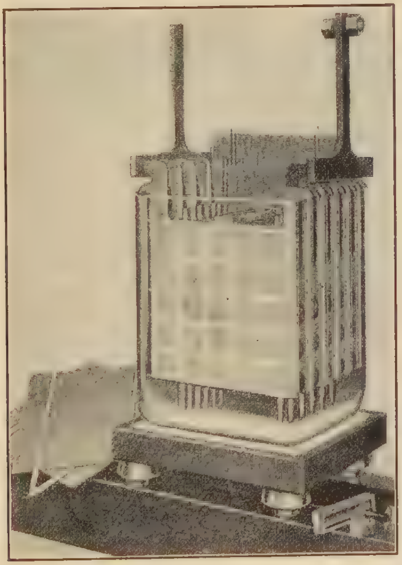


FIG. 104.—Lowering plates into position.

separately when shipped. When received the separators should be placed immediately where they may be kept wet. The jars should be set in sand trays which rest on insulators as shown in Fig. 105(a). The plates should be handled carefully and placed in the jars in the manner shown in Fig. 104. The separators should be carefully slid into position as shown in Fig. 105(a). As the active material on the plates is more or less converted into lead salts during exposure to the atmosphere, these salts must be reduced electrically before the battery is ready for service. Therefore the battery should be given an initial charge at the normal charging rate for about 40 hr. or more.

If the battery stands over a long period without being used, the active material becomes more or less converted into inactive lead sulphate, which is a non-conductor, and so is difficult to

reduce electrically. Therefore a battery if idle should be charged occasionally. If the battery is to remain idle over a long period and it is impracticable to charge it periodically the following procedure is necessary to prevent sulphation. Give the battery a full charge, then siphon off the electrolyte, which may be saved and again used. Fill the cells with water and allow them to stand 12 to 15 hr. Siphon off the water and the cells will stand indefinitely without injury to the plates. To put back in



(a) Open-type battery in position.

(b) Sealed type.

FIG. 105.—Stationary batteries.

service, fill the battery with the electrolyte having a specific gravity of 1.210 and charge for 35 hr. or more at the normal rate or its equivalent.

109. Vehicle Batteries.—In the design of batteries for propelling vehicles and for automobile starting and lighting it is necessary to obtain a very high discharge rate with minimum weight and size. Therefore pasted plates are used for both positives and negatives. These are made extremely thin and are insulated from one another by very thin grooved wooden separators. They are then packed tightly into a hard rubber jar as

shown in Fig. 106. This jar is sealed in with an asphaltum compound to prevent the liquid splashing out. There is a hole in the top of the jar which is closed with a cap. This permits the replenishing of the electrolyte and a vent in the cap allows the gases to escape. The discharge rates of this type of battery must be high as, for example, when doing starting duty. Furthermore, the ampere-hour capacity of the battery for its weight and size must be large. Hence, the volume of the electrolyte is small and the specific gravity must vary between wide limits. When the battery is fully charged, the specific gravity is as high as 1.280 and 1.300, and when it is completely discharged, the specific gravity is as low as 1.100.

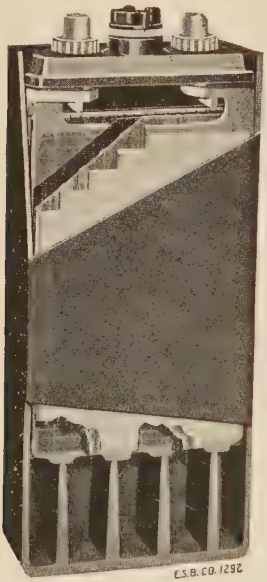


FIG. 106.—Assembly of an Exide vehicle cell.

The individual cells are mounted beside one another in boxes or crates and are connected together on top by lead connectors which may be burned or held by lead nuts. The number of cells in such a unit depends upon the voltage which is desired.

Vehicle batteries are usually shipped assembled, charged and complete with the electrolyte so that they are ready for use when received. However, a preliminary charge is advisable.

Because of its ruggedness, the "Exide Ironclad" (see Par. 101) is used to a large extent in electric vehicles.

As the space for the electrolyte is very limited in vehicle batteries, and considerable gassing occurs, the level of the electrolyte falls quite rapidly, so that frequent additions of water are necessary.

110. Rating of Batteries.—Practically all batteries have a nominal rating based on the 8-hr. rate of discharge. Thus, if a Planté battery can deliver a current of 40 amp. continuously for 8 hrs., the battery will have a rating of $40 \times 8 = 320$ amp.-hr. The normal charging rate of such a battery would be 40 amp.

Assuming the above battery is just capable of delivering 40 amp. for 8 hr., it would not be able to deliver 64 amp. for 5 hr. (= 320 amp.-hr.) but only 88 per cent. of this or 56.4 amp. for 5 hr. 56.4 amp. is called the 5-hr. rate.

Below is given a table showing the percentage capacity with various discharge rates.

	Hours				Minutes	
Discharge rate	8	5	3	1	20	6
Percentage of capacity at 8-hr. rate:						
Planté type	100	88	75	55.8	37	19.5
Pasted type	100	93	83	63	41	25.5

This falling off in capacity with higher rates of discharge is due to the inability of the free solution to penetrate the pores of the active material. Consequently it is not possible to reduce all the active material during the short periods of discharge. After such a battery has stood a short time it will be found to have recovered to some extent and is therefore capable of delivering more current, after apparently having become exhausted. This is due to the free solution finally penetrating the pores of the active material.

Batteries are able to discharge at enormous rates for very short intervals. For instance, a starting battery having an 8-hr. rating of 10 amp. is often called upon to supply 450 amp. when doing starting duty.

111. Charging.—The following rule may be observed in charging a lead battery: The charging rate in amperes may always be made equal to the number of ampere-hours out of the battery. For example, if 200 amp.-hrs. are out of the battery, a charging rate of 200 amp. may be used. As the battery charges, the amp.-hrs. out of the battery decrease and the charging rate must correspondingly decrease. The rate should never be such that violent gassing takes place.

Gassing represents a waste of energy because a considerable portion of the charging energy is used in merely breaking up the water into hydrogen and oxygen. In addition, gassing causes the battery to become heated, the acid is carried out in a fine spray by the bubbles and active material may be carried from the plates by the mechanical agitation of the bubbles.

When a battery is fully charged, any rate will produce gassing but the rate may be reduced to such a low value that the gassing

is not excessive and is practically harmless. This rate is called the "finishing rate."

A battery may be charged in 5 hr. by beginning at a rate several times the finishing rate and then tapering off to the finishing rate. On the other hand, a constant current of moderate value may be used continuing for a much longer time, even as long as 16 hr. (constant-current method).

A common example of the constant-current method is the charging of low-voltage batteries from 110-volt mains. This is illustrated by Fig. 107, which shows the charging of a 6-volt starting battery. It should be definitely determined that the mains

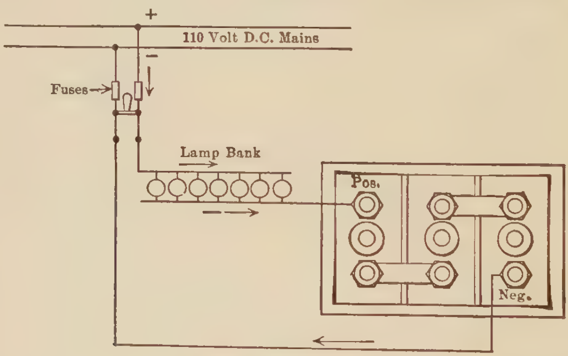


FIG. 107.—Charging a starting battery from 110-volt mains.

supply *direct* current and it is also necessary to know which main is positive. If doubt exists as to the polarity and a voltmeter is not available, dip the two ends of the wires which connect the mains to the battery into a glass of slightly acidulated water or in salt water. Bubbles form about the *negative* wire. When using the constant-current method of charging one must reduce the charging rate as the battery approaches the fully charged condition.

The constant-potential method of charging is to be preferred as the charging current automatically tapers off due to the rise in the cell electromotive force as the cell approaches the charged condition. The source of potential should be about 2.3 volts per cell when there is no series resistance in the circuit.

With 2.3 volts per cell and no series resistance, the current at the beginning of charge is usually too large, so that it is advisable

to use a low series resistance. If a series resistance is used, a voltage of 2.4 or 2.5 volts per cell is desirable, or else adjustments during the charging period must be made.

When a battery *floats* on constant-potential bus-bars, ready to take load as occasion demands, it is necessary to have a series booster for raising the charging potential to a value sufficiently high to force current into the battery. The booster ordinarily consists of a low-voltage, separately-excited shunt generator, driven by a shunt motor. Figure 108 shows the connections of the set when the battery is being charged. The booster raises the voltage just enough to send the necessary current into the battery.

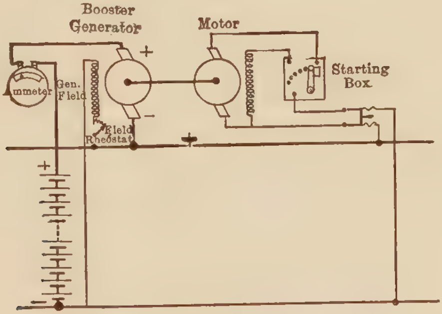


FIG. 108.—Booster method of charging a storage battery.

As an example, consider a 110-volt installation with a floating battery. As the average cell voltage is about 2 volts, 55 cells are necessary. Assume that the battery has a 320 amp.-hr. rating. The charging current will be 320/8 or 40 amp. (the normal 8-hr. rating). The voltage of each cell should be boosted to 2.3 volts on charge. Therefore the total voltage necessary will be 2.3 $\times$ 55 = 126.5 volts. Of this 126.5 volts, the bus-bars can supply 110 volts. The booster supplies the remaining 16.5 volts and its rating will be

$$\frac{16.5 \times 40}{1{,}000} = 0.66 \text{ kw.}$$

The *total* power utilized in charging the battery is, however,

$$\frac{126.5 \times 40}{1{,}000} = 5.06 \text{ kw.}$$

(Also see p. 464.)

The terminal voltage of a cell rises on being charged, as is shown in Fig. 109. The terminal voltage is about 2 volts at the beginning of charge and rises slowly to about 2.4 volts, after which it rises very rapidly to 2.6 volts. This last rise occurs in the gassing period. This final rise of voltage also indicates that the cell is nearing the completion of charge. It is this rise of voltage which automatically cuts down the charging rate when the constant-potential method is used. The voltage does not rise

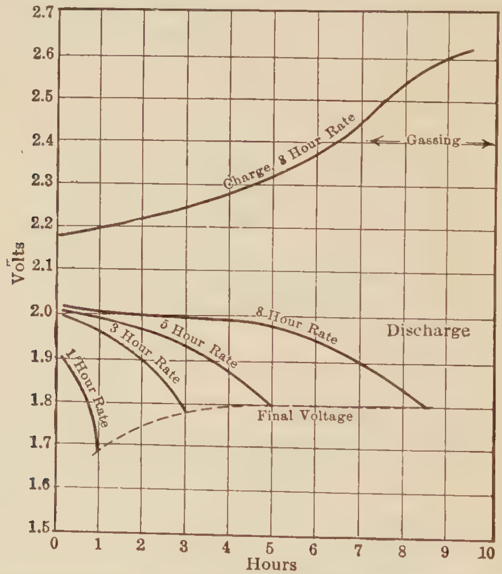


FIG. 109.—Voltage curves on charge and discharge for lead cell.

so rapidly when the charging rate is reduced toward the end of charge, because of the lesser IR drop in the cell itself. The *drop* of voltage at various rates of discharge is shown in Fig. 109. It will be noted that the battery voltage curve at the 8-hr. discharge rate is fairly flat, which is a very distinct advantage if the battery is used to supply incandescent lamps.

112. Battery Installations.—Batteries should be installed in dry, well-ventilated rooms. Small glass jars may be mounted on wooden racks painted with asphaltum paint (see Fig. 105). The jars are set in glass trays containing sand, which are in turn set on glass insulators. The larger battery jars should be set

on porcelain pedestals 6 in. or so above the floor. The floor should preferably be of acid-resisting material such as mastic asphalt. A plain cement floor is preferable to tile or vitrified brick as the acid seeps into the cracks which inevitably occur in the grouting when these last are used. All wooden surfaces should be covered with asphaltum paint. The room should be well ventilated, as the spray which is carried out of the jars on charge settles on horizontal surfaces and attracts other moisture. Therefore it is desirable to have a stream of air sweeping along the floor. As hydrogen gas is given off, no flame should be allowed in the room and no switches should be installed in the room. In addition to the danger of explosion caused by arcing at the switch contacts, the acid in the air will corrode the copper. Lead-covered insulated copper wire and lead lugs should be used for making connections.

113. Capacities and Weights of Lead Cells.—Below are given the relations of weights to kilowatt capacity for the various types of cells which have just been described.

Kilowatt Capacity (as Related to Weight) of Representative Cells Manufactured by

The Electric Storage Battery Co.

Size of plates	H	G	MV
Number of plates per cell	81	41	13
Type of plates	"Exide"	"Chloride Accumulator"	"Exide Ironclad"
Weight of plates in cell, pounds	2,102	886	28.5
[1] Weight of cell, pounds	3,660	1,864	42.75
Kilowatts—per cell:			
For 1 hr	10.80	2.80	0.235
For 4 hrs	4.14	1.23	0.0895
For 8 hrs	2.41	0.772	0.0541
Kilowatts—per pound of plates:			
For 1 hr	0.00513	0.00315	0.00825
For 4 hrs	0.00197	0.00138	0.00314
For 8 hrs	0.00115	0.000871	0.00190
Kilowatts—per pound of cell:			
For 1 hr	0.0029	0.00150	0.00549
For 4 hrs	0.001096	0.000659	0.00209
For 8 hrs	0.000638	0.000414	0.00127

[1] The necessary insulating supports for the H and G cells and tray for the MV cell are not included—these add approximately 2 and 10 per cent. respectively to the cell weights.

THE EDISON BATTERY

114. The Nickel-iron-alkaline Battery.—Instead of using acid as an electrolyte, the Edison cell uses an alkali, consisting of a 21 per cent. potassium hydrate solution. The positive plate consists of nickel pencils about ¼ in. diameter and 4½ in. long, filled with green nickel oxide. As the nickel oxide is a very poor electrical conductor, very fine metallic nickel flakes

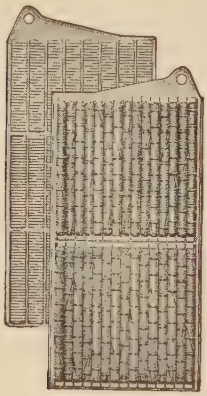


Fig. 110.—Positive and negative plates of an Edison storage cell.

Fig. 111.—Assembly, Edison battery plates removed from container.

are mixed with it to produce sufficient conductivity. The negative plate consists of flat perforated nickel-plated steel stampings, containing iron in a very finely divided form. These flat pockets are mounted on a nickel-plated steel frame for support. Both the positive and the negative plates are shown in Fig. 110.

The chemical reaction in the cell is complex, but its nature is indicated by the following chemical equation:

Type H "Exide" batteries used in central station stand-by service. Largest battery 150 cells, 169 plates per cell, capacity 3,460 kw. for 1 hr.; two other batteries 3,420 kw. each.

Type G "Chloride Accumulator" used in power plants for peak, regulating and exciter bus service, also in telephone exchanges, large isolated plants, etc. Largest battery 288 cells, 85 plates per cell, capacity 1,700 kw. for 1 hr., two other batteries same size and capacity.

Type MV "Exide Ironclad" used in electric vehicles, locomotives, industrial trucks and tractors, and for yacht lighting, etc.

Positive Plate *Negative Plate*

$$8KOH + 6NiO_2 + 3Fe = Fe_3O_4 + 2Ni_3O_4 + 8KOH$$

The above read from left to right indicates discharge, and read from right to left indicates charge. It is to be noted in the above reaction that the same quantity of potassium hydrate solution (KOH) appears on both sides of the equation. This indicates that ultimately all the reaction occurs between the electrodes themselves, and also that no water is formed. Therefore the specific gravity of the solution does not change during charge or discharge.

The plates all have a perforated lug by which they are fastened together with a steel bolt and to a binding post. The bolt is

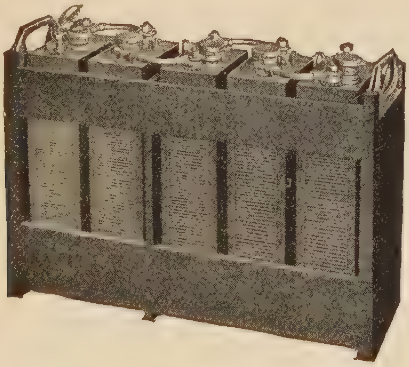


Fig. 112.—Five Edison storage cells mounted in a tray.

threaded and steel nuts clamp the plates together. Steel washers between the plates act as spacers. The positive and negative plates are insulated from one another by hard rubber grids. An Edison cell assembly is shown in Fig. 111. The positive and negative assembly is placed in a corrugated, nickel-plated, welded steel tank. The top is then welded to the rest of the container. The binding posts are insulated from the cover by hard rubber bushings. In the top is a valve which allows the gases to escape during charge and through which water may be added to the electrolyte. This valve should never be allowed to become so encrusted with a potash deposit that it sticks, because the internal pressure may become sufficient to cause the sides of the container to bulge.

The individual cells are usually mounted in wooden racks, as shown in Fig. 112, the cells being connected together by nickel-plated steel connectors.

115. Charging and Discharging.—The Edison cell is rated on the basis of a 5-hr. charging rate. Figure 113 shows typical charge and discharge curves for the Edison battery. It will be noted that the average voltage on discharge is about 1.2 volts per cell. The specific gravity of the electrolyte changes but slightly so that it cannot be used to indicate the condition of charge, as with the lead cell. Moreover, there is no sharp voltage rise near the completion of charge. If doubt exists as to the condition of charge, it is advisable to give an overcharge in order

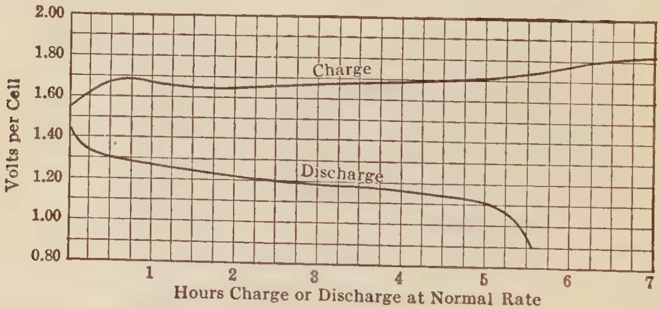


FIG. 113.—Voltage changes during the charge and discharge of an Edison cell

to be on the safe side. The overcharge does not injure the cell although it may slightly reduce the efficiency.

The electrolyte in an Edison cell evaporates rapidly and frequent additions of water are necessary. As the electrolyte is changed to potassium carbonate very readily, only freshly distilled water should be used in replacing the electrolyte, as tap water usually contains carbonates in solution. In spite of the usual precautions, the electrolyte is slowly converted into potassium carbonate by contact with the air, and it should be replaced by fresh electrolyte every 250 complete cycles of charge and discharge.

The advantages of the Edison battery are its ruggedness, long life, and small weight per watt-hour capacity. It will withstand, without injury, severe vibration, shock, short-circuits, and

standing idle for long periods in any condition of charge or discharge. The plates do not buckle and the active material does not "flake" or drop from the plates.

116. Applications.—Edison cells are used for vehicle lighting and ignition, and are also much used in motor boats. They are also used in various types of electric trucks and for battery street cars. In automobiles they are not generally used for starting, as their comparatively large internal resistance does not permit a sufficiently high discharge rate.

Below is given the relation between the battery weight and capacity.

The figures are based upon the capacity obtainable on normal charge:

Capacity ampere-hours	Positive plates	Weight pounds per cell		Watt-hours per pound	
		Single cell only	With tray connectors	Net	Gross
37.5	2	4.5	5.1	9.6	8.5
75	4	7.8	8.5	11.1	10.2
112.5	6	11.3	12.1	11.5	10.7
150	4	13.8	14.9	12.5	11.6
300	8	26.8	29.0	12.9	11.9

117. Efficiency of Storage Batteries.—The efficiency of a storage battery is the ratio of the watt-hour output to the watt-hour input.

For example, a fully charged cell is discharged at a uniform rate of 38 amp. for 6 hr. at an average voltage of 1.95 volts. The cell is then charged at a uniform rate of 40 amp. for 6 hr., the average voltage being 2.3 volts. The cell is then in its original condition of charge. What is the efficiency of this cell?

$$\text{Watt-hours output} = 38 \times 1.95 \times 6 = 445.$$

$$\text{Watt-hours input} = 40 \times 2.3 \times 6 = 552.$$

$$\text{Efficiency} = \frac{445}{552} \text{ or } 80.7 \text{ per cent.}$$

One often hears of the ampere-hour efficiency of a storage battery. As amperes do not represent energy, the ampere-hour efficiency is not a measure of a battery's ability to store energy.

In the above example the ampere-hour efficiency may be found as follows:

$$\text{Ampere-hours output} = 38 \times 6 = 228.$$
$$\text{Ampere-hours input} = 40 \times 6 = 240.$$
$$\text{Ampere-hour efficiency} = \frac{228}{240} \text{ or } 95 \text{ per cent.}$$

The much lower watt-hour efficiency is due to the difference between the voltage of charge and that of discharge, as shown in Figs. 109 and 113, pages 128 and 132.

The efficiency of a storage battery varies with the rate, both of charge and discharge, and somewhat with the temperature. As high charge and discharge rates produce relatively high I^2R and polarization losses, the efficiency is lowered under these conditions. Further, a cell may be charged at the 8-hour rate and discharged at the 3-hr. rate and have an apparent efficiency of 60 per cent. This does not represent the true efficiency as the cell actually will not be completely discharged, even though it appears to be. Owing to the inability of the free acid to permeate the active material, much of the active material has not been reduced, and after a short time the cell will be found to have recuperated to a considerable extent and to be able to deliver more energy.

The ampere-hour efficiency of a storage battery is of the order of magnitude of 95 per cent. For a complete cycle the watt-hour efficiency of a stationary battery of moderate size is about 75 per cent. at the 8-hr. charge and discharge rates. The watt-hour efficiency of a large stationary battery is about 85 per cent. under the same conditions. Where a battery merely "floats" and the cycle of charge and discharge is a matter of minutes or perhaps of seconds even, the watt-hour efficiency may be as high as 95 or 96 per cent.

The ampere-hour and the watt-hour efficiency for the Edison cell are less than for the lead cell. This is due partly to the fact that the Edison cell has a lower electromotive force and the IR drop is proportionately greater. For the Edison cell the ampere-hour efficiency is about 82 per cent. and the watt-hour efficiency about 55 per cent.

In selecting a battery, the efficiency is but one of the factors to be considered. The first costs and the maintenance of batteries

are high so that these factors, as well as the efficiency, should be given due consideration.

Note.—The uses of storage batteries in the generation and distribution of power are considered in Chap. XIV.

118. Electroplating.[1]—Electroplating is a very important electrical industry and is closely related to the subject of batteries. The principle is very simple. Assume that it is desired to copper plate a carbon dynamo brush. The portions of the brush to be plated are immersed in a solution of copper sulphate as shown in Fig. 114. A copper strip is also immersed in the solution and is connected to the + terminal of a dynamo or some other source of direct-current supply. The article to be plated is connected

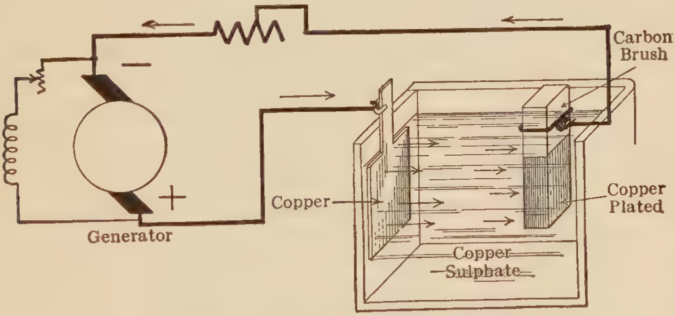


Fig. 114.—Copper plating bath.

to the negative terminal of this supply. Under these conditions the current will carry copper from the solution and deposit it on the carbon brush. This copper which leaves the solution is replaced by copper which is carried from the copper strip (the anode) into the solution so that there is no change in the solution itself. The current should be such that the density is about 0.02 amp. per square inch of the surface to be plated.

It is not necessary that the anode be of the metal which it is desired to deposit. Other metals may be used. Under these conditions, however, the solution in time becomes contaminated by the going into solution of the anode. If an inert substance such as carbon is used, as anode, acid is formed in the solution.

[1] See "Standard Handbook," 5th Ed., Sec. 19, Pars. 186 to 206, for a more complete discussion.

The *only opposing* electromotive force in the bath just described is the IR drop in the solution. This may be reduced by bringing the electrodes close together, but if the electrodes are too close together the deposit will not be uniform. The amount of metal deposited per second is proportional to the current. Because of the nature of electroplating baths, they are naturally low-voltage devices. When practicable, several are connected in series. A low voltage and high current generator is generally used for plating purposes. In practice there are many refinements to be observed.

Acid is added to the solution to prevent impurities from depositing. A cyanide solution of copper is found to give better results than the sulphate. Nickel, tin, zinc, silver, gold, etc., may be deposited by the use of suitable baths and electrodes.

A gravity cell is an example of electroplating in which the source of current is derived from the bath itself. The current flows from the zinc to the copper within the solution, zinc is carried into the solution as sulphate and copper is deposited or plated from its sulphate on the positive electrode.

Electrotyping is another common example of electroplating. An impression is made in wax with the type or object to be reproduced. The surface of the wax is made conducting by applying a thin coat of graphite. Copper is then plated on this surface. It is later backed by type metal to give it the necessary mechanical strength.

CHAPTER VII

ELECTRICAL INSTRUMENTS AND ELECTRICAL MEASUREMENTS

119. Principle of Direct-current Instruments.—If a coil like that shown in Fig. 115 carries a current, a magnetic field results (Chap. II) with a north and a south pole at opposite ends of the coil. If the coil when carrying current be placed in a magnetic field, the coil will tend to turn in such a direction that:

The ampere-turns of the coil will increase the strength of the magnetic field and the total flux of the system will be a maximum (see Par. 17, Chap. I).

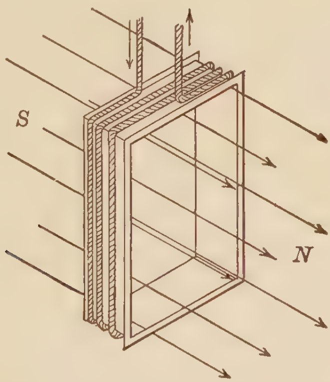


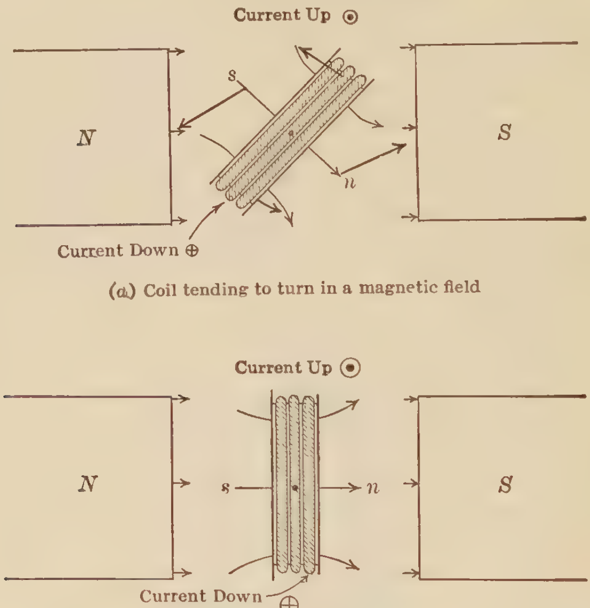
FIG. 115.—Magnetic field produced by an instrument coil.

The north pole of the coil will be attracted toward the south pole of the magnetic field and the south pole of the coil will be attracted toward the north pole of the magnetic field.

This tendency of the coil to turn is shown in Fig. 116 (*a*) where the coil attempts to turn in the direction indicated by the arrows. If the coil is pivoted and free to turn it will reach the position shown in Fig 116 (*b*). Under these conditions the coil has placed itself in such a position that its flux is acting in the same

direction as that of the main field. Also the unlike poles are as near each other as possible and the like poles are as far away from each other as possible.

This behavior of such a coil carrying a current and placed in a magnetic field should be thoroughly understood, for it is the underlying principle of most current-measuring instruments and is in addition the principle upon which all electric motors operate.



(a) Coil tending to turn in a magnetic field

(b) Ultimate position of coil

Fig. 116.—Turning moment of an instrument coil.

120. The D'Arsonval Galvanometer.—A galvanometer is a sensitive instrument used for measuring and detecting small electric currents. The D'Arsonval galvanometer, which is based on the principle of a coil turning in a magnetic field, is the most common type of galvanometer. Due to its simplicity, it has superseded practically all other types. In addition it is comparatively rugged and is not appreciably affected by stray magnetic fields. Figure 117 shows the principle of its construction. A coil of very fine wire is suspended between the poles of a permanent magnet by means of a filament, usually a flat strip

of phosphor-bronze. The coil may be wound with or without a bobbin. The bobbin is usually of fiber, or of aluminum. The advantage of an aluminum bobbin will be considered later.

Between the poles of the magnet a soft-iron core is usually placed (Figs. 117 and 118). The addition of this core results in two distinct advantages. The length of the air path is reduced so that the amount of flux linking the coil is increased, thus making the galvanometer more sensitive; the flux tends to enter the core radially. This last effect makes the deflections of the galvanometer almost directly proportional to the current flowing in the galvanometer coil.

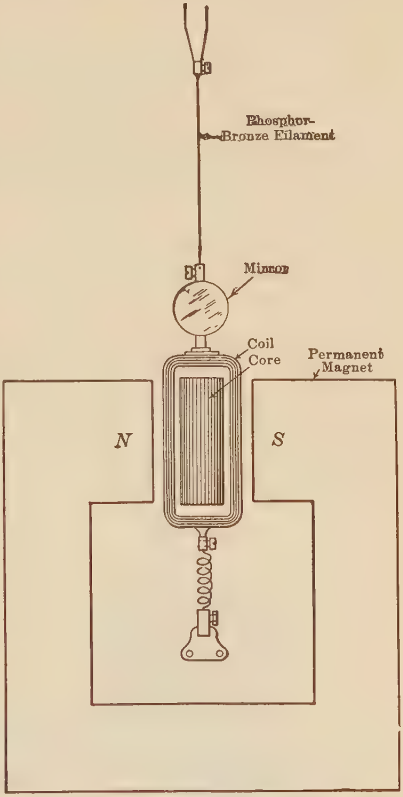


Fig. 117.—Principle of the D'Arsonval galvanometer.

The coil is usually suspended by a phosphor-bronze filament. Any turning of the coil produces torsion in the filament which opposes the turning of the coil and is called the restoring force. When the moment of the restoring force and the turning moment due to the current are equal, the galvanometer assumes a steady deflection. For all practical purposes the galvanometer deflection is proportional to the current. This phosphor-bronze filament usually serves as one of the leading-in wires which carry current to the coil. The other leading-in wire consists of a very flexible spiral filament fastened to the bottom of the coil, as shown in Fig. 117.

There are two common methods of reading the deflection of a galvanometer. A plane mirror is mounted on the coil system

and a scale and telescope are mounted about ½ m. from the galvanometer. The reflection of the scale in the mirror can be seen with the telescope (Fig. 119). When the mirror turns, the reflection of the scale in the mirror deflects. The value of this deflection is determined by means of a cross hair in the telescope.

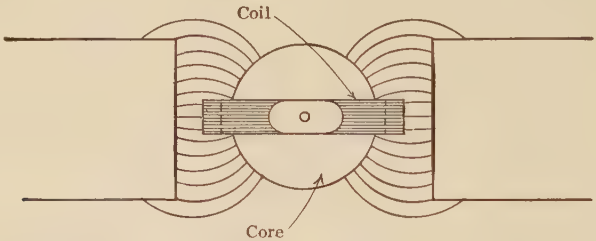


Fig. 118.—Effect of core upon the magnetic field of a galvanometer.

Another method is to use a concave mirror on the galvanometer moving system. A lamp filament is placed some distance from the mirror and its image is focused on a ground glass to which a scale graduated in centimeters is fastened. As the mirror deflects, the beam of light travels across the scale.

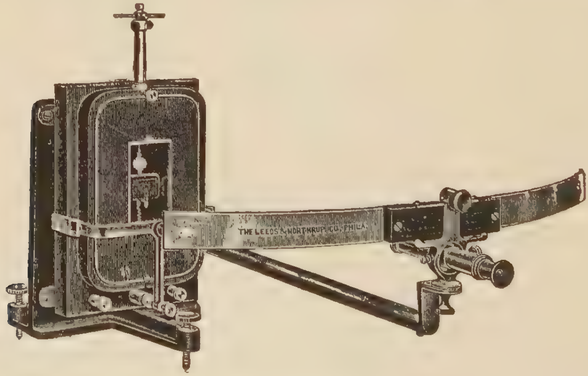
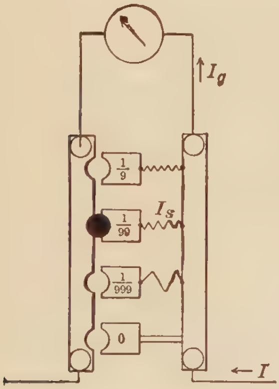


Fig. 119.—Telescope and scale method of reading a galvanometer.

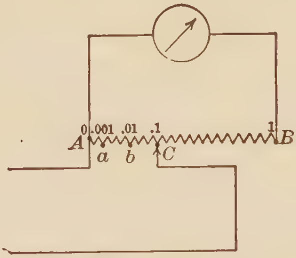
Damping.—If a galvanometer coil, which is hung freely, starts to swing, it will continue swinging for some time unless it is in some way retarded or damped. One method of damping is to attach an air vane to the coil. This air vane is enclosed so

that it swings in a restricted space and damps any swinging movement of the coil. The most satisfactory method is electrical damping. If the coil be wound on an aluminum bobbin, the motion of the bobbin through the magnetic field will induce currents within itself, and these will be in such a direction as to put an electric load on the moving coil as in an electric generator. This opposes the motion of the coil. The same result may be obtained by binding short-circuited copper coils on the main coil, by shunting the galvanometer externally with a resistance (see Ayrton Shunt, Par. 121), or even by short-circuiting it.

121. Galvanometer Shunts.—When galvanometers are used to detect small currents as in null methods (see Wheatstone



(a) Galvanometer shunt



(b) Ayrton shunt

FIG. 120.—Types of galvanometer shunt.

Bridge), the apparatus may be so far out of adjustment that a comparatively large current flows through the galvanometer. This causes a violent deflection of the coil, and may result in injury to the galvanometer. In certain other measurements the current that it is desired to measure with the galvanometer may be so large that the deflection is considerably beyond the scale.

In either case the galvanometer may be protected by the use of a shunt, or a resistance which by-passes a certain known proportion of the current from the galvanometer. There are two common types of shunt. One type is shown in Fig. 120 (a).

It consists of three or four separate resistances which are plugged across the galvanometer one at a time. These are so adjusted in value that with a given current to be measured the successive galvanometer currents are in the ratio of 10 to 1. For example, if the galvanometer is to measure $\frac{1}{10}$ the external current, the top resistance, (Fig. 120 (a),) is of such a value that it shunts nine-tenths of the current away from the galvanometer when it is plugged across the galvanometer, etc.

To determine the values of these resistances proceed as follows:

Let R_g = galvanometer resistance.
I_g = galvanometer current for full-scale deflection.
I = circuit current.
I_s = shunt current.
R_s = shunt resistance.

To reduce the galvanometer deflection to one-tenth the value which it would have if all of the current I passed through the galvanometer, I_g must be one-tenth of I. That is,

$$\frac{I_g}{I} = \frac{1}{10}. \tag{1}$$

The shunt current

$$I_s = I - I_g. \tag{2}$$

But the shunt current and the galvanometer current are inversely as their respective resistances. Hence:

$$\frac{R_g}{R_s} = \frac{I_s}{I_g} = \frac{I - I_g}{I_g}. \tag{3}$$

as

$$I_g = \frac{I}{10} \text{ from Eq. (1).}$$

$$\frac{R_g}{R_s} = \frac{I - I/10}{I/10} = 9.$$

$$R_s = \frac{1}{9}R_g.$$

For a reduction of 100 to 1

$$\frac{R_g}{R_s} = \frac{I - I/100}{I/100} = 99.$$

$$R_s = \frac{1}{99} R_g.$$

Example.—A galvanometer has a resistance of 600 ohms. What resistances should be used to shunt it in order to reduce its deflections in the ratio of 10 to 1 and 100 to 1?

$$R_1 = \frac{600}{9} = 66.7 \text{ ohms.} \quad Ans.$$

$$R_2 = \frac{600}{99} = 6.06 \text{ ohms.} \quad Ans.$$

Ayrton Shunt.—The *Ayrton shunt* is shown in Fig. 120 (*b*). A permanent resistance AB is connected across the galvanometer terminals. One line terminal, A, is permanently connected to one end of this resistance, and the other line terminal, C, is movable and can be connected to various points along AB. With a fixed line current the maximum deflection is obtained when C is at B. If point C be moved to a, where resistance Aa is 1/1,000 of the total resistance AB, the galvanometer deflection will be 1/1,000 of its maximum value. If C be moved to b, where Ab is $\frac{1}{100}$ of the resistance AB, the galvanometer deflection will be $\frac{1}{100}$ of its maximum value, etc.

The advantages of the Ayrton shunt are:

1. The shunt is applicable to any galvanometer, regardless of the galvanometer resistance.

2. A fixed resistance is shunted across the galvanometer, which gives a constant value of damping in open-circuit ballistic measurements (see Par. 186).

When the shunt is adjusted to give the maximum galvanometer deflection, this deflection for the same value of external current is less than it would be were the shunt not used. That is, the maximum sensitivity of the galvanometer is reduced by the addition of the shunt. If the shunt has a resistance of only five times that of the galvanometer the sensitivity will be reduced only in the ratio of 6 to 5, which is not usually objectionable.

122. Ammeters.—An ammeter is an electrical instrument which measures the current flowing in an electric circuit.

There were many early types of ammeters, most of which depended for their operation upon the pull exerted by a solenoid on some type of iron plunger. The amount of pull is dependent upon the current strength in the solenoid, so by restraining the motion of the plunger by gravity or by means of a spring, the deflection of a pointer attached to the plunger might be made to read amperes. Figure 121 shows a typical instrument of

this class. Such an instrument is inaccurate, due: (1) To magnetic hysteresis or lag, which for a given current results in a higher reading for decreasing values of current than for increasing values. (2) The weight of the plunger makes it impossible to mount the moving system so that the friction error is negligible. (3) The instrument is not damped, and fluctuates violently on slightly fluctuating loads.

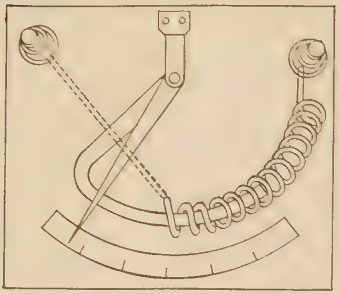


Fig. 121.—Early type of plunger ammeter.

For direct-current measurements, the Weston instrument, developed by Edward Weston, has come into almost universal use. The instrument is based on the principle of the D'Arsonval galvanometer, but it is so constructed that it is easily portable and it is provided with a pointer and scale for indicating the deflections of the moving coil.

The essential parts of the instrument are shown in Fig. 122. As in the D'Arsonval galvanometer, a permanent magnet is necessary, being made in horseshoe form. Two soft-iron pole-pieces are fitted to the magnet poles and a cylindrical core is held between these pole pieces by a strip of brass. This gives uniform air-gaps and a radial field. The length of the air-gap is very much shorter than is usual with D'Arsonval galvanometers. The moving coil consists of very fine silk-covered copper wire wound on an aluminum bobbin. The aluminum bobbin, besides supporting the coil mechanically, also makes the instrument highly damped. This damping is due to the currents set up in the aluminum because of its cutting the magnetic field.

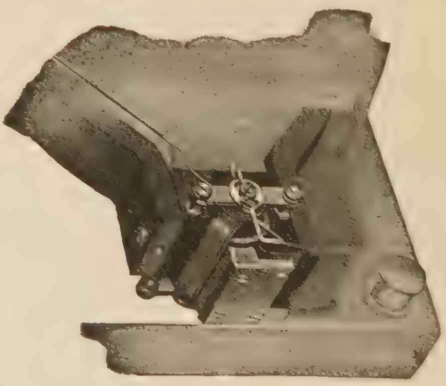


Fig. 122.—Movement of a Weston instrument.

Instead of suspending the coil by a filament, it is supported at the top and bottom by hardened steel pivots turning in cup-shaped jewels, usually sapphire. This method of supporting the moving coil is almost frictionless and makes the instrument portable, whereas the D'Arsonval galvanometer is not so. The current is led in and out of the coil by two flat spiral springs, one at the top of the coil and the other at the bottom. These springs also serve as the controlling device for the coil. That is, any tendency of the coil to turn is opposed by these two springs. The top and the bottom springs are coiled in opposite directions so that the effect of change of temperature, which causes a spiral spring to coil or uncoil, will not cause the needle to change its zero position. A very light and delicate aluminum pointer is attached to the moving element to indicate the deflection of the coil. This is carefully balanced by very small counterweights so that the whole moving element holds its zero position very closely, even if the instrument is not level. The pointer moves over a graduated scale, which may be marked in volts or in amperes as the case may be. Because of the radial field, the deflection of the moving coil in this type of instrument is practically proportional to the current in the moving coil, so that the scale of the instrument has substantially uniform graduations, which is desirable. The internal connections of a Weston instrument are shown in Fig 123.

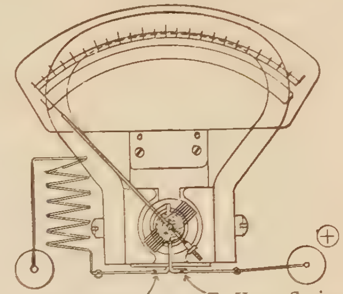


FIG. 123.—A typical Weston direct-current millivoltmeter.

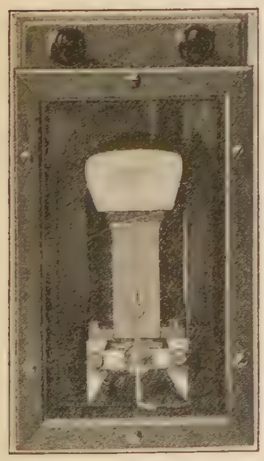


FIG. 124.—Weston portable galvanometer.

Instruments of this construction having very weak springs are often used for portable galvanometers. Although lacking the

extreme sensitivity of the suspended type, they can be made sufficiently sensitive for certain classes of work and their ruggedness and portability make them very useful. Such a galvanometer is shown in Fig. 124.

123. Ammeter Shunts.—The moving coils of Weston portable ammeters deflect to the full scale value with from 0.02 to 0.05 amp. in the coil. Therefore, to measure currents greater than this, the larger portion of the current must be diverted from the moving coil by a *shunt*. The shunt is merely a low resistance, usually made of manganin strip M brazed to comparatively heavy

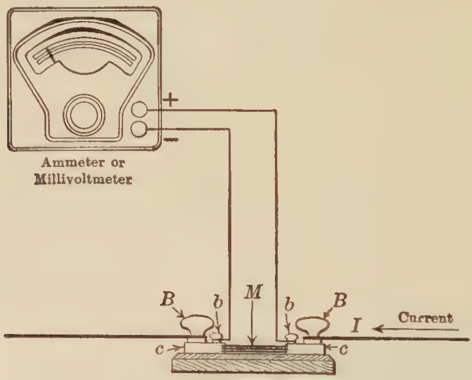


FIG. 125.—Ammeter with an external shunt.

copper blocks (c) as shown in Fig. 125. Two sets of binding nuts are fastened to the copper blocks. The heavy wing nuts BB are for carrying the *main* current through the shunt. The small posts bb are used to connect the ammeter leads. The copper blocks serve two purposes. They are an excellent conductor of heat, so carry the heat away from the manganin strip, and their low resistance keeps all parts of each copper block at very nearly the same potential. The ammeter is in reality a voltmeter reading the voltage drop across a resistance. A complete set of shunts, with their current ratings, is shown in Fig. 126. The heavy copper terminals for connection to bus-bars should be noted.

The voltage drop across the shunt is

$$V_{sh} = I_{sh}R_{sh},$$

where I_{sh} and R_{sh} are the shunt current and the shunt resistance respectively. If R_{sh} is constant, the voltage drop across the shunt is proportional to the current in the shunt, so that the instrument readings are proportional to the current in the shunt. For this reason the ammeter itself (Fig. 123) is often marked "Millivoltmeter." For full-scale deflection the drop across a shunt is about 50 millivolts. The current taken by the instru-

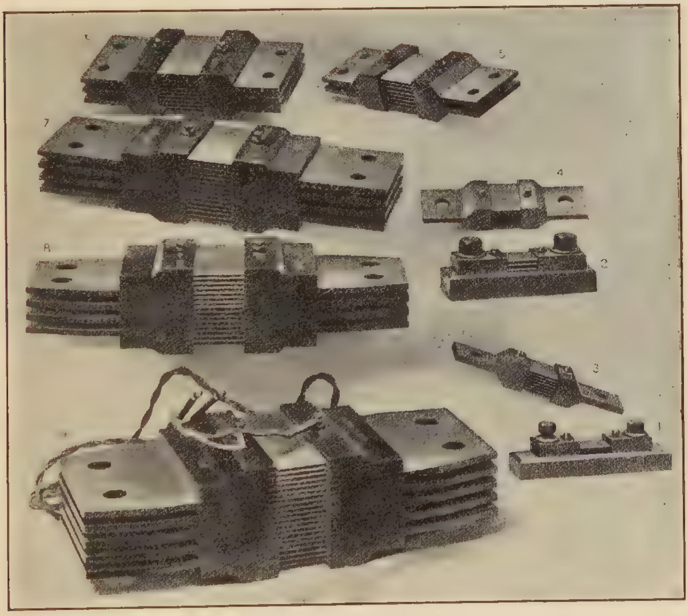


Fig. 126.—Ammeter shunts.

No. 1 from 25 to 200 amp.
No 9 from 4,500 to 6,000 amp.

ment itself is usually from 0.02 to 0.05 amp. so that it is almost always negligible as compared with the main current. Therefore, in most cases the line current equals the shunt current, practically.

An ammeter and its shunt may also be considered as a divided circuit. In Fig. 127 let R_{sh} and I_{sh} be the shunt resistance and the shunt current respectively, and let R_m and I_m be the instru-

ment resistance and the instrument current respectively. By the law of divided circuits:

$$\frac{I_{sh}}{I_m} = \frac{R_m}{R_{sh}}.$$

That is, the current divides between the instrument and the shunt inversely as their resistances.

Example.—Assume that an instrument has a resistance of 4 ohms, the shunt a resistance of 0.0005 ohm, and that the line current is 90 amp. What is the value of the instrument current?

As the current in the line differs from the shunt current by a very small amount, the two may be assumed equal. Then,

$$\frac{90}{I_m} = \frac{4}{0.0005}$$

$$I_m = 0.0113 \text{ amp.} \quad \textit{Ans.}$$

For accuracy, the current must always divide between the instrument and the shunt in a fixed ratio. This means either that the resistance of the shunt and the resistance of the instrument must not change at all or that both must change in the same ratio. As the shunt operates at a higher temperature than the instrument, it should be made of a metal whose resistance does not change appreciably with the temperature, such as manganin (see p. 43). The resistance of the instrument circuit should also remain constant. The resistance of the leads connecting the shunt to the instrument should remain constant and the leads with which the instrument is calibrated should always be used to connect the shunt to the instrument. The lugs and binding post contacts should be kept clean from oxide and dirt. A low adjustable resistance (the spiral, Fig. 123) is connected inside the instrument. By varying this resistance the instrument is adjusted to its shunt.

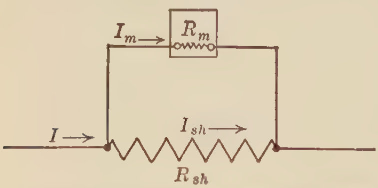


FIG. 127.—Division of current between an ammeter and its shunt.

An ammeter with an external shunt may be made to have a large number of scales or ranges. Assume that an instrument gives full-scale deflection when the instrument current is 0.02 amp. and that its resistance is 2.5 ohms. The resistance of the

shunt is 0.0005 ohm. The volts across the instrument terminals are $0.02 \times 2.5 = 0.050$ volt or 50 millivolts. Dividing this voltage by the shunt resistance, the shunt current is

$$I = \frac{0.05}{0.0005} = 100 \text{ amp.}$$

The instrument then deflects full scale with 100 amp. in the line.

If a shunt having a resistance of 0.005 ohm be substituted, the 50 millivolts drop across the shunt may be obtained with 10 amp. ($10 \times 0.005 = 0.050$). Therefore a 10-scale ammeter results. By the choice of suitable shunts the same instrument may be made to give full-scale deflection with 1 amp., and with 5,000 amp. For example, all the shunts shown in Fig. 126 could be used with the same instrument and as many different scales thereby obtained.

In the smaller sizes of instruments up to 50 amp. and where only one scale is desired, the shunt is usually placed within the instrument. For ranges between 50 and 100 amp. the use of an internal or an external shunt is optional. Above 100 amp. it is usual to have the shunt external to the instrument on account of its size and its heating loss.

An ammeter can usually be distinguished from a voltmeter by the fact that its binding posts are heavy and are of bare metal, except in the case of an instrument having an external shunt. The posts of millivoltmeters and voltmeters are of much lighter construction and the metal posts are covered with hard rubber, mostly for insulation purposes.

Any instrument when connected in a circuit should disturb the circuit conditions as little as possible. An ammeter shunt, as it goes in series with the line, should have as low a resistance as is practicable, so that when it is connected, very little additional resistance is introduced into the circuit. To protect ammeters from heavy currents, etc., provision may be made for *short-circuiting* them when readings are not being taken.

124. Voltmeters.—The construction of a voltmeter does not differ materially from that of an ammeter in so far as the movement and magnet are concerned (see Fig. 122). The moving coil of the voltmeter is usually wound with more turns and of finer wire than that of the ammeter and so has a higher resist-

ance. The principal difference, however, lies in the manner of connecting the instrument to the circuit. As a voltmeter is connected directly across the line to measure the voltage, it is desirable that the voltmeter take as little current as is practicable. Because of its comparatively low resistance, the moving coil of the voltmeter cannot be connected directly across the line, as it would ordinarily take an excessive current and might be burnt out. Therefore it is necessary to connect a high resistance in series with the moving coil. This is shown in Fig. 128. By Ohm's law the current through the instrument is proportional to the voltage, so that the instrument scale can be graduated in volts. The resistance required is easily determined. Assume

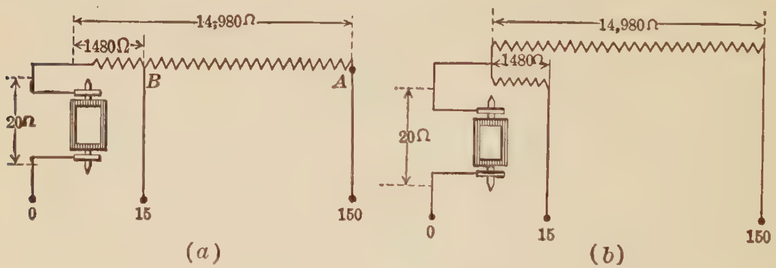


FIG. 128.—Methods of connecting resistance in a voltmeter.

that an instrument gives full-scale deflection with 0.01 amp. in the moving coil, and that the coil resistance is 20 ohms. If it is desired that the instrument indicate 150 volts, full scale, the total resistance of the instrument circuit must be

$$R = \frac{V}{I} = \frac{150}{0.01} = 15{,}000 \text{ ohms.}$$

As the instrument has a resistance of 20 ohms, this means that 14,980 ohms additional are necessary (Fig. 128 (a)).

If it be desired that this same instrument also have a full-scale deflection with 15 volts, the resistance of 14,980 ohms may be tapped so that the resistance OB (Fig. 128 (a)) $= 15/0.01 =$ 1,500 ohms, and this tap can be brought to a binding post. Another method of securing the same result is shown in Fig. 128 (b). Wind another resistance equal to $1{,}500 - 20 = 1{,}480$ ohms and connect it from a binding post to the junction of the resistance and the moving coil. This last method is advantage-

ous as it permits independent adjustment of each resistance; also injury or repair in one resistance does not affect the other.

125. Multipliers or Extension Coils.—The range of a voltmeter having its resistance incorporated within the instrument, may be increased by the use of external resistance connected in series with the instrument.

Example.—A 150-scale voltmeter has a resistance of 17,000 ohms. What external resistance should be connected in series with it so that its range is (*a*) 300 volts? (*b*) 600 volts?

(*a*) In order to maintain the same current through the instrument at 300 volts as flows at 150 volts, the resistance of the circuit must be doubled.

Therefore, a total resistance of

$$17{,}000 \times 2 = 34{,}000 \text{ ohms is necessary.}$$

As the instrument already has 17,000 ohms, the added resistance will be,

$$34{,}000 - 17{,}000 = 17{,}000 \text{ ohms.} \quad \textit{Ans.}$$

(*b*) The total resistance must now be,

$$\frac{600}{150} \times 17{,}000 = 68{,}000 \text{ ohms.}$$

As 17,000 ohms is already within the instrument, 68,000 − 17,000 = 51,000 ohms must be added external to the instrument. *Ans.*

External resistances used in this manner are called *multipliers*, or sometimes *extension coils*. They are usually placed within a perforated box and the terminals brought out to binding posts. The multiplying power of the multiplier is marked near a terminal.

The equation giving the relation among the resistance of the multiplier R_x, the resistance of the instrument R_m, and the multiplying power M is as follows:

$$M = \frac{R_x + R_m}{R_m} \qquad (49)$$

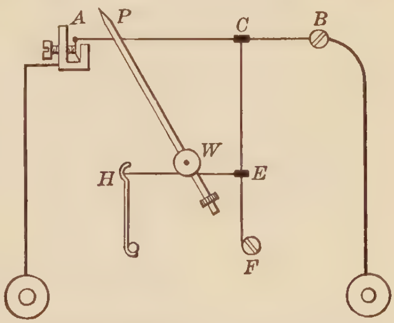


Fig. 129.—Principle of Hartmann and Braun hot-wire instruments.

Example.—In the above example (*b*) the multiplying power of the multiplier is as follows:

$$M = \frac{51{,}000 + 17{,}000}{17{,}000} = 4.$$

126. Hot-wire Instruments.—In the instruments heretofore considered, the action of the instrument depends on the electro-

magnetic action of the current. There is another type of instrument which depends for its indications upon the *heating* action of the current. A diagram of this instrument is shown in Fig. 129. AB is a fine wire of platinum-silver through which the current passes. At C, a wire CF is attached to AB. At E, on CF, a silk fiber EH is attached. This passes around the pulley W, and is held in tension by the spring H. When a current flows through AB, the heat expands the wire AB, reducing the tension in the wire CF, and allowing the spring H to pull the silk fiber to the left. This fiber, acting on the pulley W, moves the pointer P over the scale.

When used as an ammeter, a shunt is necessary unless the current is very small. When used as a voltmeter, a high resistance is connected in series with the wire AB.

This type of instrument is "dead beat," that is, it is very sluggish in its behavior and only reaches its ultimate deflection after the lapse of considerable time. This is an advantage in the measurement of fluctuating currents, as the needle follows the fluctuations very slowly and can be accurately read. Another advantage of the hot-wire type of instrument is that it can be used for alternating as well as for direct currents. It is often used as a transfer instrument to measure alternating currents in terms of direct current. This type of instrument is particularly useful for the measurement of high-frequency alternating currents, as its indications are independent of the frequency if a shunt is not used. For this reason this type is very useful in radio telegraphy. Such instruments are affected by temperature and do not hold their calibration for very long periods. Therefore, for accurate work they should be calibrated at the time of using.

ELECTRICAL MEASUREMENTS

Measurement of Resistance

127. Voltmeter-ammeter Method.—The resistance of any portion of an electric circuit which does not contain a source of electromotive force is, by Ohm's law,

$$R = \frac{V}{I}$$

where V is the voltage across that portion of the circuit and I is the steady current flowing in that portion of the circuit. Obviously, the voltage V may be measured with a voltmeter, the current I measured with an ammeter, and the resistance R computed.

Let it be required to determine the resistance R in the circuit shown in Fig. 130. The source of power is the 110-volt supply. The resistance R is comparatively small and if connected directly across 110 volts would take an excessive current. Therefore, it is necessary to insert a resistance R' in series with R to limit the current. The voltmeter, however, must be connected directly across R as it is desired to know the resistance of this portion of the circuit only.

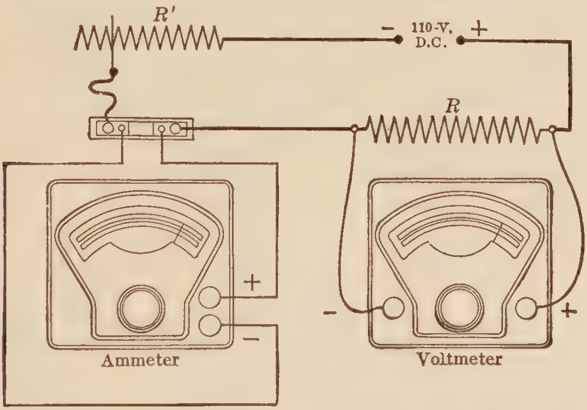


Fig. 130.—Voltmeter-ammeter method of measuring resistance.

Example.—The voltmeter (Fig. 130) reads 19 volts when the ammeter reads 24 amp. What is the value of the resistance R?

The resistance:

$$R = \frac{19}{24} = 0.792 \text{ ohm.} \quad \textit{Ans.}$$

As a matter of interest let it be required to determine the resistance of R'. The voltmeter terminals are transferred from across R to across R'. Under these conditions the voltmeter reads 91 volts and the ammeter still reads 24 amp. Therefore:

$$R' = \frac{91}{24} = 3.79 \text{ ohms.}$$

Elimination of Contact Resistance.—It is sometimes desired to measure resistances of such low value that, if a voltmeter were

connected directly across their terminals, the contact resistance, which may be comparatively large, would introduce considerable error and might even exceed in magnitude the resistance which it is desired to measure. To eliminate this error due to contact resistance, the voltmeter terminals are connected well inside the terminals BB (Fig. 131) through which the current is led to the specimen. As the voltmeter takes but a very small current, small sharp-pointed contacts CC may be used. As the resistance of the voltmeter is comparatively high, it is only necessary that the contact resistances at CC be negligible compared to the resistance of the instrument. This condition is easily met. As these

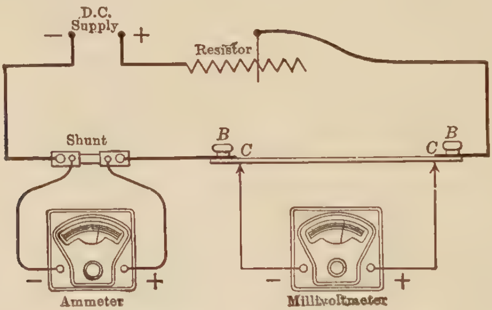


FIG. 131.—Measuring the resistance of a metal rod.

contacts are small and sharp the points of contact on the specimen can be determined very accurately.

Example.—When the ammeter (Fig. 131) reads 50 amp., the millivoltmeter indicates 40 millivolts. The contacts CC are 23 in. apart. What is the resistance per inch length of the rod?

The resistance for 23 in. is:

$$R = \frac{0.040}{50} = 0.00080 \text{ ohm.}$$

The resistance per inch:

$$R = \frac{0.00080}{23} = 0.0000348 \text{ ohm.} \quad \textit{Ans.}$$

128. The Voltmeter Method.—It is possible to measure a resistance by means of a voltmeter alone provided the resistance to be measured is comparable with that of the voltmeter. In Fig. 132 (a) let it be required to measure the resistance R. The voltmeter is first connected across the source of supply and a

reading V_1 taken. It is then transferred so that the resistance R is in series with it across the source of supply and the voltmeter reading is again taken. Let this reading be V_2.

As V_1 is the total circuit voltage and V_2 is the voltage across the instrument, the voltage across the unknown resistance R is obviously $V_1 - V_2$. When the voltmeter is in series with R, the same current i must flow through each so that the voltages are as follows:

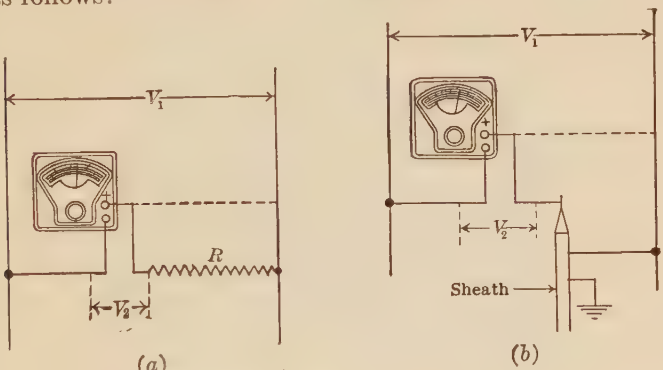


FIG. 132.—Measurement of resistance by the voltmeter method.

$$V_2 = iR_v \tag{1}$$
$$V_1 - V_2 = iR \tag{2}$$

where R_v is the resistance of the voltmeter.

Dividing Eq. (2) by Eq. (1) and solving for R,

$$R = R_v \frac{V_1 - V_2}{V_2} \tag{50}$$

This method of measuring resistance is particularly useful in determining insulation resistance of dynamo windings, cables, etc. As such resistances are very high they are usually expressed in megohms (1 megohm = 1,000,000 ohms). It will be seen from Eq. (50) that the greater the value of R_v, the greater the resistance that can be measured by this method. For this reason special 150-scale voltmeters, having resistances of 100,000 ohms (one-tenth of a megohm) are available. These give a sensitivity about six times as great as can be obtained with the ordinary 150-scale voltmeter.

Figure 132 (b) shows the application of this method to the measurement of the insulation resistance of a cable.

Example.—When a 100,000-ohm voltmeter is connected across a direct current line it reads 120 volts. One terminal of the voltmeter is then connected to the core of a lead-covered cable and the sheath of the cable is connected to the other side of the line as in Fig. 132 (*b*). The voltmeter now reads 10 volts. What is the insulation resistance of the cable?

$$X = 0.1 \frac{120 - 10}{10} = 1.1 \text{ megohms.} \quad \textit{Ans.}$$

129. The Wheatstone Bridge.—In distinction to the foregoing methods of measuring resistance, the Wheatstone Bridge method is one in which the unknown resistance is balanced against other known resistances. The bridge in its simplest form, is shown in Fig. 133. Three known resistances M, N, P, and the unknown resistance X are connected to form a diamond. Current from a battery B feeds the two opposite corners o and c of the diamond. Across the other two corners, a and b, is connected a galvanometer.

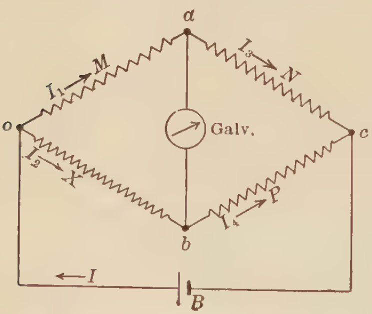


FIG. 133.—Elementary Wheatstone bridge.

To make a measurement, the two arms M and N are each set at some fixed value of resistance, usually 1, 10, 100, 1,000 ohms, etc. The arm P is then adjusted until the galvanometer does not deflect. If the galvanometer does not deflect, no current flows through it and therefore the two points a and b must be at the *same potential.* Also, the currents $I_1 = I_3$ and $I_2 = I_4$, as no current passes through the galvanometer.

If the points a and b are at the same potential, the voltage drop $oa = ob$ and:

$$I_1M = I_2X \tag{1}$$

Also the voltage drop $ac = bc$ and

$$I_3N = I_4P$$

And since

$$I_1 = I_3 \text{ and } I_2 = I_4$$

$$I_1N = I_2P \tag{2}$$

Dividing Eq. (1) by Eq. (2)

$$\frac{I_1M}{I_1N} = \frac{I_2X}{I_2P} \text{ or } \frac{M}{N} = \frac{X}{P}$$

$$X = \frac{M}{N}P \tag{51}$$

which is the equation of the Wheatstone bridge. M and N are called the ratio arms and P the balance or rheostat arm. Obviously the battery and the galvanometer may be interchanged without affecting the relation given in Eq. (51).

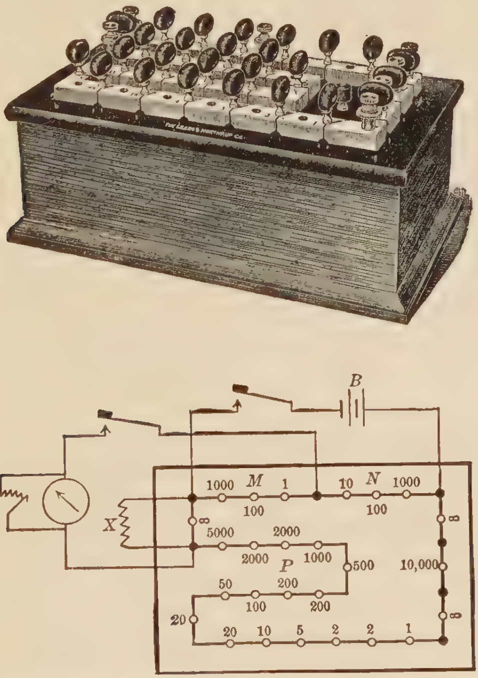


FIG. 134.—Massachusetts Institute of Technology pattern of Wheatstone bridge.

The many types of Wheatstone bridge found in practice do not differ in principle from that shown in Fig. 133. The differences lie in the positions of the arms M, N, and P on the bridge as well as in the manner in which the coils in these arms are cut in and out of circuit.

A common plug type of bridge is shown in Fig. 134. M consists of three resistances of 1,000, 100, and 1 ohms respectively,

and N consists of three of 10, 100, and 1,000 ohms respectively. P consists of a number of resistances ranging from 5,000 ohms to 1 ohm and of such values that with the proper combinations P may be made equal to any whole number between 1 and 11,110 ohms. Between the outer ends of N and P are two infinity plugs (∞) and a 10,000-ohm coil. The infinity plugs mean that the bridge can be open-circuited at these points and by their position the 10,000-ohm coil may be made a part of N or a part of P. The unknown resistance X may be connected across any one of the infinite resistances, if it is found advisable to do so. Between M and P is another infinite resistance, across which the unknown resistance may also be connected, the infinity plug being removed.

In this type of bridge the resistance coils are connected across gaps cut in heavy brass or composition bars. When it is desired to insert a resistance the plug is removed, and when it is desired to remove a resistance it is short-circuited by the plug. These plugs have hard rubber tops and are tapered. As the principle source of error in this type of bridge lies in the contact resistance of these plugs, they should be made to fit tightly when used. This is accomplished by exerting a slight pressure and simultaneously twisting them, thus giving a wiping contact. As dirt and oxide are a frequent source of error the plugs should be kept clean.

In using the bridge, much time may be saved if a systematic procedure is followed in obtaining a balance. Assume that it is desired to measure a certain unknown resistance. Connect the bridge as shown in Fig. 134, placing keys in the battery and in the galvanometer circuits and a shunt, S, around the galvanometer to protect it from deflecting violently when the bridge is considerably out of balance. Make the ratio arms M and N each 1,000 ohms, a 1 to 1 ratio. With the galvanometer well shunted and all the plugs in P (Res. = 0), depress first the battery and then the galvanometer key. The galvanometer is observed to deflect to the left. Now remove the 5,000-ohm plug and the galvanometer deflects to the right. From these observations, two facts are determined. The unknown resistance is less than 5,000 ohms and when the galvanometer deflects to the left the value of resistance in P is too small, and when it deflects to the right the value of P is too large. By inserting the 5,000-ohm plug and removing the 1,000-ohm plug the galvanometer still deflects to the right, indicating that 1,000 ohms in P is too large. This is repeated with 500 ohms, 200 ohms, etc. By proceeding in this manner, it is found that the galvanometer does not reverse until a 2-ohm plug is removed. This means that the unknown resistance lies between 2 and 5 ohms. By removing the two 2-ohm plugs

and then a 1 and a 2 the unknown resistance is narrowed down to between 2 and 3 ohms. To obtain a more precise value, the ratio arms must be changed. M is now made 1 ohm and 2,000 ohms are unplugged in P. By successive trials, all the time reducing the shunt S, a balance is obtained at 2,761 ohms in P. Then:

$$X = \frac{M}{N}P = \frac{1}{1{,}000}\,2{,}761 = 2.761 \text{ ohms.}$$

In obtaining a balance the battery key should always be depressed before the galvanometer key, so that the current in the bridge has time to reach a constant value. Otherwise the electromotive force of self-induction may introduce an error.

A more convenient arrangement of the resistance units of the rheostat arm P is shown in Fig. 135. The resistances are arranged in groups of

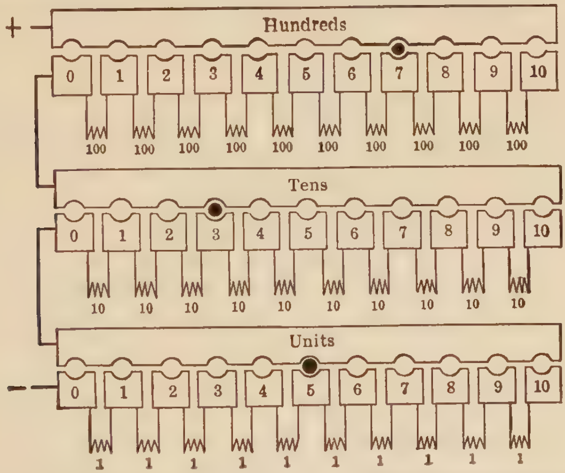
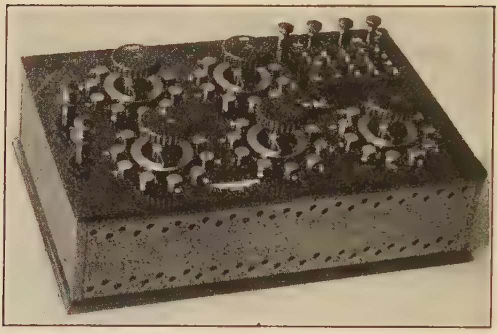


FIG. 135.—Arrangement of rheostat arm resistances in a decade bridge.

equal resistances, one group consisting of ten 1-ohm coils, the next of ten 10-ohm coils, the next of ten 100-ohm coils, etc. Each group is called a *decade*. Only one plug per decade is necessary. This arrangement has the advantage that the plugs are always in service, so are not so likely to be mislaid or to become dirty; there is less probability of error in reading; it is a simple matter to see that the few plugs used are fitting tightly, and a balance can be quickly obtained. It is obvious that nine coils per decade are sufficient for obtaining any desired resistance, although ten coils per decade are often used.

The decade principle has been extended to an even more convenient type of bridge, the dial bridge. Instead of using plugs, a dial arm similar to the type used in rheostats is employed to select the required resistances. Because of its ease of manipulation this type has come into extensive use. Care should be taken to keep the dials and contacts free from dirt and oxides. Figure 136 shows a dial bridge of the Leeds and Northrup type.



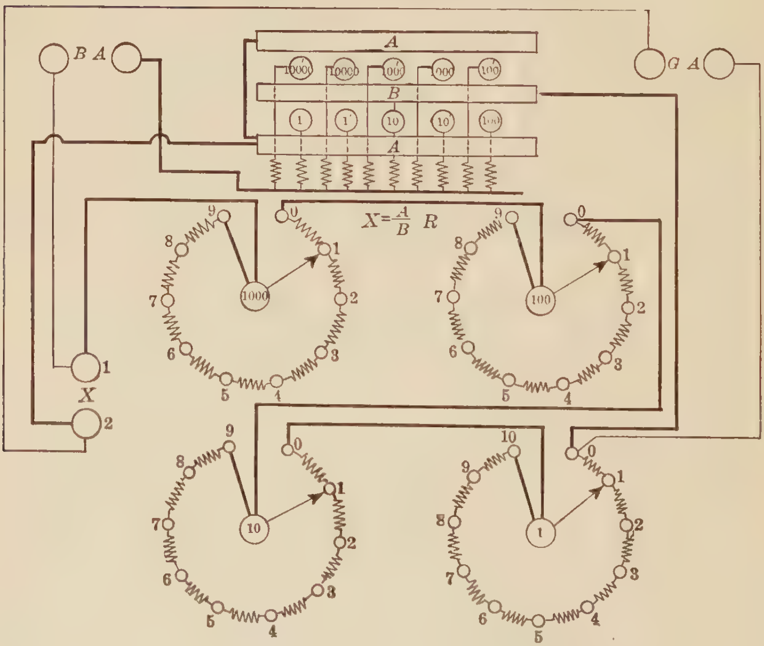


FIG. 136.—Leeds and Northrup dial bridge.

130. The Slide-wire Bridge.—The slide-wire bridge is a simplified Wheatstone bridge, in which the balance is obtained by means of a slider which moves over a German silver or manganin resistance wire. A typical slide-wire bridge is shown in Fig. 137. The resistance wire AB, 100 cm. long, is stretched tightly between two heavy copper blocks CD, 100 cm. apart. A meter scale is placed along this wire. A contact key K' is movable along the scale and when the key K' is pressed a knife edge makes contact with the wire. The rest of the bridge consists of a heavy copper bar E, a known resistance R, and the unknown resistance X. R is connected between D and E and X between C and E, although the positions of R and X are interchangeable.

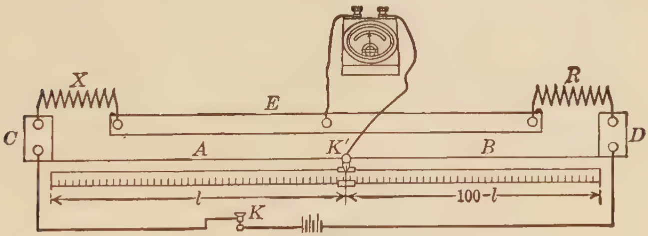


FIG. 137.—Slide-wire bridge.

The galvanometer is connected between the key K' and E and the battery terminals are connected to C and D. A balance is obtained by moving K' along the wire until the galvanometer shows no deflection.

Let l be the distance in centimeters from one end of the scale to K' when a balance is obtained. Then $100 - l$ is the distance from K' to the other end of the scale. Let r be the resistance per unit length of the wire. Then the resistance of l is lr and that of the remainder of the wire is $(100 - l)r$.

By the law of the Wheatstone bridge:

$$\frac{X}{lr} = \frac{R}{(100 - l)r}. \tag{52}$$

r cancels out and Eq. (52) becomes:

$$X = l\frac{R}{(100 - l)}. \tag{53}$$

Eq. (52) may also be written

$$\frac{X}{R} = \frac{l}{100 - l} \tag{54}$$

This is equivalent to stating that when a balance is obtained the slide wire is divided into two parts which are to each other as X is to R.

The slide wire is not as accurate as the coil bridge, because the slide wire may not be uniform; the solder at the points of contact at C and D makes the length of the wire uncertain although it is possible to correct for this error; the slide wire cannot be read as accurately as the resistance units of a bridge can be adjusted.

Example.—Assume that R (Fig. 137) equals 10 ohms and that a balance is obtained at 74.6 cm. from the left-hand end of the scale. Find the unknown resistance X.

From Eq. (53)

$$X = 74.6\frac{10}{100 - 74.6} = 74.6\frac{10}{25.4} = 29.38 \text{ ohms.} \quad \textit{Ans.}$$

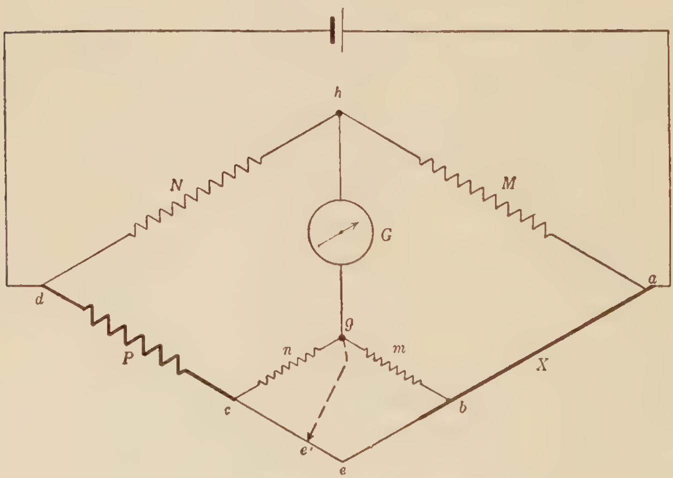


FIG. 138.—Wiring diagram of the Kelvin double bridge.

131. The Kelvin Bridge.—In the measurement of very low resistances with the Wheatstone bridge, the contact resistance of the sample and the bridge terminals may be so large as compared with the resistance of the sample itself that values of resistance obtained for the sample are practically worthless (also see p. 154). The effect of these contact resistances is eliminated in the Kelvin bridge (Fig. 138). M and N are two ratio arms, similar to those of the usual type of bridge. Their values may be made sufficiently high so that the contact resistances at a and d are negligible

in comparison. Let ab or X be the unknown resistance, such as a copper rod, and let cd or P be the rheostat arm of the bridge, which must necessarily have a low value. The contact resistance included between b and c ordinarily would cause large error. The galvanometer, however, instead of being connected to the junction e, as in the ordinary bridge, is connected to g, the junction of two resistances m and n which in turn connect to b and c, as shown in Fig. 138. If m and n are adjusted so that $m/n = M/N$ the contact resistance is eliminated and $X/P = M/N$ as in the ordinary bridge.

This may be proved by writing Kirchhoff's laws for the network and solving the equations when $M/N = m/n$. The following demonstration, however, is a more simple and obvious proof. Assume that the bridge is in balance. There must be some point e' between b and c which has the same potential as the point g at the junction of m, n, and the galvanometer. If, therefore, g and e' are connected as shown by the dotted line, no effect is produced in the bridge. As points e', g, and h are all at the same potential, the ordinary equation of the Wheatstone bridge can be applied. That is,

$$\frac{M}{N} = \frac{ae'}{e'd} = \frac{ab + be'}{cd + e'c} \tag{1}$$

The four resistances m, n, be', and $e'c$ form a miniature Wheatstone bridge in balance, since g and e', are at the same potential. Hence

$$\frac{m}{n} = \frac{be'}{e'c} = \frac{M}{N}. \tag{2}$$

From Eqs. (1) and (2)

$$\frac{ab + be'}{cd + e'c} = \frac{be'}{e'c}. \tag{3}$$

Eq. (3) may be written

$$\frac{ab + be'}{be'} = \frac{cd + e'c}{e'c}. \tag{4}$$

Treating Eq. (4) by division,

$$\frac{ab + be' - be'}{be'} = \frac{cd + e'c - e'c}{e'c} = \frac{ab}{be'} = \frac{cd}{e'c}. \tag{5}$$

Eq. (5) may be written

$$\frac{ab}{cd} = \frac{be'}{e'c} = \frac{M}{N}.$$

Hence,

$$\frac{X}{P} = \frac{M}{N} \qquad \text{Q.E.D.}$$

The actual arrangement of the bridge, as manufactured by Leeds and Northrup Company, is shown in Fig. 139. The lettering corresponds to the diagram of Fig. 138.

In order to obtain sufficient sensitivity, it is necessary to use from 30 to 150 amp., depending on the range of resistance for which the bridge is designed.

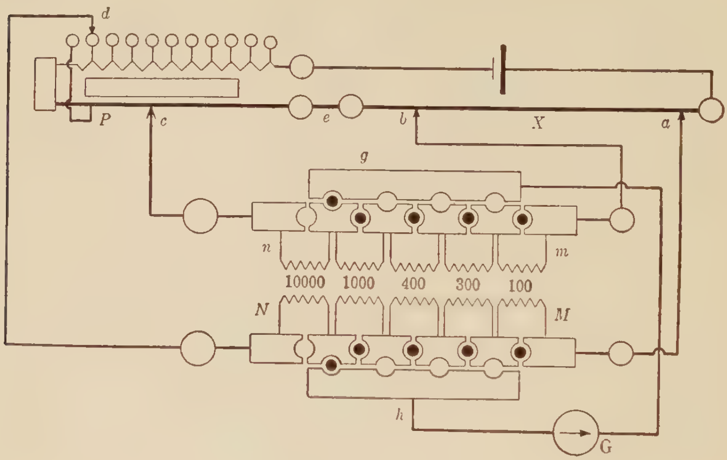


FIG. 139.—Actual connections of the Kelvin double bridge.

CABLE TESTING

132. The Murray Loop.—The slide-wire bridge offers a very convenient method of locating grounds in cables and wires. Figure 140 shows a cable AB which has become grounded at the

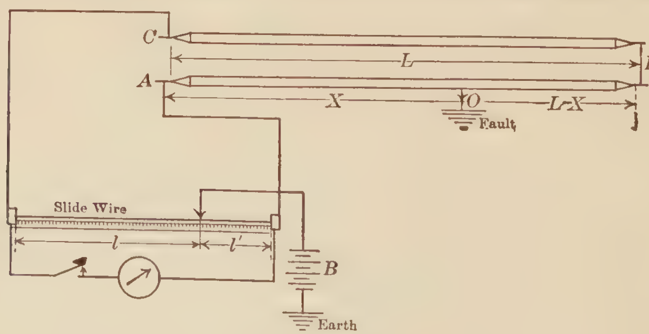


FIG. 140.—Murray-loop test.

point O, owing to a defect in the insulation. CB is the return conductor and is similar to AB except that it has no ground or "fault." The two conductors are looped together at B, the far

end of the two conductors, which may be at some power station, telephone exchange, etc.

The slide wire is then connected to the home ends of the cable as shown. It will be noted that the battery and the galvanometer are not in the positions shown in Fig. 137, but have been interchanged. This is done in order that earth currents shall not disturb the galvanometer readings. Also if the resistance of the fault to ground is high, the electromotive force of the battery B may be increased until sufficient current to operate the bridge is sent through this resistance. The resistance of the fault to ground does not produce any error in the measurement so long as the conductor is not broken. If the conductor is broken with both ends lying on the ground, the resistance of the conductor is increased and a false location of the fault may result.

In Fig. 140, the distance X to the fault may be found as follows:

$$\frac{X}{l'} = \frac{L + (L - X)}{l} \qquad (55)$$

where L is the length of one cable.

The slide wire is divided into two sections which are to each other as the two lengths of cable on each side of the fault.

Solving Eq. (55) for X,

$$X = \frac{2Ll'}{l + l'} \qquad (56)$$

This assumes that the resistance per foot of both conductors is the same, and is uniform. The jumper tieing the cable ends together at B should make good connection, as contact resistance at this point may introduce an appreciable error. A ratio and rheostat arm of a bridge box may obviously be used instead of the slide wire.

Example.—A cable 2,000 ft. long consists of two conductors. One conductor is grounded at some point between stations. A Murray loop test, with a 100-cm. slide-wire bridge, is connected as in Fig. 140 to locate the fault. A balance is obtained at 85 cm. How far from the station is the ground? From Eq. (56):

$$L = 2{,}000 \quad l' = 15 \quad l = 85$$

$X = \dfrac{4{,}000 \times 15}{100} = 600$ ft. from the station at which the measurement is made. *Ans.*

133. The Varley Loop.—The Varley loop is also used to locate cable faults. It is similar in principle to the Murray loop, a

bridge box being necessary, however. The connections are shown in Fig. 141. M and N are the two ratio arms of a bridge and P is the rheostat arm. It is necessary that the battery and the galvanometer occupy the positions shown, in order to avoid disturbances in the galvanometer due to earth currents. A balance is first obtained by means of P, with the switch S at a. Let r be the resistance per foot length per conductor, assumed uniform. (It will be noted that P and X together form one arm of the bridge.)

$$\frac{M}{N} = \frac{r(L + L - X)}{P + rX}. \tag{57}$$

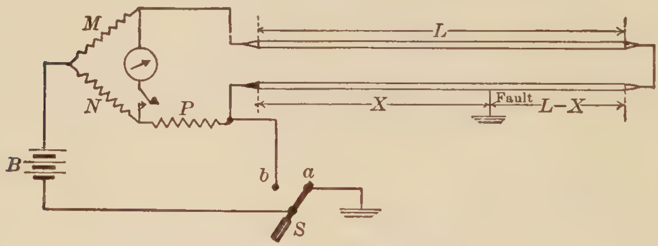


FIG. 141.—Varley-loop test.

Before X can be found it is necessary to know r. To obtain this, the switch S is thrown to position b. This connects both lengths of cable in series and makes them the fourth arm of a bridge. A simple bridge measurement is then made of the total loop resistance. Call this resistance R. Then the resistance per foot of cable

$$r = \frac{R}{2L}.$$

(This measurement is not necessary if the resistance per foot or the total resistance of the cable is already known.)

Substituting this value of r in Eq. (57)

$$\frac{M}{N} = \frac{R/2L(2L - X)}{P + RX/2L}.$$

Solving for X,

$$X = \frac{2L}{R}\left(\frac{NR - MP}{M + N}\right). \tag{58}$$

This equation gives the distance in feet to the fault. The equation is frequently given as follows:

$$R_x = \frac{NR - MP}{M + N}. \tag{59}$$

In this case R_x is not the *distance* to the fault, but rather the *resistance* along the grounded conductor to the fault.

If $M = N$ in Eq. (58)

$$X = \frac{L}{R}(R - P) \tag{60}$$

which is simpler in form than Eq. (58).

Example.—In locating a fault by the Varley-loop test, the connections shown in Fig. 141 were used. Each conductor is 2,800 ft. long. With the switch at (*a*) and $M = 10$, $N = 1{,}000$, P was found to be 137 when a balance was obtained. Switch S was then thrown over to (*b*). Under these conditions, a balance was obtained when $M = 10$, $N = 1{,}000$, $P = 221$, making $R = 2.21$.

By Eq. (58) the distance in feet to the fault

$$X = \frac{2 \times 2{,}800}{2.21}\left(\frac{1{,}000 \times 2.21 - 10 \times 137}{1{,}010}\right) = 2{,}100 \text{ ft.} \quad \textit{Ans.}$$

134. Insulation Testing.—In practice it is necessary to measure the resistance of the insulation of cables, both at the factory and after the cable is installed. A low value of insulation resist-

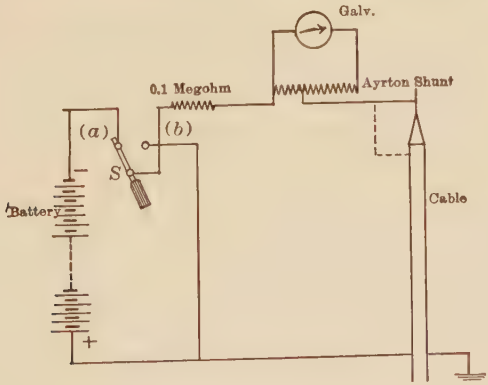


FIG. 142.—Measurement of the insulation resistance of a cable.

ance may indicate that the insulation is of an inferior grade. A low insulation resistance after installation may indicate improper handling or faulty installation. The voltmeter method described in Par. 128 is applicable in many cases, but where the insulation resistance is high even a high-resistance voltmeter is not sufficiently sensitive.

To make the measurement, a sensitive galvanometer is utilized. A considerable source of potential, from 100 to 500 volts, is

usually necessary. Such potential may be secured from direct-current mains, although dry cells, silver-chloride cells, and test-tube batteries connected in series are more satisfactory. A simple diagram of connections is shown in Fig. 142.

The method is one of substitution. A known resistance, usually 0.1 megohm (100,000 ohms), is first connected in the circuit and the galvanometer deflection noted. The unknown resistance X is then substituted and the galvanometer reading again noted. As the currents in the two cases are inversely proportional to the circuit resistances, the unknown resistance can be determined, the galvanometer deflections being used rather than actual values of current. Let D_1 be the deflection with the 0.1 megohm and D_2 be the deflection with the unknown resistance.

$$\frac{X}{0.1} = \frac{D_1}{D_2}$$

$$X = 0.1\frac{D_1}{D_2} \tag{61}$$

Under ordinary circumstances it would not be possible to obtain accurate results under these conditions alone, because the unknown resistance may be in the hundreds of megohms and the known resistance is but 0.1 megohm. This would make the deflection D_2 so many times smaller than D_1 that it would not be readable.

This difficulty is overcome by the use of the Ayrton shunt described in Par. 121. When the 0.1 megohm only is in circuit, the galvanometer sensitivity ordinarily is such that it would deflect off the scale unless the galvanometer were shunted. Therefore the shunt is adjusted to some low value as 0.0001. Call this reading of the shunt S_1 and the galvanometer deflection D_1. The multiplying power of the shunt equals $M_1 = 1/S_1$. The cable is now introduced into the circuit and the shunt adjusted until a reasonable deflection is obtained. Call this deflection D_2 and the value of the shunt S_2. Its multiplying power is now $M_2 = 1/S_2$.

The ratio of the currents in the circuit in the two cases

$$\frac{I_1}{I_2} = \frac{M_1D_1}{M_2D_2},$$

therefore the unknown resistance, from Eq. (61), is

$$X = 0.1\frac{I_1}{I_2} = 0.1\frac{M_1D_1}{M_2D_2}. \qquad (62)$$

In practice, instead of substituting the cable for the 0.1 megohm, the cable is first short-circuited by the wire shown dotted (Fig. 142) and the constant determined. This wire is then removed, placing the cable in circuit. The 0.1 megohm is left permanently in circuit to protect the galvanometer in case of accidental short-circuit of the cable. Its resistance is usually not appreciable compared to that of the cable, so that no correction is ordinarily necessary for it.

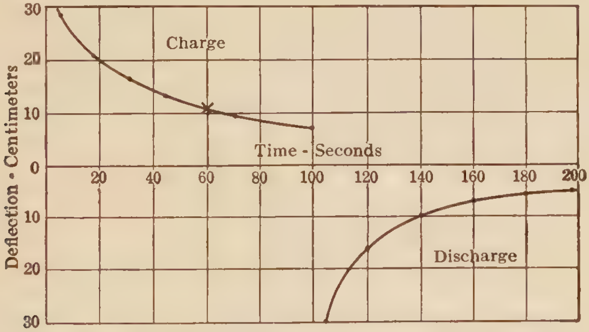


FIG. 143.—Charge and discharge curves of a cable.

A switch or key S is ordinarily provided. When in position (a) the circuit is closed through the cable. When thrown over to (b), the cable, which is charged electrostatically, discharges through the galvanometer.

When the switch (a) is first closed there is a rush of current which charges the cable electrostatically (see Par. 179, Chap. IX). It takes time to charge the cable, so for some time this charging current flows, decreasing continuously. This is shown in Fig. 143, giving the relation of the galvanometer deflection to the time. As it is often inconvenient to wait for the galvanometer to reach a steady deflection, it has been arbitrarily agreed to take the deflection at the end of one minute as the value to be used in determining insulation resistance.

When the switch S is thrown to (b), the electrostatic charge in the cable rushes out through the galvanometer in the reverse

direction. Due to absorption it requires considerable time for the cable to become totally discharged. This is also shown in Fig. 143.

In making insulation-resistance measurements, precautions must be taken to insulate thoroughly the apparatus itself. Hard-rubber posts should be used for supports and, wherever possible, the leads should be carried through the air rather than be allowed to rest on the ground. The insulation resistance varies enormously with temperature, so the temperature at which the measurements are made should be carefully determined and stated.

Example.—The cable whose insulation curves are shown in Fig. 143 was tested for insulation. The deflection with 0.1 megohm only in circuit was 20 cm. and the shunt read 0.0001. When the curve shown in Fig. 143 was obtained the shunt read 0.1. The cable was 2,200 ft. long.

(*a*) What is its insulation resistance?

(*b*) What is its insulation resistance per mile?

$$M_1 = 1/0.0001 = 10{,}000$$

$$M_2 = 1/0.1 = 10.$$

$$D_2 \text{ (from curve)} = 11 \text{ cm.}$$

$$(a)\ X = 0.1\frac{10{,}000 \times 20}{10 \times 11} = 182 \text{ megohms.} \quad \textit{Ans.}$$

(*b*) The resistance per mile will be *less* than that of the 2,200-ft. length because the amount of leakage current is directly proportional to the length of the cable. Therefore the resistance of this leakage path is inversely proportional to the length of cable. The cross-sectional area of the leakage path for the mile length is greater than it is for the 2,200-ft. length. Therefore the resistance per mile

$$R = \frac{2{,}200}{5{,}280}182 = 75.9 \text{ megohms.} \quad \textit{Ans.}$$

135. The Guard Wire.—In determining the insulation resistance of cables, precautions against leakage over the ends of the cable must frequently be taken. Moisture, combined sometimes with dust, provides comparatively low resistance paths over the surface of the insulation between the core and sheath. These conducting paths are in shunt with the insulation, so that this end leakage may give values of insulation resistance which are entirely too low. To prevent such leakage, the ends of the cable are frequently dried with a torch, care being taken not to carbonize the insulation, and hot paraffin is then poured over the ends.

Where both of the ends of the cable are accessible, the end leakage may be shunted around the galvanometer by the use of *guard wires*, and its effect, so far as the measurement is concerned, is thus eliminated. The method is shown in Fig. 144. A few turns of bare wire are wrapped tightly around the insulation at *a* and *b* between the sheath or ground and the core of the cable. A study of Fig. 144 shows that any leakage current is conducted back to the negative terminal of the battery without passing through the galvanometer. That is, when leakage current reaches *a* and *b*, it has the choice of two paths to the negative

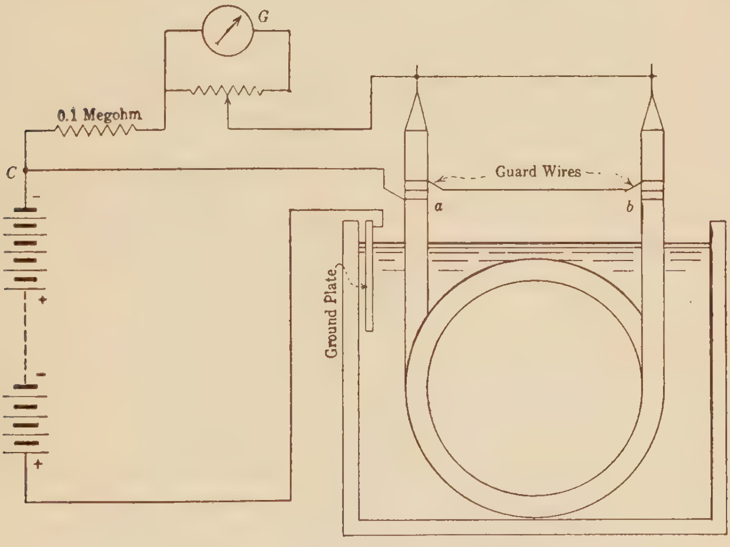


Fig. 144.—The use of guard wires.

terminal of the battery, one through the low resistance wire *ac*, and the other over the remainder of the insulation through the galvanometer to the battery. Practically all the leakage current will pass through the wire *ac*, and will not affect the galvanometer deflection.

POTENTIOMETERS

136. The Potentiometer.—The potentiometer is an instrument for making accurate measurements of voltage. Its standardiza-

tion depends primarily upon the Weston standard cell (see Par. 96, Chap. VI). The principle is as follows:

Assume in Fig. 145 (*a*) that a standard cell *S* has an electromotive force of exactly 1 volt. Let a storage cell *Ba* supply current to a wire *AB* through a rheostat *R*. Let the wire *AB* be divided into 15 divisions each of 1 ohm resistance, making the total resistance of *AB* equal to 15 ohms. The standard cell is connected with its negative terminal to the negative terminal of

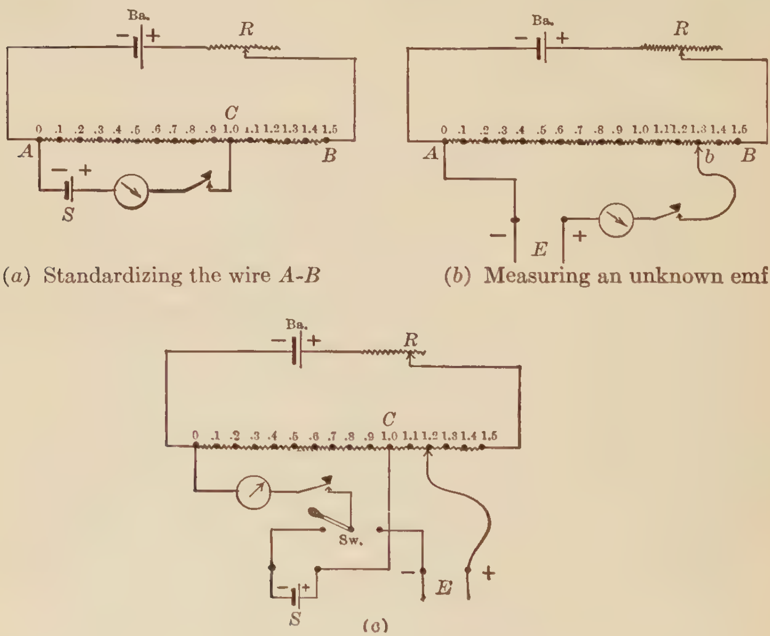


Fig. 145.—Simple potentiometer.

the storage cell and its positive terminal is connected to the tenth 1-ohm coil *C* through a key and galvanometer. If 0.1 amp. flows through the wire *AB*, the voltage drop through each resistance will be 0.1 volt and the voltage drop across *AC* will be 1.0 volt. If the key be depressed no current will flow through the galvanometer, as the standard cell e.m.f. is in exact opposition to this 1-volt drop. If, however, the current in *AB* is not exactly 0.1 amp., current will flow through the standard cell circuit.

due to the voltage drop from A to C being either greater or less than 1 volt. If the current is less than 0.1 amp., the galvanometer deflects in one direction, and if it is greater than 0.1 amp., the galvanometer deflects in the reverse direction. Obviously it is possible to adjust the current in AB to such a value that the galvanometer deflection is zero. Under these conditions the current in AB is exactly 0.1 amp. and the potential drop across each resistance in AB is 0.1 volt. Therefore AB may be marked in volts as shown.

Let it be required to measure some unknown electromotive force E whose value is known to be less than 1.5 volts. The negative terminal of E is connected to the end A of the wire AB, Fig. 145 (*b*). The positive terminal of the electromotive force is connected through the galvanometer and key to a movable contact b. It is assumed that the current in AB has been adjusted to exactly 0.1 amp. as just described. Contact b is moved along AB until the galvanometer deflection is zero. This means that the electromotive force E is just balanced against an equal drop in the wire AB. As AB is calibrated in volts, the value of E may be read directly on AB. This method of measuring voltage is the *Poggendorff Method* and is the fundamental principle of the potentiometer.

The two diagrams (*a*) and (*b*) (Fig. 145) may be combined into one by the use of the single-pole, double-throw (S.-P.D.-T.) switch *Sw*, Fig. 145 (*c*). When the switch is in its left-hand position the standard cell is in circuit for calibration as in (*a*). When it is in its right-hand position the unknown e.m.f. is in contact with the wire AB so that its value may be determined.

137. The Leeds and Northrup Low-resistance Potentiometer. Figure 146 shows the Leeds and Northrup low-resistance potentiometer. In most respects it is similar to the simple potentiometer shown in Fig. 145. The battery W supplies current through the rheostat R and then through the low resistance OA, to the potentiometer wire AB. AB consists of two parts, fifteen 5-ohm coils and a slide wire DB of 5.5 ohms. As each contact represents 0.1 volt, the working current is $0.1/5 = 1/50$ amp. As the slide wire DB is 5.5 ohms, the voltage drop across it when in adjustment is 0.11 volt. This slide wire consists of 11 turns of resistance wire mounted on a marble cylinder.

Each turn represents 0.01 volt and the entire wire is divided into 1,100 divisions.

The standard cell has a voltage slightly in excess of 1.0 volt so that instead of connecting the standard cell exactly as in Fig. 145 (*a*), an added resistance *AO* is necessary to allow for this small excess voltage. A contact *T* is movable on *AO* so that the setting can be made to correspond with the electromotive force of the standard cell used. *M* and *M′* are the movable contacts, which are adjusted to balance the unknown e.m.f. *M* moves over the 15 contacts, each corresponding to 0.1 volt,

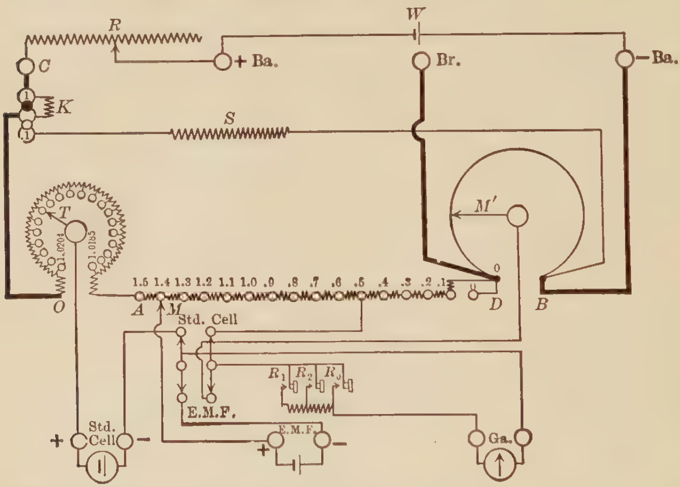


FIG. 146.—Connections of Leeds and Northrup low-resistance potentiometer.

and *M′* moves over the slide wire. A double-pole, double-throw (D.-P.D.-T.) switch (corresponding to *Sw*, Fig. 145 (*c*)) changes the connection of the galvanometer from the standard cell to the unknown e.m.f. There are three galvanometer keys, R_1, R_2, and R_0. R_1 should first be depressed as it inserts a high resistance in series with the galvanometer and prevents a violent deflection if there is considerable unbalancing. R_2 inserts less resistance and there is no resistance in series with R_0 which is depressed when the final balance is obtained.

A resistance *S* shunts nine-tenths of the current from *OB*, when the plug at *K* is changed. The resistance *K* is automatically put

in circuit, keeping the total potentiometer resistance, and therefore the load on the battery, constant. By this arrangement, the readings on the potentiometer are all made one-tenth their previous values.

An external view of this potentiometer is shown in Fig. 147.

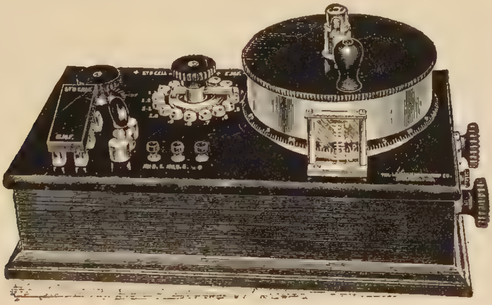


Fig. 147.—Leeds and Northrup potentiometer without accessories.

138. Other Potentiometer Methods.—With the simple potentiometer, such as the Leeds and Northrup type, it is impossible to have more than two dial or slide-wire readings. For example, in the Leeds and Northrup type the tenths of a volt are obtained on one dial with contact M (Fig. 146) and the smaller divisions, such as hundredths, thousandths, and ten-thousandths of a volt, must all be obtained on the single slide wire with the contact M'. To obtain more than two significant figures, it is necessary that the hundredths, thousandths, etc. of a volt be obtained on the slide wire. This is considered objectionable if extremely high precision is necessary, because the slide wire may not be uniform, may be subject to wear, etc. It is possible to arrange a potentiometer so that the tenths, hundredths, thousandths, etc. of a volt are each read on a separate dial. In any of these types of potentiometer, it is essential to maintain a fixed resistance in the potentiometer itself, for the current must remain constant irrespective of the positions of the dials. There are two common methods of obtaining more than two significant figures by separate dials and at the same time maintaining constant potentiometer resistance: the Thomson-Varley method and the Wolff method.

The Thomson-Varley method is illustrated in Fig. 148. It is desired that tenths, hundredths, thousandths, and ten-thousandths of a volt each be obtained on a separate dial, except for the ten-thousandths, which may be obtained on the slide wire AB. Fundamentally, it is necessary *to maintain constant resistance in the entire potentiometer circuit* AC. Assume that the resistance corresponding to a tenth-volt is 10 ohms. The resistance BC consists of sixteen 10-ohm resistances. Two contacts b and c, fastened rigidly together, always span two of the 10-ohm coils in BC. Between b and c there are connected eleven 2-ohm coils, $B'C'$. Two contacts b' and c', fastened rigidly together, always

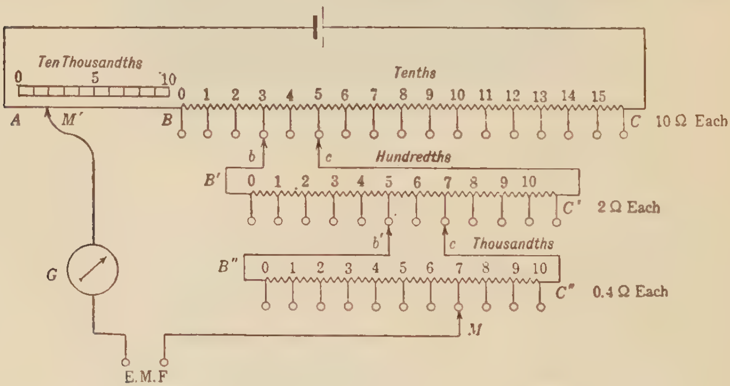


Fig. 148.—Principle of Thomson-Varley slide.

span two of the 2-ohm coils in $B'C'$. Between b' and c' there are connected ten 0.4-ohm coils $B''C''$. One contact arm M operates over the contacts in the dial $B''C''$, since these resistances are usually arranged in circular dials. The total resistance of $B''C''$ is 4 ohms and it is connected across two 2-ohm resistances in $B'C'$. Hence, the equivalent resistance between the contactors $b'c'$ is 2 ohms and the total resistance of $B'C'$ is 20 ohms. Likewise, the 20 ohms in $B'C'$ is connected across two 10-ohm resistances in BC by means of the contactors bc. Hence, the equivalent resistance between contactors bc is 10 ohms and the total resistance of BC is 150 ohms. With the normal current of 0.01 amp. flowing, the total voltage across BC is 1.5 volts; the total voltage across $B'C'$ is 0.1 volt; and the total voltage across $B''C''$ is 0.01 volt. Hence, the tenths of volts are read at b on dial BC;

the hundredths of volts are read at b' on dial $B'C'$, and the thousandths of volts are read at M on scale $B''C''$. The ten-thousandths of volts may be read at contact M' on a slide wire AB having a total resistance of 0.1 ohm. The reading with the dials set as in Fig. 148 is 0.3572 volt. It is obvious that the resistance between points A and C remains constant at 150.1 ohms, irrespective of the position of any of the contact arms.

Since contact resistance, except at M and M', introduces error, it is desirable that potentiometers of this type have a high inherent resistance, so that the contact resistance is negligible. This reduces the sensitivity. Another disadvantage of this type is the fact that the contacts must be kept clean from oxide and dust.

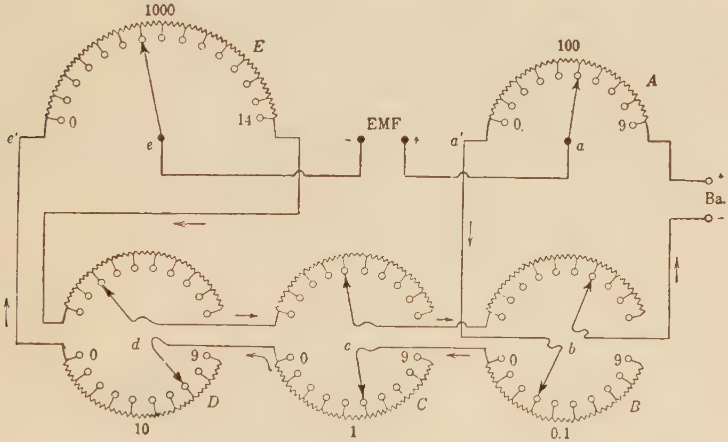


FIG. 149.—Principle of the Wolff potentiometer.

The Wolff Method.—The principle of the Wolff potentiometer is illustrated in Fig. 149. The potentiometer has a total resistance of 15,000 (actually 14,999.9) ohms and the working current is 0.0001 amp., so that it is a high-resistance potentiometer. The dial E consists of fourteen 1,000-ohm coils, and the dial A consists of nine 100-ohm coils. The dials B, C, D are double, and have double dial switches, the purpose of which is to maintain constant resistance in the battery circuit. There are nine 10-ohm coils in each half of dial D, nine 1-ohm coils in each half of dial C, and nine 0.1-ohm coils in each half of dial B. The path of the current, shown by arrows, is from the battery positive terminal

through the entire dial A, through those portions of the lower halves of dials B, C, and D which are not cut out by the lower parts of the dial switches b, c, and d, thence through the entire dial E and so through those portions of the upper halves of dials D, C, and B which are not cut out by the upper parts of the dial switches d, c, and b, and thus to the battery negative terminal.

A study of Fig. 149 shows that the resistance of the battery circuit is constant irrespective of the positions of any of the dial switches, for each double dial switch always cuts out as much resistance in the upper half of the dial as it inserts in the lower half. On the other hand, the moving of any of the double dial switches changes the resistance between points a' and e', and hence causes a change in the potential difference between points a and e.

With the setting of the dials shown in Fig. 149, the potentiometer reads 0.65753 volt.

139. Voltage Measurements with the Potentiometer.—Potentiometers are designed to measure potentials up to 1.6 volts

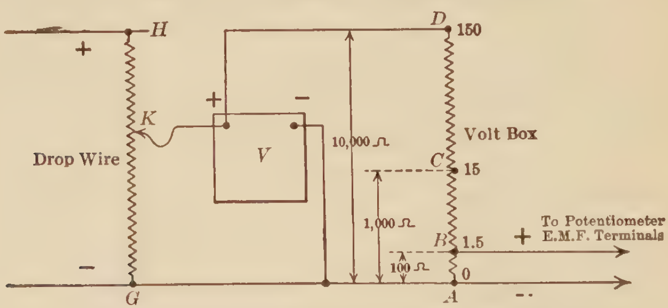


FIG. 150.—Volt-box and drop-wire connections.

only. For the measurement of potentials in excess of this value a *volt box* is necessary. A volt box is merely a very high resistance from which suitable taps are brought. This is illustrated by the resistance AD (Fig. 150). Assume AD to have a resistance of 10,000 ohms and AB a resistance of 100 ohms. If no current leaves the wire at B, the voltage drop across AB will be $100/10{,}000 = \frac{1}{100}$ that across AD. If leads be carried from AB to the potentiometer, the potentiometer will measure $\frac{1}{100}$ the voltage across AD, since the potentiometer principle is an opposition method so that no current is taken from B. Therefore, if a

voltmeter V is being calibrated it should be connected in parallel with AD. If the voltmeter reads 119.0 volts and the potentiometer reads 1.184 volts, the true line voltage across the voltmeter will be $1.184 \times 100 = 118.4$ volts. Therefore the correction to the voltmeter is -0.6 volt.

In a similar manner, voltages from 1.5 to 15 volts are connected across AC, the multiplying factor in this case being 10.

The Drop Wire.—GH is a resistance connected directly across the line. One voltmeter terminal and one terminal of the volt box are connected to the end G of this wire. The other terminal of the voltmeter and the remaining terminal of the volt box are connected to a movable contact K. By sliding K along GH any desired voltage may be obtained. When used in this manner, GH is called a *drop wire*. It is not necessary to the operation of the volt box, but is merely a convenient means for adjusting the voltage.

140. The Measurement of Current with Potentiometer.—As has just been pointed out, a potentiometer is designed primarily to measure *voltage*. By merely applying Ohm's law, the

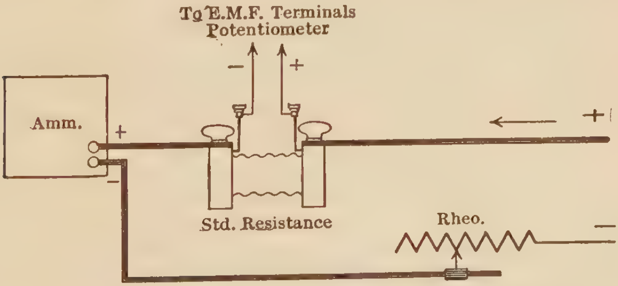


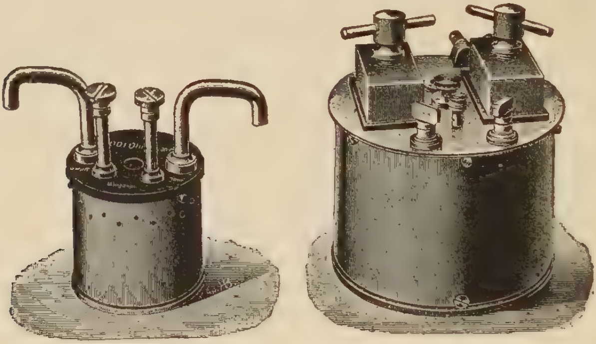
Fig. 151.—Calibration of an ammeter with a potentiometer.

current in a circuit may also be determined with the potentiometer. Let an unknown current I flow through a known resistance R. If E, the voltage drop across R, be measured, the current I is immediately determined, since for this part of the circuit both the voltage and the resistance are known. Therefore:

$$I = \frac{E}{R}.$$

The method of making the measurement is shown in Fig. 151. It is desired to know the exact current passing through the

ammeter, in order to determine its errors, if any exist. The ammeter is connected in series with the standard resistance, and also with a rheostat to control the current. Standard resistances are provided with four terminals as a rule, two heavy ones for



(*a*) 0.01 ohm. (*b*) Self-contained 0.001 ohm.
FIG. 152.—Standard resistances.

current and two smaller binding posts for potential (see Fig. 152). The two potential binding posts are connected to the potentiometer, the proper polarity being observed. The voltage across the standard resistance is then measured by means of the potentiometer.

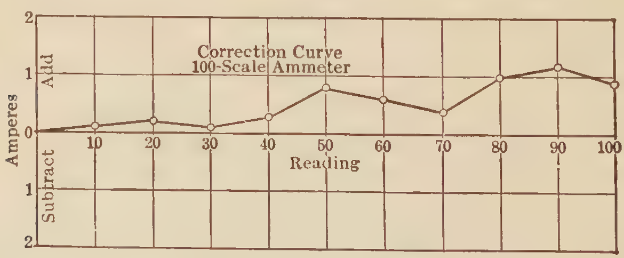


FIG. 153.—Ammeter calibration curve.

Standard resistances are usually adjusted to even decimal values such as 10, 1, 0.1, 0.01, etc., ohms. They are ordinarily rated to carry a current that will give 1.0 volt drop. Thus the 1 ohm can carry 1 amp., the 0.001 ohm, 1,000 amp., etc. To keep the resistances cool they are often immersed in oil. The type shown in Fig. 152 (*a*) is set in a water-jacketed oil bath

provided with a motor-driven stirrer. The type shown in (*b*) is rated for larger currents, 1,000 amp. and more. The water jacket, the stirrer, etc., are included within the unit itself.

Knowing that the potentiometer is limited to 1.5 volts, it is easy to select the proper standard resistance. An instrument having a range of 100 amp., would require $1.5/100 = 0.015$ ohm. 0.01 ohm would be used. Likewise a 15-scale instrument would require $1.5/15 = 0.1$ ohm.

When instruments are calibrated, they should be checked at ten or fifteen points on the scale and the corresponding corrections at each point are plotted as ordinates. (The instrument readings are plotted as abscissas.) As an instrument scale is subject to scale errors, etc., it is customary to connect successive points by straight lines, as shown in Fig. 153. For example (Fig. 153), the correct current when the instrument reads 50 amp. is $50 + 0.8 = 50.8$ amp.

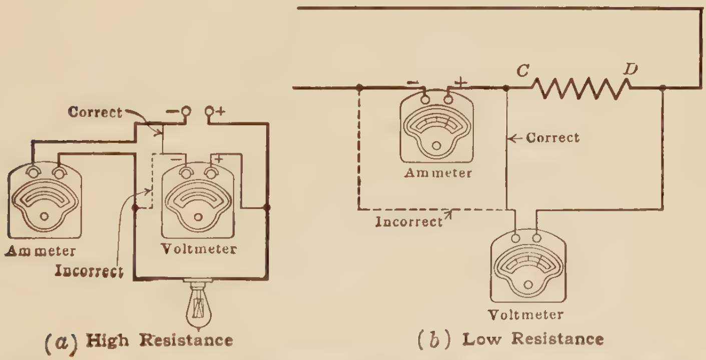


FIG. 154.—Correct and incorrect methods of connecting voltmeters and ammeters in power measurements.

141. Measurement of Power.—Direct-current power is usually measured by means of a voltmeter and an ammeter. Since the power is the product of the volts and the amperes ($P = EI$), it is merely necessary to multiply the volts by the amperes to obtain the power in watts. Certain precautions may be necessary in measuring the power, however.

Assume that it is desired to measure the power taken by an incandescent lamp. If the voltmeter is connected as shown by the dotted line in Fig. 154 (*a*), the current taken by the voltmeter

is being measured by the ammeter. In other words, the voltmeter is a load connected in parallel with the lamp. As the current taken by the lamp is small, this voltmeter current, although of itself small, may introduce a very appreciable error into the measurement. That is, the power taken by the voltmeter will be included in the measurement. There are three methods of eliminating this error. The voltmeter power may be calculated, knowing the voltmeter resistance, and proper correction made. The voltmeter may be open-circuited when the ammeter is being read if it is certain that this will not alter the voltage across the lamp. The voltmeter lead may be connected as shown by the solid line so that the voltmeter current does not pass through the ammeter. In this last case the voltmeter is not reading the true voltage across the lamps, but its reading is too high by the drop through the ammeter. As the resistance of the lamp is high and that of the ammeter low, this last error is usually negligible.

However, if a low resistance CD is being measured, Fig. 154 (b), the drop across the resistance is necessarily low, and if the voltmeter in this case is connected outside the ammeter, a very appreciable error may be introduced, as the voltmeter reading includes the voltage drop in the ammeter. The voltmeter should now be connected *inside* the ammeter. This will not introduce an appreciable error, for presumably a large current is required for the measurement of the low resistance, and the addition of the very small voltmeter current to the ammeter reading is negligible.

The above precautions should be observed also in making resistance measurements.

Example.—It is desired to measure the power taken by a 40-watt tungsten lamp. A 0.5-scale ammeter having a resistance of 0.15 ohm and a 150-scale voltmeter having a resistance of 16,000 ohms are used for the measurement. When the voltmeter is connected inside the ammeter it reads 120 volts and the ammeter reads 0.35 amp. What is the true power taken by the lamp and what is the apparent power if the voltmeter loss is neglected?

Apparent power $= 120 \times 0.35 = 42$ watts. *Ans.*

$$\text{Power taken by voltmeter} = \frac{(120)^2}{16{,}000} = 0.9 \text{ watt.}$$

True power to lamp $= 41.1$ watts. *Ans.*

The voltmeter introduces a 2 per cent. error in this case.

If connected outside the ammeter, the ammeter will now read:

$$0.35 - \frac{120}{16{,}000} = 0.3425 \text{ amp.}$$

The voltmeter will now read:

$$120 + (0.15 \times 0.3425) = 120.05$$

and the apparent power $= 120.05 \times 0.3425 = 41.12$, an error of 0.05 per cent., which is negligible.

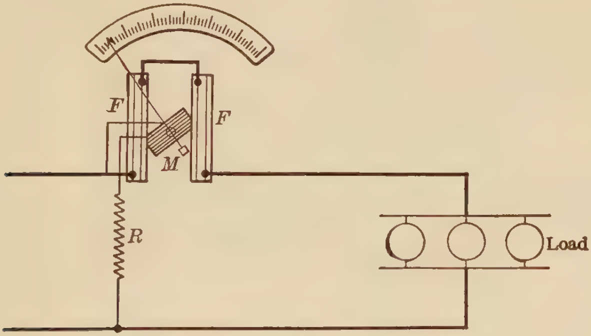


Fig. 155.—The indicating wattmeter.

142. The Wattmeter.—The wattmeter measures power directly. It consists of fixed coils *FF* and a pivoted coil *M*, free to turn within the magnetic field produced by coils *FF*, as shown in Fig. 155. The coils *FF* are wound with comparatively few turns of wire and are capable of carrying the entire current of the circuit. The moving coil *M* is wound with very fine wire and the current is led into it through two control springs in the same manner that current is led into the coil of a Weston instrument. The fixed coil is connected in series with the load in the same manner as an ammeter is connected. The moving coil is connected across the line in series with a high resistance *R* in the same manner as a voltmeter coil is ordinarily connected.

The field of the coils *FF* is proportional to the current and the current in the coil *M* is proportional to the voltage. Therefore the turning moment is proportional to the power of the circuit and it also depends on the angular position of *M* with respect to *FF*, which is taken into consideration when the scale is marked.

Owing to the high degree of accuracy obtainable by the use of the voltmeter and ammeter, the wattmeter is seldom used for direct-current measurements. As it is subject to stray fields,

reversed readings should be taken, that is, both the current and voltage should be reversed and the average of the two readings used. The wattmeter is used more extensively for alternating current than for direct current. A more complete description together with its uses is found in Chap. IV, Vol. IV.

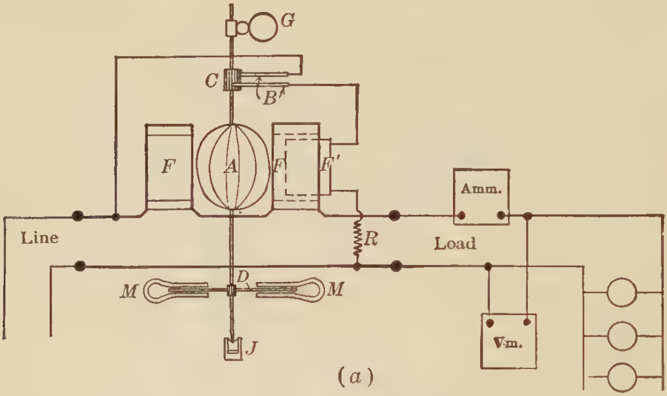


FIG. 156 (a).—Connections of the watthour meter.

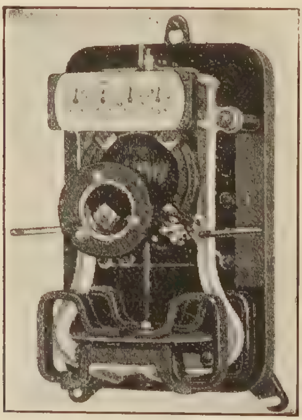


FIG. 156 (b).—Interior of Thomson watthour meter.

143. The Watthour Meter.—The watthour meter is a device for measuring *energy*. (See Par. 66, p. 67.) As energy is the product of power and time, the watthour meter must take into consideration both of these factors. As power is usually sold on an energy basis, many dollars may depend upon the accuracy of

such a meter. Therefore a proper understanding of its mechanism and the method of adjustment is very often essential.

In principle the watthour meter is a small motor whose instantaneous speed is proportional to the power passing through it, and whose total revolutions in a given time are proportional to the total energy or watthours delivered during that time.

Referring to Fig. 156 (*a*), the line is connected to two terminals on the left-hand side of the meter. The upper terminal is connected to two coils *FF* in series, wound with wire sufficiently heavy to carry the maximum current taken by the load, which should not greatly exceed the rated current of the meter. This line terminates at the upper binding post on the right-hand side of the meter. These coils *FF* are wound so that they aid each other and they supply the field in which the armature rotates. The other line wire runs straight through the meter to the load. A shunt circuit is tapped to the upper line on the left-hand side. It runs first to the armature, through the silver brushes *B*, which rest on the small commutator *C*. From the brushes the line passes through coil F', and through a resistance *R* to the lower line wire. This resistance *R* is omitted in certain types of meters.

As the load current passes through *FF*, and there is no iron in circuit, the magnetic field produced by these coils is proportional to the *load current*. As the armature, in series with resistance, is connected directly across the line, the current in the meter armature is proportional to the *line voltage*. Neglecting the small voltage drop in *FF*, the torque acting on the armature must then be proportional to the product of the load current and the load voltage or, in other words, it is proportional to the power passing through the meter to the load.

It can be proved that if the meter is to register correctly, there must be a retarding torque acting on the moving element which is proportional to its speed of rotation. To meet this condition an aluminum disc *D* is pressed on the motor shaft. This disc rotates between the poles of two permanent magnets *MM*. In cutting the field produced by these magnets, eddy currents are set up in the disc, retarding its motion. As the strength of these currents is proportional to the velocity of the disc and they are acting in conjunction with a magnetic field of constant strength,

their retarding effect is proportional to the speed of rotation, so that the condition for correct registration is fulfilled.

Friction cannot be entirely eliminated in the rotating element, even with the most careful construction. Near the rated load of the meter the effect of friction is practically negligible, but since the friction torque is nearly constant it has a much greater proportionate effect at light loads. As the ordinary meter may operate at light loads during a considerable portion of the time, it is desirable that the error due to friction be eliminated. This is accomplished by means of coil F' connected in series with the armature. F' is so connected that its field acts in the same direction as that due to coils FF. Therefore it assists the armature A to rotate. Being connected in the shunt circuit and across practically constant voltage, it is acting continuously and produces nearly constant torque. The coil is movable so that its position can be so adjusted that the friction error is just compensated.

To reduce friction and wear, the rotating element of the meter is made as light as possible. The element rests on a jewel bearing J, which is a sapphire in the smaller sizes and a diamond in the heavier types. The jewel is supported on a spring. A hardened steel pivot rests in the jewel. In time the pivot becomes dulled and the jewel roughened, which increases friction and causes the meter to run more slowly unless F' is readjusted. The moving element turns the clock work of the meter dials through a worm and the gears G.

Figure 156 (*b*) shows the interior view of a Thomson watthour meter.

144. Adjustment of the Watthour Meter.—Even if the initial adjustment be accurate the registration of a watthour meter may, in time, become incorrect. This is due to many causes, such as pitting of the commutator, roughening of the jewel, wear on the pivot, change in the strength of the retarding magnets, etc. As the cost of energy to consumers is largely based on the registration of such meters, it is important that they be kept in adjustment, as a small error in the larger sizes may ultimately mean a difference of many dollars one way or the other.

To adjust the meter it may be loaded as shown in Fig. 156 (*a*). The power taken by the load is measured by a calibrated volt-

meter and ammeter. The revolutions of the disc D are counted over a period of time which is measured with a stop watch. The relation between watt-hours and the revolutions of the *disc*, in most meters, is as follows:

$$W \times H = K \times N, \tag{63}$$

where W is in watts.

H is in hours.

K is the meter "constant" usually found on the disc.

N is the revolutions of the disc.

This equation means that the meter constant multiplied by the revolutions of the disc gives the watt-hours registered by the meter. The gear ratios and clockwork take care of the dial registration.

When checking a meter, the time is usually measured in seconds.

Equation (63) then becomes

$$\frac{W \times t}{3{,}600} = K \times N \tag{64}$$

where t is the time in seconds.

When the meter is tested, the voltmeter and ammeter are read intermittently while the revolutions of the disc are being counted. A run of about a minute gives good results.

Let the average watts determined from the corrected voltmeter and ammeter readings be W_1.

The average watts as indicated by the meter during the same period are, from Eq. (64),

$$W = \frac{K \times N \times 3{,}600}{t} \tag{65}$$

The "per cent. accuracy" of the meter is

$$\frac{100W}{W_1}$$

Example.—In the test of a 10-amp. watthour meter having a constant of 0.4, the disc makes 40 revolutions in 53.6 sec. The average volts and amperes during this period are 116 volts and 9.4 amp. What is the per cent. accuracy of the meter at this load?

Average standard watts $W_1 = 116 \times 9.4 = 1{,}090$.

Average meter watts from Eq. (65)

$$W = \frac{0.4 \times 40 \times 3{,}600}{53.6} = 1{,}074.$$

$$\text{Per cent. accuracy} = \frac{100 \times 1{,}074}{1{,}090} = 98.5 \quad \textit{Ans.}$$

This means that the meter is 1.5 per cent. slow and should be speeded up slightly. With calibrated indicating instruments and careful adjustment, a meter may easily be brought within 0.5 per cent. of accurate registration.

There are two adjustments to be made. Near full load the magnets are moved. If the meter is running slow the magnets are moved nearer the center of the disc where the effect of the retarding currents is reduced, and if the meter is running fast the magnets are moved farther from the center. If the meter has been correctly adjusted near full load, and is found to be in error near light load, the error is obviously due to friction. The light load adjustment (made at from 5 to 10 per cent. rated load) is effected by moving the friction compensating coil F'. If the meter is slow the coil F' is moved in nearer the armature, and if the meter is fast it is pulled out further from the armature. This adjustment of F' may affect the full-load adjustment slightly so that the meter should be rechecked at full load and then again at light load.

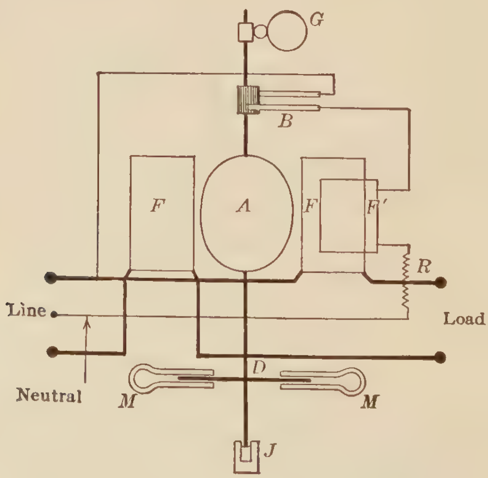


FIG. 157.—Diagram of a 3-wire watthour meter.

145. Other Types of Watthour Meters.—The three-wire meter is designed to register energy upon a three-wire system. It does not differ materially from the meter shown in Fig. 156 except that the two coils FF are connected in opposite sides of the line as shown in Fig. 157. The armature circuit may be connected

to the neutral as shown or it may be connected across the outer wires. If this latter connection is used the neutral connection to the meter is omitted. In the former case the meter does not register accurately unless the voltages between the two outer lines and neutral are equal. This error is usually small.

The meters already described should not be installed near bus-bars, carrying heavy currents because the strength of the meter field and of the retarding magnets may be affected by the stray fields. To eliminate the effect of stray fields an astatic type of meter is used, (Fig. 158). There are two armatures on

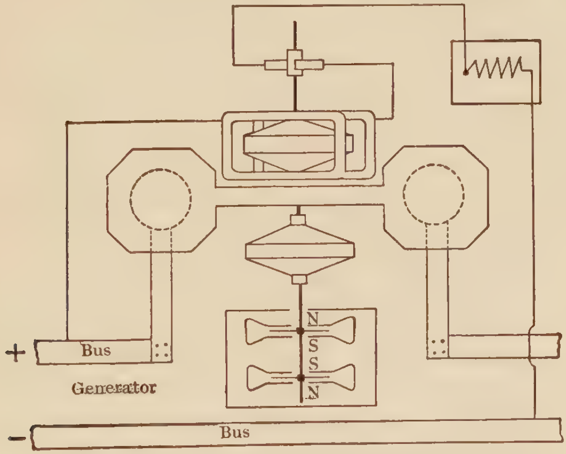


FIG. 158.—Astatic heavy current watthour meter.

the spindle which rotate in the magnetic field created by a single heavy conductor. One armature is above and the other is below the conductor. Any stray field will presumably strengthen the field in which one armature rotates as much as it will weaken the field in which the other armature rotates so that the resulting effect will be nil. There are two sets of retarding magnets. These magnets are so placed that if a stray field increases the strength of one it simultaneously reduces the strength of the other. For further protection these magnets are surrounded by an iron box.

CHAPTER VIII

THE MAGNETIC CIRCUIT

146. The Magnetic Circuit.—Although the general nature and characteristics of magnetism were discussed in both Chaps. I and II, no quantitative relations were considered. If the magnetic properties of a circuit and the ampere-turns linked with this circuit be known, the magnetic flux can be calculated in the same manner that the current in the electric circuit may be calculated if the resistance and voltage be known. In this respect the two circuits are similar. The magnetic circuit differs from the electric circuit in three respects, which makes it difficult to attain the same degree of precision in magnetic calculations as are obtained in electrical calculations.

The electric current has been considered as confined to a definite path, for example, a wire. The surrounding air and the insulating supports for the wire have an extremely high resistance, so that any leakage current which escapes from the wire is negligible compared with the current flowing in the wire or conductor. In the magnetic circuit there is no known insulator for magnetic flux. In fact, the air itself is a fairly good magnetic conductor. Therefore it is impossible to restrict magnetic lines to definite paths in the same way that electric currents are restricted. This is illustrated by the fact that even in the best designed dynamos from 15 to 20 per cent. of the total flux produced leaks across air paths where it cannot be utilized. The presence of this leakage flux may be detected with a compass, and its intensity is often sufficient to magnetize watches even when they are several feet distant from the machine.

Magnetic paths are usually short and have large cross-sections in proportion to their length. They are often so complicated in their geometry that only approximations to their magnetic resistance can be obtained. This often causes errors of considerable magnitude in magnetic calculations. This is very well

illustrated by the air-gap of a dynamo. The geometry of the magnetic path between the armature teeth and slots and the pole-face is extremely complicated, and at best only approximations to the actual distribution of the magnetic flux may be made (see Fig. 41, p. 31).

Under ordinary conditions of use the resistance of most electric conductors is substantially constant, although temperature changes may cause variations of several per cent. Correction for the effect of temperature changes can be accurately made. The magnetic resistance of materials, however, is not constant but varies over wide ranges. This resistance depends to a large extent on the magnetic history of the material. The magnetic resistance of iron may easily increase fifty times when the flux alters from a low to a high magnetic density.

MAGNETIC UNITS

147. Ampere-turns (IN).—The ampere-turns acting on a circuit are given by the product of the turns linked with the circuit and the amperes flowing through these turns. For example, 10 amp. flowing through 150 turns give 1,500 amp.-turns. The same result is produced by 15 amp. flowing through 100 turns. If any ampere-turns act in opposition, they must be subtracted.

In magnetic calculations the c.g.s.[1] (cm.-gm.-sec.) or absolute system is almost always used. Hence the following units are c.g.s. units.

Magnetomotive Force (m.m.f. also F).—Magnetomotive force tends to drive the flux through the circuit and corresponds to e.m.f. in the electric circuit. It is directly proportional to the ampere-turns of the circuit and only differs from the value of the ampere-turns by the constant factor $0.4\pi = 1.257$. That is, $F = 0.4\pi IN = 1.257IN$.

In the c.g.s. magnetic system the magnetomotive force of a circuit is the work in ergs done in carrying a unit north pole once through the entire circuit.

The unit of magnetomotive force is the *gilbert*. The gilberts acting on a circuit are obtained by multiplying the ampere-turns by 0.4π or 1.257.

[1] Par. 54, p. 52.

Reluctance ($\mathcal{R}$).—Reluctance is resistance to the passage of magnetic flux and corresponds to resistance in the electric circuit. The unit of reluctance is that of a centimeter-cube of air. This unit is called the *oersted.*

Permeance ($\mathcal{P}$).—The permeance of a circuit is the reciprocal of the reluctance $\left(\mathcal{P} = \frac{1}{\mathcal{R}}\right)$ and may be defined as that property of the circuit which permits the passage of the magnetic flux or of the lines of induction. It corresponds to conductance in the electric circuit.

Permeability (μ).—The permeability of a material is the ratio of the flux or of the number of lines of induction existing in that material to the flux or the number of lines of induction which would exist if that material were replaced by air, the m.m.f. acting on this part of the circuit remaining unchanged. The permeability of air is taken as unity and with the exception of iron, steel, nickel, liquid oxygen, and certain iron oxides, all materials may be considered as having a permeability of unity. The permeability of commercial iron and steel ranges from 50 and even lower to about 2,000. In special investigations, vacuum-treated iron has attained a permeability of 5,000 and even greater. (Also see Permalloy, p. 204.)

Example.—In a ring solenoid wound on a core similar to that of Fig. 15 (*a*), page 14, the magnetic flux is found to be 4,000 lines or maxwells. When the iron core is removed the flux in air is but 20 lines. What is the permeability of the iron?

Removing the iron core does not change the ampere-turns and the flux path does not change appreciably. Therefore

$$\mu = \frac{4{,}000}{20} = 200. \quad \textit{Ans.}$$

Flux (ϕ).—The magnetic flux is equal to the total number of lines of induction existing in the circuit and corresponds to current in the electric circuit. The unit of flux is the maxwell, but "line of induction" or simply "line" is more often used.

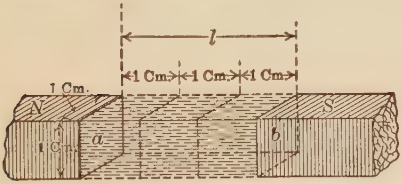
Magnetic Induction or Flux Density (B).—The flux density is the number of maxwells or of lines of induction per unit area, the area being taken at right angles to the direction of the flux. The unit of flux density in the c.g.s. system is one line per square centi-

meter and is called the *gauss*. Flux density is usually expressed in "lines per square centimeter" or "lines per square inch."

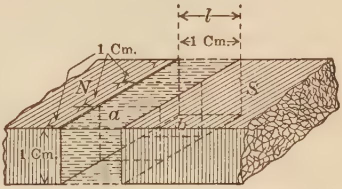
$$B = \frac{\phi}{A}$$

where A is the area and ϕ the flux through and normal to this area.

148. Reluctance of the Magnetic Circuit.—The unit reluctance is defined as that of a centimeter-cube of air. If a portion of a magnetic circuit between pole-faces a and b, Fig. 159 (a),[1] con-



(a) Path whose reluctance is 3 oersteds.



(b) Path whose reluctance is ⅓ oersted.

FIG. 159.—Reluctance of simple magnetic paths.

sists of a path in air having a length of 3 cm. and a cross-section of 1 sq. cm. as shown in the figure, this path is equivalent to three centimeter-cubes placed in series. As the total flux must pass successively through each cube, it is evident that the total reluctance is 3 units (oersteds). *The reluctance is proportional to the length of the flux path.*

On the other hand, if the path has a length of 1 cm. and a cross-section of 3 sq. cm., as shown in Fig. 159 (b), the reluctance of the path through which the flux passes is one-third that of one cube alone, or ⅓ oersted. The reluctance is inversely proportional to the cross-section of the path.

Moreover, if these paths were in iron, having a permeability μ, the flux would be μ times its value in air, provided the same m.m.f. were maintained between the two pole-faces. This means a lower reluctance.

It then follows that *the reluctance of any portion of a magnetic circuit is proportional to its length, inversely proportional to its cross-section and inversely proportional to the permeability of the material.* The constant of proportionality is unity, since the

[1] The actual flux path between pole-faces would not exist as shown in Fig. 159 (a), but the flux would "fringe" as shown in Figs. 15 (b) and 16, pages 14 and 15.

reluctance of a path in air 1 cm. long and 1 sq. cm. cross-section is one oersted. Hence,

$$\mathcal{R}_1 = \frac{l_1}{A_1\mu_1},$$

where l_1 = length in cm. of that part of the circuit under consideration; A_1 = the uniform cross-section in square centimeters of that portion of the circuit; and μ_1 = the permeability of that portion of the circuit.

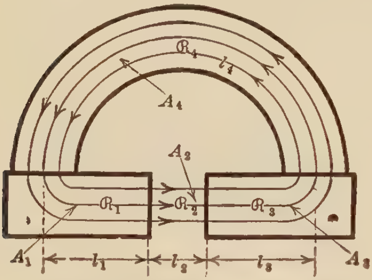


FIG. 160.—Reluctances in series.

If a magnetic circuit consists of several parts in series as shown in Fig. 160, the total reluctance is:

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4$$
$$= \frac{l_1}{A_1\mu_1} + \frac{l_2}{A_2\mu_2} + \frac{l_3}{A_3\mu_3} + \frac{l_4}{A_4\mu_4} \qquad (66)$$

Permeances in parallel are added together to find the total permeance just as conductances in parallel are added together to find the total conductance.

The total permeance

$$\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3 + \mathcal{P}_4$$

and reluctances in parallel combine just as resistances in parallel.

$$\frac{1}{\mathcal{R}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \frac{1}{\mathcal{R}_4}$$

149. Permeability of Iron and Steel.—The permeability of iron or steel depends on the quality of the material, the flux density and the previous magnetic history.

The relation of the flux in iron or steel to the magnetomotive force cannot be expressed by a simple equation. It is necessary to show this relation by a curve called the "magnetization curve." Such a curve for one grade of cast steel is shown in Fig. 161. Abscissas are magnetomotive force in gilberts per centimeter (H), and ordinates are the corresponding flux densities (B).

From O to A the curve concaves upwards slightly. According to Weber's Theory (see p. 4), this is due to the fact that before the application of magnetizing force, the molecular magnets of which the iron is formed are arranged in a haphazard manner,

and a considerable magnetizing force is required before these molecular magnets all commence to take directions corresponding to the applied magnetomotive force.

From A to B the curve is practically a straight line. Beyond B the flux density increases much less rapidly for a given increase

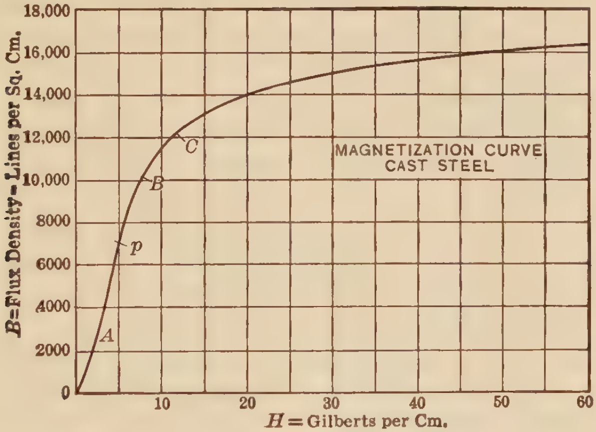


Fig. 161.—Magnetization curve for cast steel.

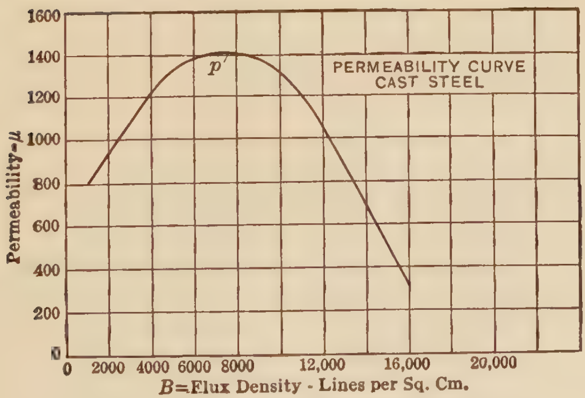


Fig. 162.—Permeability curve for cast steel.

in magnetomotive force and the iron approaches saturation. The point C, where the bend in the curve is very pronounced, is the "knee of the curve." Beyond C the flux can be increased but slightly even with a very great increase in the magnetomotive

force. This, according to Weber's Theory, is due to the fact that all the molecular magnets are now nearly parallel to the direction of the magnetizing force, and a large increase in magnetizing force has little further effect on their direction. The steel is then said to be *saturated.* The type of curve shown in Fig. 161 which starts with zero induction and is taken with increasing values of magnetizing force is called the *normal* saturation or induction curve. Figure 164 shows normal induction curves for other commercial grades of iron.

Figure 162 shows the permeability curve for this same steel. Each ordinate is obtained by dividing B by H for each point of the curve in Fig. 161.

That H, the gilberts per centimeter (also field intensity) is equal numerically to the lines per square centimeter in the air, if the iron were removed and the m.m.f. remains unchanged, may be demonstrated as follows:

Consider a magnetic path in air having a length of l cm. and a uniform cross-section of A sq. cm. The unit of reluctance, the oersted, is equal to the reluctance between opposite faces of a centimeter cube of air. Hence, the reluctance of this path, $\mathcal{R} = l/A$ oersteds. (See Eq. (66), p. 194.) A m.m.f. of F gilberts is impressed on this magnetic path. Hence, the resulting flux

$$\phi = \frac{F}{l/A} = \frac{FA}{l} \text{ maxwells.}$$

The flux density, $B = \dfrac{\phi}{A} = \dfrac{F}{l}$ gausses.

But F/l is equal to the gilberts per centimeter or H,

That is, $H = \dfrac{F}{l}$.

Hence in air, $B = H$ (numerically).

Since, by definition, permeability is equal to the ratio of the flux in the iron to that in air, the m.m.f. remaining unchanged, $\mu = B/H$.

It will be noted that the permeability varies over a wide range. It begins at a comparatively low value, corresponding to the portion OA of the curve (Fig. 161), increases to a maximum at the point p and then decreases with increasing saturation to about one-fifth its maximum value.

150. Law of the Magnetic Circuit.—The relation among flux, magnetomotive force, and reluctance, for the magnetic circuit, is identical with the relation among current, e.m.f., and resistance for the electric circuit.

$$\phi = \frac{F}{\mathcal{R}}. \tag{67}$$

The flux is directly proportional to the magnetomotive force and inversely proportional to the reluctance of the circuit.

If the magnetic circuit consists of several distinct parts having reluctances $\mathcal{R}_1$, $\mathcal{R}_2$., etc., in series and magnetomotive forces F_1, F_2, from Eq. (66)

$$\phi = \frac{F_1 + F_2 + F_3 + \cdots}{\dfrac{l_1}{A_1\mu_1} + \dfrac{l_2}{A_2\mu_2} + \dfrac{l_3}{A_3\mu_3} + \cdots} = \frac{0.4\pi IN}{\dfrac{l_1}{A_1\mu_1} + \dfrac{l_2}{A_2\mu_2} + \dfrac{l_3}{A_3\mu_3} + \cdots} \quad (68)$$

Example.—The ring magnet, Fig. 163, is wound with 250 turns of wire, through which a current of 1.5 amp. flows. Assume the permeability of the iron to be 800. Neglecting fringing, determine the flux in the ring, and also the flux density.

$F = 0.4\pi \times 1.5 \times 250 = 471.$

$l_1 = 18$ in. $= 18 \times 2.54 = 45.7$ cm.

$l_2 = \frac{3}{16}$ in. $= \frac{3}{16} \times 2.54 = 0.476$ cm.

$A_1 = A_2 = 0.2$ sq. in. $= 0.2 \times 2.54 \times 2.54 = 1.29$ sq. cm.

From Eq. (68)

$$\phi = \frac{471}{\dfrac{45.7}{1.29 \times 800} + \dfrac{0.476}{1.29 \times 1.0}} = \frac{471}{0.0443 + 0.369} =$$

1,140 lines (maxwells). *Ans.*

The flux density:

$$B = \frac{1,140}{1.29} = 884 \text{ lines per square centimeter (gausses)} \qquad Ans.$$

$$= 5,700 \text{ lines per square inch.}$$

151. Method of Trial and Error.—Magnetic problems cannot be solved readily by the method used in Par. 150. This is due to the fact that the permeability (which is a variable but is given in the problem as a constant value of 800) is not ordinarily known until the flux density is known and curves similar to those of Figs. 161 and 162 have been consulted. Therefore the permeability is not known until the answer has been determined. As the answer in turn depends upon the permeability, it is usually necessary to resort to trial and error.

Example.—The iron ring of Fig. 163 and Par. 150 is made of cast steel whose permeability curve is given in Fig. 162. The air-gap is reduced to $\frac{1}{16}$ in. Determine the flux and the flux density.

Assume that the permeability is 800.

$$\mathcal{R}_1 = \frac{18.13 \times 2.54}{1.29 \times 800} = 0.0446.$$

$$\mathcal{R}_2 = \frac{\frac{1}{16} \times 2.54}{1.29} = 0.123.$$

$$\phi = \frac{471}{0.0446 + 0.123} = 2{,}810 \text{ maxwells.}$$

$$B = \frac{2{,}810}{1.29} = 2{,}180 \text{ gausses.}$$

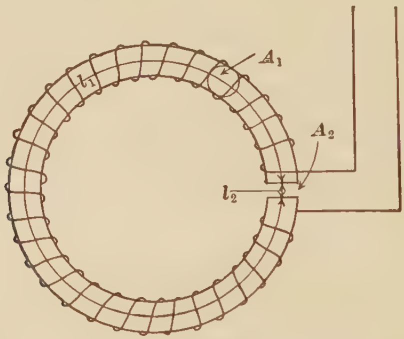
From Fig. 162 the permeability at this density is 980. Therefore $\mathcal{R}_1$ must be recalculated using the new value of permeability.

$$\mathcal{R}_1 = \frac{18.13 \times 2.54}{1.29 \times 980} = 0.0365.$$

$$\phi = \frac{471}{0.0365 + 0.123} = 2{,}950 \text{ maxwells.}$$

The new value of $B = 2{,}290$ gausses.

As the value of μ corresponding to this flux density is 990 or sufficiently close to the value 980 just used, the last two values of flux and flux density are substantially correct.



$l_1 = 18$ in. $A_1 = A_2 = 0.2$ sq.in.
$l_2 = \frac{3}{16}$ in.

Fig. 163.—Ring-type electromagnet.

With magnetic problems which are usually met in practice, the problem is simplified since either the flux or the flux density is known, and it is required to determine the corresponding ampere-turns.

152. Determination of Ampere-turns.—It was shown in Par. 71, Chap. IV, that the voltage drop per unit length of a conductor is independent of the total current but depends only upon the *current density* and the resistivity of the conductor. In a similar manner the magnetomotive force per unit length depends only upon the *flux density* and the reluctivity of the material. This is proved as follows:

Writing Eq. (68) for one portion of the circuit,

$$\phi = \frac{F}{\frac{l}{A\mu}}.$$

Since $\phi = BA$, where A is the cross-section of the path,

$$BA = \frac{F}{\frac{l}{A\mu}}.$$

$$\frac{F}{l} = \frac{B}{\mu}.$$

$$F = \frac{Bl}{\mu}. \qquad (69)$$

The magnetomotive force is equal to the product of the *flux density* and the length of the magnetic path, divided by the permeability of the material. To determine the magnetomotive

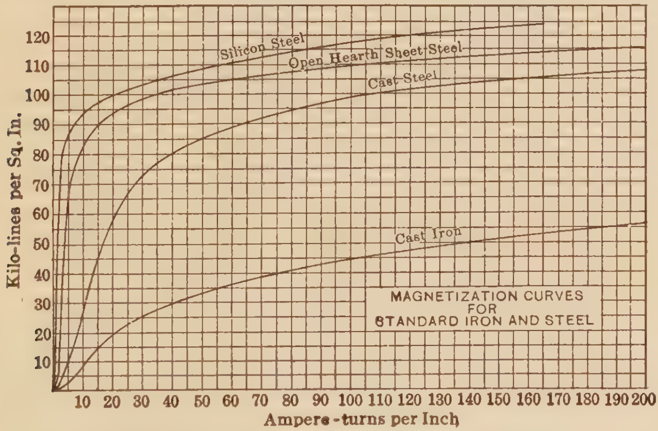


Fig. 164.—Typical magnetization curves.

force for a unit length of a circuit it is only necessary to know the flux density and the permeability. Instead of plotting the permeability against flux density the magnetization curve is usually plotted with ampere-turns per unit length as abscissas and the corresponding flux density as ordinates. This is more convenient and avoids using 0.4π and also the permeability. Such curves are shown in Fig. 164 for various commercial steels used in the manufacture of electrical machinery.

In problems where the flux and the cross-section of the magnetic paths are known, and it is desired to find the requisite ampere-turns to produce this flux, the curves just referred to enable the solution to be readily obtained.

153. Ampere-turns for a Simple Air-gap.—Let it be required to determine the ampere-turns IN necessary to produce a flux ϕ in a simple air-gap between two parallel planes having areas of A sq. cm. and spaced a distance of l cm.

The flux $\phi = BA = \dfrac{0.4\pi IN}{\dfrac{l}{A}}$ (from Eq. 68, p. 197; $\mu = 1.0$)

where B is the flux density. Hence

$$IN = \frac{1}{0.4\pi}Bl = 0.796Bl \tag{70}$$
$$= 0.8Bl \text{ (nearly).}$$

Hence, the ampere-turns for an air-gap depend only on the *flux density* and the *effective length* of gap.

Since the dimensions of machines are frequently given in inch units and the flux densities are best known in lines per square inch, it is often convenient to express Eq. (70) in inch units. Let B' be the flux density in lines per square inch and l' the length of the gap in inches.

$$B' = (2.54)^2B,\ l' = \frac{l}{2.54}$$

$$\text{Hence } IN = 0.796\frac{B'}{(2.54)^2}\cdot 2.54l' = 0.313B'l' \tag{71}$$

Example.—The effective length of the air-gap of a dynamo, after correcting for the effect of the teeth and slots,[1] is 0.25 cm. and the pole-faces have an area of 400 sq. cm. If the total flux is 2,400,000 maxwells, determine the ampere-turns necessary to produce this flux in the gap.

$$B = \frac{2,400,000}{400} = 6,000 \text{ gausses.}$$

$IN = 0.796 \times 6,000 \times 0.25 = 1,194$ (from Eq. 70). *Ans.*

If inch units are used, A' = the area

$$A' = \frac{400}{6.45} = 62.0 \text{ sq. in.}$$

$$B' = \frac{2,400,000}{62.0} = 38,700.$$

[1] The methods of making this correction may be found in "Principles of Direct-current Machines," by A. S. Langsdorf, Chap. IV.

The length

$$l' = \frac{0.25}{2.54} = 0.0984 \text{ in.}$$

Using Eq. (71),

$$IN = 0.313 \times 38{,}700 \times 0.0984 = 1{,}192 \text{ (check)}$$

154. Use of the Magnetization Curves.—To illustrate the use of the magnetization curves the following problem is given.

Example.—Determine the ampere-turns necessary to produce an air-gap flux of 750,000 lines in the electromagnet of Fig. 165. The cores are cast iron and the yoke and pole-pieces are cast steel. Neglect fringing and leakage.

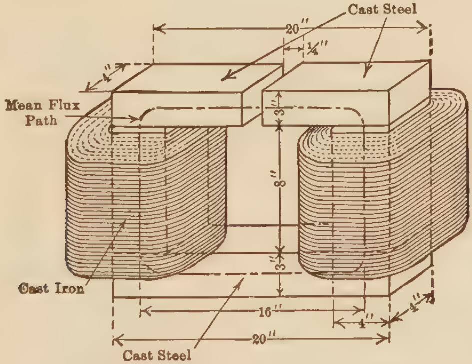


Fig. 165.—Typical electromagnet.

The flux density in the lower yoke:

$$B_1 = \frac{750{,}000}{3 \times 4} = 62{,}500.$$

The ampere-turns per inch for a density of 62,500, from Fig. 164 (cast steel), is 23.

The mean length of flux path is (approximately) 16 in.

$$I_1N_1 = 16 \times 23 = 368.$$

or 368 ampere-turns is required to produce a flux of 750,000 lines in the lower yoke.

The density in the cores is

$$B_2 = \frac{750{,}000}{4 \times 4} = 46{,}900.$$

From the curve (cast iron) the ampere-turns per inch = 118.

As there are two cores, the total length will be 16 in.

$$I_2N_2 = 16 \times 118 = 1{,}890.$$

The pole-pieces are in every way identical with the yoke, except that the path is 0.25 in. shorter. This small difference will not make any appreciable error, so the ampere-turns for the two pole-pieces are:

$$I_3N_3 = 368.$$

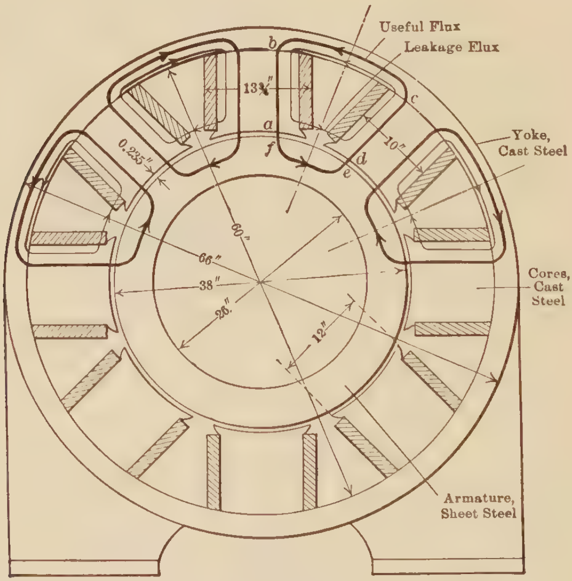
The ampere-turns for the air-gap (from Eq. 71).

$$I_4N_4 = 0.313 \times 62{,}500 \times 0.25 = 4{,}900.$$

As all the various parts are in series the total ampere-turns =

$$368 + 1{,}890 + 368 + 4{,}900 = 7{,}526. \quad \textit{Ans.}$$

155. Magnetic Calculations in Dynamos.—The magnetic circuits of dynamos have already been discussed in Chap. II. The calculation of the exciting ampere-turns is somewhat complicated by the irregular nature of the air-gap, due to the armature teeth, air ducts, fringing, etc. The amount of leakage flux between poles introduces another factor which must be considered.



Axial length of armature stampings and pole-faces = 16 in.

FIG. 166.—8-pole, 100 r.p.m., 250-volt d.c. generator.

As a simple example of such calculations, consider the dynamo shown in Fig. 166. It is desired to send a flux of 7,500,000 lines from each pole into the armature. The air-gap has an effective length of 0.235 in., after correction has been made for armature teeth, fringing, etc.[1] The leakage coefficient (ratio of core flux to armature flux) is equal to 1.15.

The paths of the fluxes from the various poles, including the leakage flux, are shown in the figure. The lengths of path are easily determined. Consider the flux path *abcdef*.

[1] See footnote, p. 200.

The length $ab = \frac{60 - 38}{2} - 0.235 = 10.8$ in. (approximately); bc (approximately one-eighth the mean circumference of the yoke, less 5 in.) $= \frac{\pi 63''}{8} - 5 = 24.7 - 5 = 19.7$ in.

$$fe = \frac{\pi 32}{8} = 12.6 \text{ in. (approximately).}$$

The flux densities are as follows:

Flux in cores $= 7{,}500{,}000 \times 1.15 = 8{,}630{,}000$ as the flux in the core is equal to the armature flux plus the leakage flux.

Flux density in cores $= \frac{8{,}630{,}000}{16 \times 10} = 54{,}000.$

Flux density in yoke $= \frac{8{,}630{,}000}{2(16 \times 3)} = 90{,}000$ as the pole flux divides, one half going each way in the yoke.

Flux density in armature $= \frac{7{,}500{,}000}{2(6 \times 16)} = 39{,}000.$

This must be increased about 25 per cent. to allow for the air duct space and the spaces between laminations.

This makes the density in the armature:

$$39{,}000 \times 1.25 = 48{,}800.$$

The air-gap density $= \frac{7{,}500{,}000}{16 \times 12} = 39{,}000.$

Knowing the above factors, and utilizing the magnetization curves of Fig. 164, it is a comparatively simple matter to determine the total ampere-turns per pole.

For 54,000 lines per square inch, 19 ampere-turns per inch are necessary for cast steel (Fig. 164). Therefore for ab:

Core ab I_1N_1	$= 19 \times 10.8$	$=$ 205 (cast steel).
Yoke bc I_2N_2	$= 64 \times 19.7$	$=$ 1,260 (cast steel).
Core cd I_3N_3	$= I_1N_1$	$=$ 205 (cast steel).
Gap de I_4N_4	$= 0.313 \times 39{,}000 \times 0.235$	$=$ 2,870 (air). (See Eq. 71)
Arm. ef I_5N_5	$= 3 \times 12.6$	$=$ 38 (O. H. sheet steel).
Gap fa I_6N_6	$= I_4N_4$	$=$ 2,870 (air).
	Total	$=$ 7,448 ampere-turns.

As two poles in series supply the excitation for this flux the ampere-turns per pole are

$$IN = \frac{7{,}448}{2} = 3{,}724. \quad \textit{Ans.}$$

As this machine is symmetrical, each complete magnetic circuit requires this same number of ampere-turns per pole. The design of the exciting coils themselves is not a difficult matter.

156. Permalloy.—Permalloy is a recently discovered nickel-iron alloy, which has abnormally high magnetic properties at very low values of magnetizing force. Its composition is approximately 78.2 per cent. nickel, 21.4 per cent. iron, the remainder being impurities.

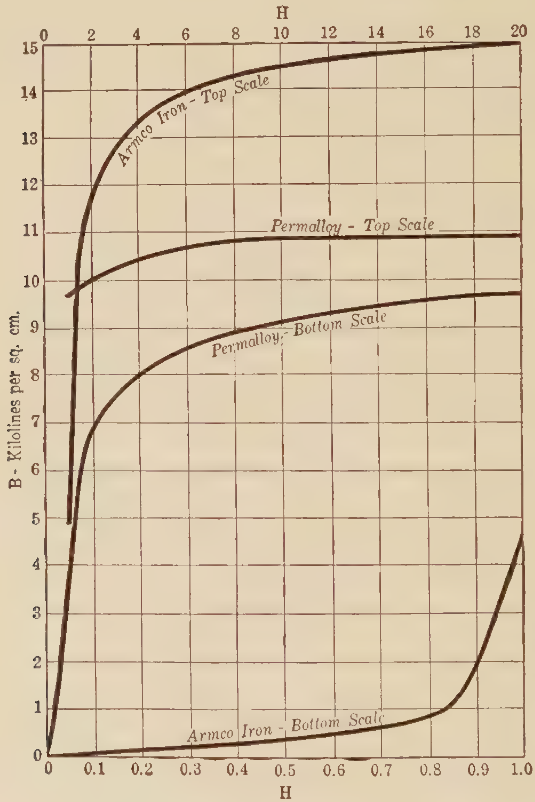


FIG. 167.—*B-H* curves of permalloy and Armco iron.

The maximum permeability of ordinary permalloy is approximately 87,000, although values much higher than this have been found in specially treated samples. This is very much higher than any values of permeability which have heretofore been found for iron or for any of the other iron or steel alloys. This maximum permeability in permalloy occurs at a flux density of 5,000 gausses, corresponding to a magnetizing force of 0.058

gilberts per centimeter, a value very much lower than those at which ordinary iron and steel obtain their maximum permeabilities. In fact, permalloy becomes highly saturated in the earth's field alone ($H = 0.45$) so that extreme care must be exercised in testing it.

Figure 167 gives a *B-H* curve for permalloy, and, for comparison, the curve of Armco iron, which is almost pure iron. It will be noticed that only at low values of H is permalloy superior to Armco iron in its magnetic properties.

The hysteresis loop of permalloy, when carried to a maximum induction of 5,000 gausses, has only one-sixteenth the area of that of iron.

The magnetic properties of permalloy are very susceptible to heat treatment, so that it must be carefully prepared and in very thin strips. Its principal use at present is in communication work, particularly in the "loading" of submarine cables.[1]

157. Hysteresis.—If the magnetomotive force per centimeter or magnetizing force acting on an iron sample begins at zero and is increased, the relation between magnetizing force and flux density (or induction) will be similar to that shown by curve Oa (Fig. 168). This curve is called the *normal saturation* or magnetization curve and has already been discussed.

If the magnetizing force is now decreased, the induction will *not* decrease along the line aO, but will decrease less rapidly along ab. When point b is reached, the magnetizing force is zero but the magnetic induction has not reached zero. The flux density Ob is called the *remanence.* Before the flux density can be reduced to zero, the magnetizing force must be reversed in direction. That is, it requires a negative magnetizing force Oc to reduce the flux density to zero. The magnetizing force Oc is called the *coercive force.*

If now the magnetizing force be increased in the negative direction to d' where $Od' = Oa'$, the flux density will be carried to a negative maximum $d'd$. The negative maximum flux density

[1] For further details see "Permalloy, an Alloy of Remarkable Magnetic Properties," by H. D. Arnold and G. W. Elmen, *Jour.* of the Franklin Institute, p. 621, May 1923. Reprinted July 1923 in *Bell System Technical Journal;* A. E. Kennelly, "The Reluctivity of the Recently Discovered Magnetic Metal Permalloy," *Jour. of the Franklin Institute*, May 1924.

$d'd$ is equal to $a'a$. If the magnetizing force is now increased toward zero, the curve will pass through point c when the magnetizing force is again zero and the negative remanence $Oc = Ob$. A positive coercive force $Of = Oc$ is necessary to bring the flux density again to zero. When the magnetizing force again becomes Oa' the flux density will return to its original value at a, closing the loop.

It is thus seen that the induction always lags the magnetizing force. For example, at b, the magnetizing force has reached zero; the induction, however, does not reach zero until the iron has been carried further along the cycle of magnetization to c. This lag of the induction behind the magnetizing force has been given the name *hysteresis*. The cycle of magnetization *abcdefa* (Fig. 168) is called a *hysteresis loop*.

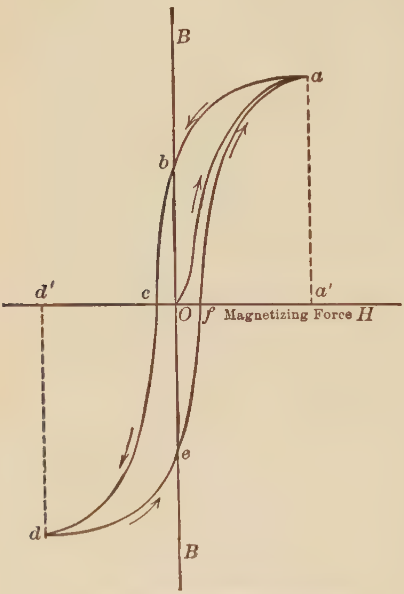


FIG. 168.—Hysteresis loop.

The iron does not return to a previous condition of magnetization without the application of a magnetizing force. For example, it requires a magnetizing force oc to bring the iron to zero induction at c. (The state of magnetization at c is also different from that of O). Hence an expenditure of energy is required to carry the iron through the cycle of magnetization. According to the Weber theory of molecular magnetism (see Par. 7) this expenditure of energy is due to the friction of the molecular magnets in changing their directions with change of magnetizing force.

If several loops are taken, each having different maximum flux densities, they will have the appearance of the three loops shown in Fig. 169. The maximum points a, a_1, a_2 all lie along the normal saturation curve Oa_2.

158. Hysteresis Loss.—The hysteresis loss is proportional to the area of the hysteresis loop (Figs. 168 and 169). In fact the hysteresis loss may be obtained by finding the area of the loop to scale, and dividing by 4π. This gives the loss in ergs per cycle.

For example, let the area of the smallest loop (Fig. 169) be A sq. in. The scale is such that 1 in. on the abscissa scale represents 10 gilberts per centimeter, and 1 in. on the ordinate scale represent 4 kilogausses. The ergs loss per cycle per cubic centimeter is:

$$W_h = \frac{A \times 10 \times 4{,}000}{4\pi} \text{ ergs.}$$

To convert this energy loss into joules or watt-seconds divide by 10^7.

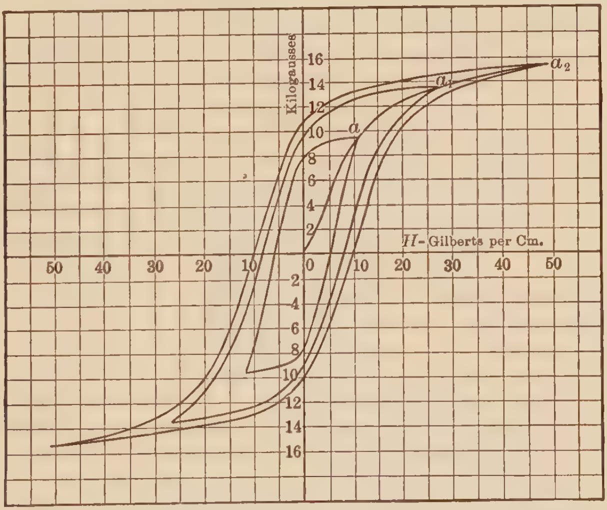


Fig. 169.—Hysteresis loops for three maximum flux densities.

The hysteresis loss per cycle depends upon two factors, the magnetic material and the maximum flux density. The loss within certain limits may be expressed by the Steinmetz law as follows:

$$W_h = \eta B^{1.6} \tag{72}$$

W_h is the hysteresis loss per cubic centimeter in ergs per cycle, η is a constant depending on the material, and B is the maximum flux density in gausses.

Below are given a few typical values of η:

Hard cast steel	0.025	Sheet iron	0.004
Forged steel	0.020	Silicon sheet steel	0.0010
Cast iron	0.013	Silicon steel	0.0009

Example.—What will be the ergs loss per cycle in a core of sheet iron having a net volume of 40 cu. cm., in which the maximum flux density is 8,000 gausses?

$W_h = 0.004 \times 8{,}000^{1.6}$

$$\log 8{,}000 = 3.9031$$
$$1.6 \times 3.9031 = 6.2449$$
$$\log 1{,}757{,}000 = 6.2449$$

$W_h = 0.004 \times 1{,}757{,}000 = 7{,}028$ ergs per cubic centimeter per cycle.
Total loss $W = 7{,}028 \times 40 = 281{,}000$ ergs per cycle. *Ans.*

INDUCTANCE

159. Linkages.—If a current flows in a conductor, a magnetic flux is set up about the conductor. This magnetic flux com-

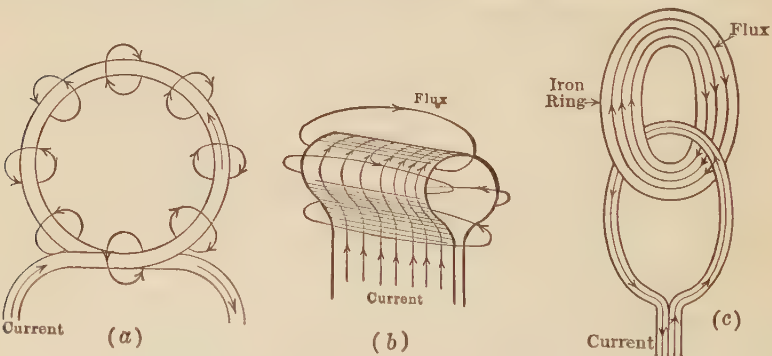


FIG. 170.—Illustrations of flux-current linkages.

pletely encircles the conductor and the current in the conductor completely encircles the flux. Some familiar examples of this are given in Fig. 170, where the currents and related fluxes are shown. As a current and the resulting flux always completely encircle each other they are said to *link* with each other. This is shown particularly well in Fig. 170 (*c*), where a conductor carrying a current is linked with an anchor ring.

The product of the turns of conductor and the number of lines of flux linking these turns is called the *linkages* of the circuit. These linkages will be given in c.g.s. units if the flux is given in maxwells.

Example.—A certain solenoid has 800 turns. A current of 5 amp. flowing in the winding produces a flux of 2,500,000 lines. What are the c.g.s. linkages?

$$800 \times 2{,}500{,}000 = 20 \times 10^8 \text{ linkages (c.g.s. units).} \quad \textit{Ans.}$$

When the reluctance of the magnetic circuit is constant[1] the number of *these linkages per unit current* is called the *inductance* of the circuit and is represented by the symbol "L," implying linkages. The unit of inductance is the *henry.*

Inductance from definition:

$$L = \frac{N\phi}{I \times 10^8} \tag{73}$$

where L is the inductance in henrys, ϕ is the flux in maxwells, and I is the current in amperes.

Note.—It is necessary to divide by 10^8 because 10^8 c.g.s. magnetic lines or maxwells are equal to *one* line in the practical system of volts, amperes, etc.

Example.—What is the inductance of the above circuit?

$$L = \frac{20 \times 10^8}{5 \times 10^8} = 4.0 \text{ henrys.} \quad \textit{Ans.}$$

The definition of self-inductance given in the Smithsonian Physical Tables follows from the foregoing relationships.

Self-inductance is for any circuit the electromotive force produced in it by a unit rate of change of current. (Also see Eq. (75), p. 216.)

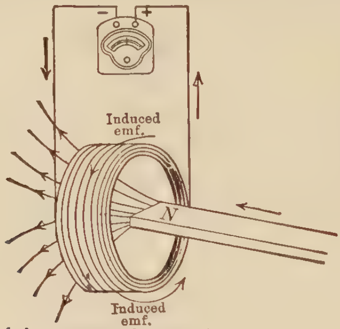
160. Induced Electromotive Force.—If the terminals of an insulated coil, Fig. 171 (*a*), be connected to a galvanometer, and a magnetic field be set up through this coil, either by thrusting a bar magnet into the coil or by some other means, the galvanometer will be observed to deflect momentarily and then to return to rest. This shows that an e.m.f. has been temporarily induced in the coil. When the flux through the coil has ceased to change, this electromotive force also ceases. If investigation be made, it will be found that the direction of this induced electromotive force is that shown in the figure and that this direction is such

[1] When the reluctance of the magnetic circuit is *not* constant, the inductance is equal to the product of the number of turns and the *rate of change* of flux divided by the rate of change of current and 10^8. For example, if $N = 500$, and the flux is changing at a rate 2×10^5 maxwells per second when the current is changing at the rate of 200 amp. per second, the inductance

$$L = 500\frac{2 \times 10^5}{200 \times 10} = 0.005 \text{ henry}$$

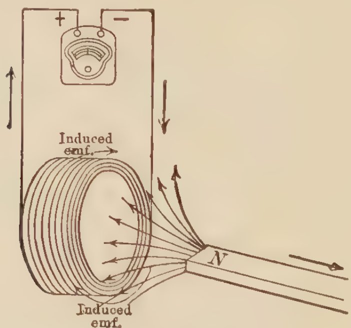
that if the e m.f. be allowed to produce a current, this current will tend to push the bar magnet *out* of the coil, or what is the same thing, will oppose its entering the coil.

If the magnet be withdrawn from the coil, Fig. 171 (*b*), the galvanometer will be observed to deflect again, momentarily as before, but the deflection is opposite to its direction in the first case. The direction of the induced electromotive force is now



(*a*) North pole inserted in coil

Fig. 171(*a*).—Induced electromotive force.



(*b*) North pole withdrawn

Fig. 171(*b*).—Induced electromotive force.

such that if the e m.f. produces a current, this current will tend to prevent the magnet from being withdrawn from the coil. The electromotive force in each case is transient and ceases when the *change* of flux through the coil ceases.

If careful measurements be made, the value of this electromotive force will be found to depend upon: (1) the number of turns in the coil; (2) the rate at which the flux linked with the coil changes.

The average electromotive force in volts is given by

$$e = -\frac{N\phi 10^{-8}}{t}, \tag{74}$$

where N is the number of turns in the coil, ϕ is the total change of flux in lines linked with the coil, and t is the time in seconds required to insert or withdraw this flux from the coil. 10^{-8} reduces the flux ϕ to practical units so that e becomes volts. The minus sign indicates that the induced e.m.f. is in opposition to the force which produces it.

$\frac{\phi}{t}$ is the average rate of change of flux, so that the induced electromotive force may be said to be proportional to the *number of turns* and the *rate of change of flux.*

Example.—A flux of 1,500,000 lines links a coil having 350 turns. This flux through the coil is decreased at a uniform rate to zero in 0.2 sec. What is the induced electromotive force during the time of withdrawal?

$$e = 350\frac{1,500,000}{0.2}10^{-8}$$

$$= 26.25 \text{ volts.} \quad \textit{Ans.}$$

The fact that the currents produced by induction oppose the motion producing them should be carefully noted, for this principle is manifest in practically all types of electric machinery. This principle was first formulated by Lenz, in a form known as Lenz's law which says:

"*In all cases of electromagnetic induction, the induced currents have such a direction that their reaction tends to stop the motion which produces them.*"

This law is also based upon the law of the conservation of energy. That is, the induced currents, which represent energy, are produced at the expense of the mechanical energy required to push the magnet in the coil against their opposition, or the energy required to withdraw the magnet against the opposition of the induced currents, which try to prevent this withdrawal.

161. Electromotive Force of Self-induction.—If a coil be connected to a battery and a switch S closed (Fig. 172), current will begin to flow in the coil. This current produces a flux linking the coil. As this flux increases it must induce an e.m.f. in the coil, the magnitude of which depends on the number of turns in the

coil and the rate at which the flux increases. By Lenz's law, and also from a consideration of Fig. 171 (*a*), the electromotive force thus induced must have such a direction as to oppose the increase in the flux linking the coil and hence must *oppose* any increase of current. Therefore this current cannot reach its

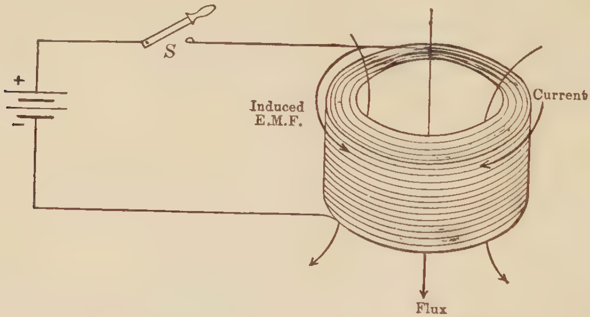


FIG. 172.—Relation of e.m.f. of self-induction to current.

maximum value at once, but is retarded in its rise by the opposing electromotive force of self-induction.

In Fig. 173 is shown the rise of current in a circuit containing resistance only, the impressed voltage being 10 volts and the resistance 20 ohms. When the switch S is closed the current reaches its maximum or Ohm's law value of 0.5 amp. at once.

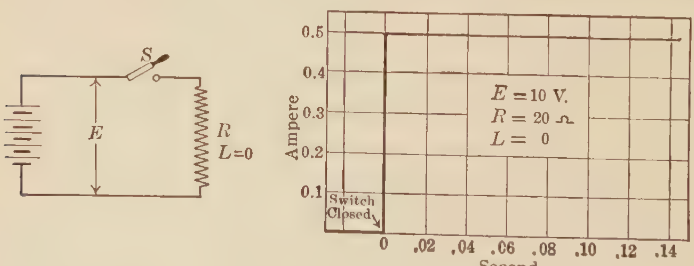


FIG. 173.—Rise of current in a non-inductive circuit.

With the inductive circuit, the current gradually approaches its Ohm's law value as shown in Fig. 174. To be exact, it takes an infinite time for the current to reach its Ohm's law value, although in a comparatively short time it reaches substantially this value. An idea of the time required to build up a current in

an inductive circuit may be obtained from the inductance and the resistance of the circuit. The ratio of the inductance in *henrys* to the resistance in ohms, L/R, is called the *time constant* of the circuit. This is the time in seconds required for the current to reach 63.2 per cent. of its final value. It is a measure of the rapidity with which the current in a circuit rises to its ultimate value. Figure 174[1] shows the rise of current in a circuit whose impressed voltage is 10 volts, resistance 20 ohms and inductance

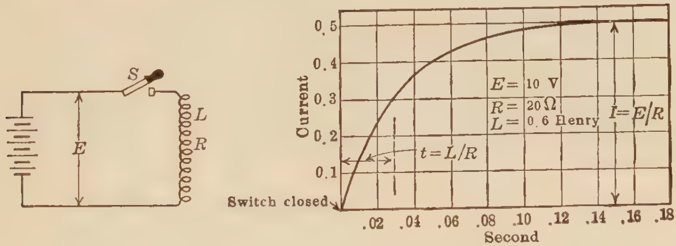


FIG. 174.—Rise of current in an inductive circuit.

0.6 henry. The time constant of *this* circuit is $0.6/20 = 0.03$ sec. and is shown on the diagram. This curve should be compared with Fig. 173, in which the circuit has the same impressed voltage and the same resistance but has no inductance.

Example.—A relay having a resistance of 400 ohms and an inductance of 0.4 henry is connected across a 110-volt circuit. What is the time constant of the relay? To what value does the current in the relay rise in this time?

$$\text{The time constant} = \frac{0.4}{400} = 0.001 \text{ sec.} \quad \textit{Ans.}$$

$$i = 0.632 \frac{110}{400} = 0.1738 \text{ amp.} \quad \textit{Ans.}$$

This delayed rise of current in a circuit due to self-inductance should be carefully kept in mind, since it accounts for some of the time lag observed in relays, trip coils, etc. When a short-circuit takes place there may be considerable delay between the time at which the short-circuit occurs and the opening of the breaker or switch controlled by the relay. The effect of induc-

[1] The equation of the curve showing the rise of current is $i = \frac{E}{R}\left(1 - \epsilon^{\frac{-Rt}{L}}\right)$ where E is the impressed voltage, i = current at time, t sec. after closing switch and ϵ the Napierian logarithmic base. The current increases at a rate of E/L amp. per second at the instant when the switch is closed.

tance is also one of the controlling factors in the initial current-rush on short-circuit.

If an inductive circuit carrying current be short-circuited, the current does not cease immediately, as it does in a non-inductive circuit under similar conditions, but continues to flow and does not become zero until an appreciable time after the instant of short-circuit. This again is due to the electromotive force of self-induction. The flux linking the coil is due to the current, and when the current decreases, this flux also decreases. In decreasing, the flux induces an electromotive force in the coil. In the same way that the current due to the induced electromotive force tended to prevent the flux being withdrawn in Fig. 171 (b), so now the electromotive force of self-induction tends to prevent the decrease of the current.

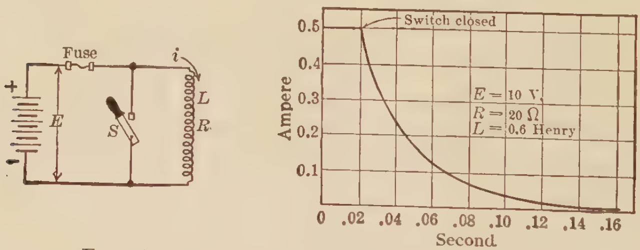


FIG. 175.—Decay of current in an inductive circuit.

A curve[1] showing the decrease of the current with time is given in Fig. 175. The circuit has the same constants as the circuit shown in Fig. 174. It is usually advisable to fuse the battery so that it will not be injured, since short-circuiting the inductive circuit also short-circuits the battery, as is shown in Fig. 175.

It thus appears that the effect of inductance is always to oppose any change in circuit conditions. If the current tends to increase, inductance opposes it; if it tends to decrease, inductance tends to oppose this decrease. Inductance corresponds to inertia in mechanics. A body having inertia opposes any force tending to set it in motion when the body is at rest, and if the body is in

[1] The equation of this curve,

$$i = I_0 \epsilon^{-\frac{Rt}{L}}$$

where i is the value of the current at a time, t sec. after the closing of the switch, and I_0 is the initial value of current.

motion, inertia opposes any force tending to bring the body to rest.

If, after having established the current in the circuit of Fig. 174, the switch S be opened, a noticeable arc will appear at the switch blades. This arc will be much greater in magnitude than that formed at the contacts of the switch in the circuit of Fig. 173, with resistance only in the circuit, although the current and circuit voltage are the same in each case. This arc is due to the electromotive force of self-induction and in some circuits may have such a value as to cause severe arcing at the switch contacts. In fact this voltage has been known to reach such values in alternator fields as to puncture their insulation when the field

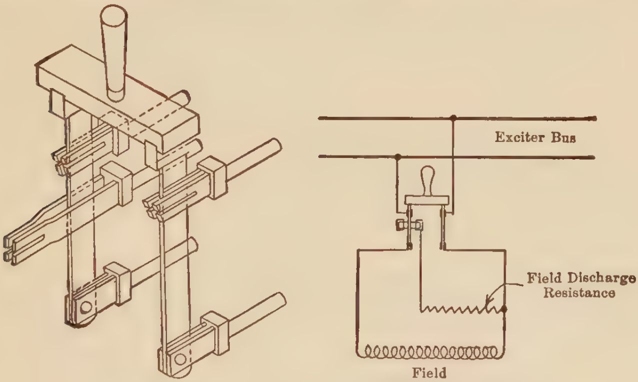


FIG. 176.—Field-discharge switch with connections.

circuit is opened. To protect the field from puncture, a field-discharge switch shown in Fig. 176 is often used. At the instant of opening the switch the field (and the line temporarily) is paralleled by the field discharge resistance. The energy of the field is dissipated partly in this resistance rather than at the switch contacts. Contact with switches opening inductive circuits, even in the case of very low voltages, should be carefully avoided. Not only is there the danger of being burned by the arc, but of being injured from the high induced voltages as well.

Calculation of the Electromotive Force of Self-induction.—From Eq. (74) page 211, the electromotive force induced in a coil due to a change in the flux linking the coil is

$$e = -N\frac{\phi}{t}10^{-8}$$

where N is the number of turns, and ϕ/t the rate at which the flux changes.

Remembering that

$$L = \frac{N\phi}{I}10^{-8} \text{ or } N\phi 10^{-8} = LI \text{ (Eq. 73, page 209),}$$

and also that the electromotive force of self-induction opposes the change in current, its value may be written:

$$e = -\frac{N\phi 10^{-8}}{t} = -L\frac{I}{t} \tag{75}$$

The electromotive force of self-induction is proportional to the product of the inductance and the rate of change of current with respect to time. The minus sign indicates that this electromotive force *opposes* the change of current.

If the inductance varies as well as the flux, Eq. (75) may be written:

$$e = -\left(L\frac{I}{t} \pm I\frac{L}{t}\right) \tag{76}$$

the additional term accounting for the electromotive force due to any change in the inductance.

Example.—The field circuit of a generator has an inductance of 6 henrys. If the field current of 12 amp. is interrupted in 0.05 sec., what is the average induced electromotive force in the field winding?

$$e = -6\frac{12}{0.05} = -1{,}440 \text{ volts.} \quad \textit{Ans.}$$

162. Energy of the Magnetic Field.—To *establish* a magnetic field energy must be expended. To maintain a constant field does not require an expending of energy even in electromagnets. The energy lost in the exciting coils of electromagnets is accounted for as heat in the copper and is not concerned with the energy of the magnetic field itself. The energy of the magnetic field is stored or potential energy and is similar to the energy of a raised weight, Fig. 177. Work is performed in raising the weight to its position, but *no expenditure of energy is required to maintain the weight in this position.* The energy of the weight due to its position is Wh ft.-lb., where W is the weight in pounds and h the height in feet through which the weight has been raised. This energy is available and can be utilized in many ways.

In the same way the energy stored in the magnetic field is available and may make itself manifest in many ways, as, for example, the arc at the switch contacts. In an alternating-current circuit this energy may all be returned to the circuit.

The energy of the field in joules, or watt-seconds, is

$$W^1 = \frac{1}{2}LI^2 \qquad (77)$$

where L is the circuit inductance in henrys and I the current flowing.

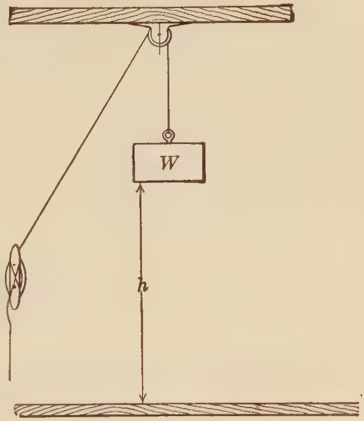


FIG. 177.—Energy of a suspended weight.

Example.—In a circuit having an inductance of 4 henrys, the current is 10 amp. What is the energy of the magnetic field? If this circuit is interrupted in 0.2 sec., what is the average value of the power expended by the magnetic field during this time?

$$W = \frac{1}{2} \times 4 \times 10^2 = 200 \text{ watt-sec.}$$

Ans.

$$P = \frac{200}{0.2} = 1{,}000 \text{ watts} = 1 \text{ kw.} \quad \textit{Ans.}$$

Equation (77) shows that the energy of the magnetic field is proportional to the *square* of the current. Therefore if the current can be reduced by a suitable resistance to one-half its initial value before opening a highly inductive circuit, the energy of the arc at the switch contacts can be reduced to one-fourth of its initial value. This fact should be remembered when opening the field circuit of a dynamo.

A very common use of the electromotive force of self-induction occurs in the so-called spark coil used for gas lighting. This coil consists of a considerable number of turns of wire wound on a laminated iron core. The core is usually made of iron wires as shown in Fig. 178. The coil is connected between the bell-ringing battery B and the grounded gas pipe. The other ter-

[1] The resemblance of this expression to that for stored energy in a moving body should be noted. For example, when a body having a mass M moves with a velocity V, the kinetic energy is $\frac{1}{2}MV^2$. With a rotating body having a moment of inertia I and angular velocity ω, the kinetic energy is $\frac{1}{2}I\omega^2$.

minal of the battery is connected directly to the insulated contact on the gas burner. When the two contacts on the burner meet, the circuit is closed, and a magnetic field is established in the laminated core of the spark coil. As the two contacts of the burner separate, they snap apart and the circuit is broken suddenly. Consequently, a high electromotive force of self-induction is produced in the spark coil. This causes a hot arc at the contacts, which ignites the gas, the gas being turned on simultaneously with the closing of the contact points and by the same mechanism.

The spark coil may be considered as having a magnetic field which is built up as the two contacts at the gas jet wipe by each

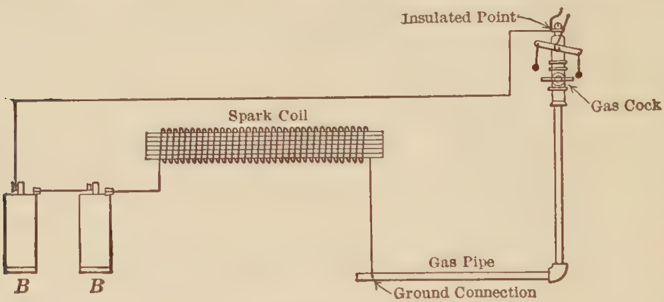


Fig. 178.—Electric gas ignition.

other. Energy is thus stored slowly in the magnetic field. When this energy is released suddenly by the contacts snapping open, considerable power is developed resulting in a hot spark at the contact points.

This same principle is applied to "make-and-break" ignition used with internal combustion engines. Two contacts within the cylinder head are in series with a low-voltage source usually six dry cells, and an "ignition coil" not unlike the spark coil (Fig. 178). The contacts are made to open and close by means of a cam mechanism. When the contacts are closed, a magnetic field is built up in the ignition coil, thus storing magnetic energy. When the contacts are made to open, this energy appears as a hot spark at the contacts, and this spark ignites the explosive mixture in the cylinder.

163. Mutual Inductance.—In Fig. 179 are shown two coils, A and B. Coil A is connected to a battery through a switch S.

Coil B is not connected to any source of voltage, but to a galvanometer. Coil B is placed so that its axis is nearly coincident with that of A and the two coils are close together. When the switch S is closed, current flows in coil A, building up a field which links the coil. The position of B with respect to A results in a considerable part of the magnetic flux produced by A linking B. Therefore, if the current in A be interrupted by opening the switch S, or if it be altered in magnitude, a *change of flux*

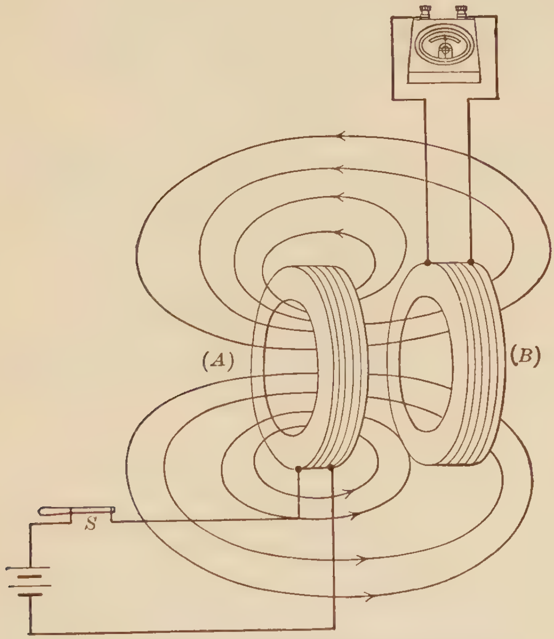


FIG. 179.—Mutual inductance between two coils.

simultaneously occurs in B inducing an e.m.f. in B. This e.m.f. is detected by the galvanometer connected across the terminals of B. Upon closing the switch S the galvanometer will deflect momentarily, and upon opening the switch S its deflection will reverse, showing that the induced voltage on opening the circuit is opposite in direction to the induced voltage on closing the circuit. Because coil B is in such a relation to A that an e.m.f. is induced in B due to the change of flux in A, these two coils are said to possess *mutual inductance*. The induced e.m.f. is an

electromotive force of mutual induction and its magnitude, Eq. (74), page 211, is

$$e_2 = N_2 \frac{\phi_2}{t} 10^{-8} \text{ volts,}$$

where N_2 is the number of turns in coil B, ϕ_2 the change in magnetic flux from coil A which links coil B, and t the time in seconds required to change the flux by ϕ_2 lines.

Even though coils A and B be brought close together, all the flux, ϕ_1, produced by coil A does not link coil B. Only a certain proportion, K, of ϕ_1 links B, K being less than unity. That is:

$$e_2 = N_2 \frac{K\phi_1}{t} 10^{-8} \text{ volts,} \tag{78}$$

K is often called the *coefficient of coupling* of the circuits A and B. As N_2 and K are constants for any given arrangement of the circuits and ϕ_1 may be assumed proportional to I_1, the current in coil A, Eq. (78), may be written

$$e_2 = M \frac{I_1}{t} \text{ volts} \tag{79}$$

where M is the *mutual inductance* or coefficient of mutual induction in henrys between coil A and coil B.

$$M = \frac{KN_2\phi_1}{I_1} 10^{-8} \tag{80}$$

In the Smithsonian Physical Tables is given the following definition:

Mutual inductance of two circuits is the electromotive force produced in one per unit rate of variation of the current in the other.

Example.—Coil A (Fig. 179) has 400 turns and coil B has 600 turns. When 5 amp. flow in coil A, a flux of 500,000 lines links with A, and 200,000 of these lines link coil B. What is the self-inductance of coil A with B open-circuited, and what is the mutual inductance of the two coils?

$$L_1 = \frac{N_1\phi_1}{I} = \frac{400 \times 500{,}000}{5} 10^{-8} = 0.4 \text{ henry.} \quad \textit{Ans.}$$

The induced voltage in B due to the current in A rising to 5 amp. in 1 sec. will be

$$e_2 = N_2 \frac{\phi_2}{t} = 600 \times 200{,}000 \times 10^{-8} = 1.2 \text{ volts.}$$

as a change of 5 amp. in coil A changes the flux in coil B by 200,000 lines.

Therefore:

$$e_2 = M\frac{I_1}{t}$$

$$1.2 = M \times \frac{5}{1}$$

$$M = 0.24 \text{ henry.} \quad \textit{Ans.}$$

or using Eq. (80)

$$M = \frac{0.4 \times 600 \times 500{,}000}{5}10^{-8} = 0.24 \text{ henry.}$$

It can also be shown that if M is the mutual inductance of coil B with respect to A, then M is also the mutual inductance of coil A with respect to B. That is, if the rate of change of current in coil B is I_2/t amp. per second, an e.m.f. is induced in A

$$e_1 = M\frac{I_2}{t} \text{ volts.}$$

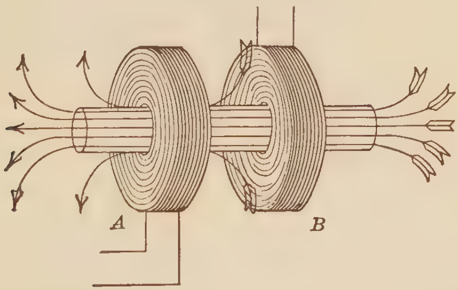


FIG. 180.—Effect of iron core upon mutual inductance.

Also it may be shown that the mutual inductance of two circuits having inductances of L_1 and L_2,

$$M = K\sqrt{L_1L_2} \tag{81}$$

where K is the coefficient of coupling.

The mutual inductance of two circuits may be materially increased by linking the circuits with an iron core. Thus, if two coils, similar to those shown in Fig. 179, be placed upon an iron core (Fig. 180) the coefficient of coupling, K, may be made very nearly unity. That is, practically all the flux linking coil A also links coil B.

A very common example of mutual inductance occurs in the induction coil (Fig. 181). A primary winding, P, of comparatively coarse wire and few turns, is wound on a laminated iron core C. This winding is connected to a battery B. The primary

current is interrupted by passing through the contact D, against which the iron armature A is held by a spring. When the core C is magnetized by the primary current, the armature A is drawn toward it and away from D, opening the circuit and causing the flux in the core to drop practically to zero. The spring then pulls the armature A against the contact D again, and the cycle is repeated. By this process the flux in the core C is continually being established and then destroyed.

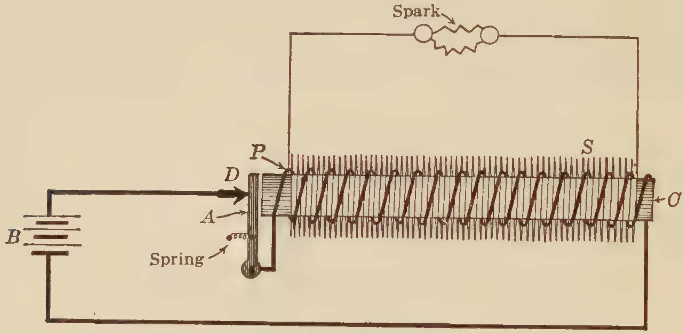


FIG. 181.—Induction coil.

On the same core is placed a secondary winding, S, consisting of many turns of fine wire. This winding is thoroughly insulated from the primary winding, but as it is wound on the same core as P, the two coils have a high value of mutual inductance. Because of the change of flux in the core, due to the interruptions of the primary current, a high alternating e.m.f. is induced in the secondary. This induced electromotive force may be considered as due to the mutual inductance existing between the primary and the secondary coils. The induction coil has many practical applications. Its wide use in automobile and gas-engine ignition systems is important.

164. Magnetic Pull.—It has been shown that a force exists between magnetized surfaces. This force can be accurately calculated if the surfaces are parallel and quite close together, being given by

$$f = \frac{B^2A}{8\pi}, \tag{82}$$

where f is the force in *dynes*, A the area of each of the two surfaces in square centimeters, and B the flux density in gausses.

This becomes:

$$F = \frac{B^2A}{24.64}\text{ kg.} \tag{83}$$

if B is expressed in *kilolines* per square centimeter.

$$F = \frac{B^2A}{72{,}130{,}000}\text{ lb.} \tag{84}$$

if B is in lines per square inch and A in square inches.

Example.—The core of a solenoid is 2 in. in diameter and a total flux of 200,000 lines passes from the end of the core into an iron armature of equal area. What is the pull on the armature in pounds?

$$A = \frac{\pi}{4}(2)^2 = 3.14\text{ sq. in.}$$

$$B = \frac{200{,}000}{3.14} = 63{,}800\text{ lines per square inch.}$$

$$F = \frac{63{,}800^2 \times 3.14}{72{,}130{,}000} = 177\text{ lb.} \quad \textit{Ans.}$$

MAGNETIC-FLUX MEASUREMENTS

Magnetic flux cannot be measured as readily as electric current, voltage, and power, that is, by the mere insertion of instruments in a circuit. Magnetic flux is ordinarily measured in two ways; *indirectly* by reading of the ballistic deflection of a galvanometer or the indication of a voltmeter; and *directly* by permeameters of the Koepsel type. The first method depends on the fact that an e.m.f. is induced in a coil when the flux linking the coil is changed; the second method depends on the fact that the deflections of Weston-type direct-current instruments are proportional to the flux if the current in the moving coil is maintained constant. In the first method if the voltmeter is used it must be of the alternating-current type and the value of an alternating flux is determined by measuring the e.m.f. it induces in a coil of a known number of turns (see Vol. II, Chap. VIII).

165. The Ballistic Galvanometer.—It is shown in Par. 186 page 257 that the ballistic throw of a galvanometer is proportional to the quantity Q of electricity discharged through it, provided the discharge all takes place before the galvanometer coil has commenced to move.

If a galvanometer is connected in series with a coil (Fig. 182) and the flux linking this coil be in any manner changed, a current

will flow and hence a quantity of electricity will be discharged through the galvanometer. If can be shown that if the flux is changed by an amount $\Delta\phi$, the quantity of electricity discharged

$$Q = N\frac{\Delta\phi}{R} \tag{85}$$

where N is the number of turns in the coil and R is the combined resistance of the coil and galvanometer. Since the galvanometer deflection is proportional to Q, it must also be proportional to the *change of flux*, provided the resistance of the entire circuit R is kept constant. Therefore, a ballistic galvanometer may be used to measure flux. The change of flux

$$\phi_1 - \phi_2 = KD \tag{86}$$

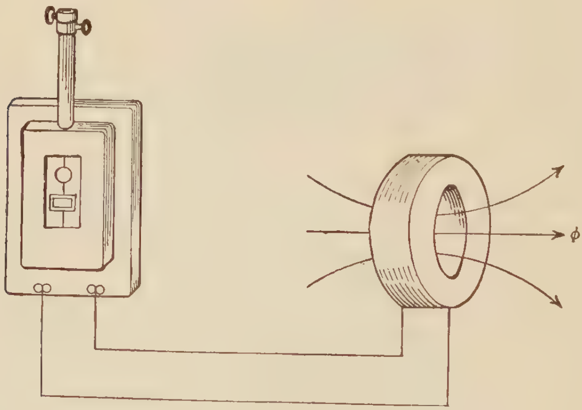


FIG. 182.—Ballistic galvanometer used to measure flux.

where ϕ_1 is the initial flux, ϕ_2 the final flux, K the galvanometer constant, and D the ballistic deflection of the galvanometer.

166. The Standard Solenoid.—In order to determine the constant K in Eq. (86) a flux of known value is necessary. This flux which links a secondary coil of known number of turns is caused to change by a known amount, either by decreasing it to zero or by building it up from zero. Simultaneously, the ballistic deflection of the galvanometer in series with the coil is read. This gives sufficient data for obtaining the galvanometer constant.

By means of a long, straight air solenoid (Fig. 183), it is possible to obtain a flux whose value is readily calculated. Consider first the ring solenoid (Fig. 184), which has a circular cross-section

and a core of non-conducting non-magnetic material such as wood. Let the cross-sectional area be A sq. cm. and the mean circumferential length l cm. (Fig. 184). The solenoid is wound

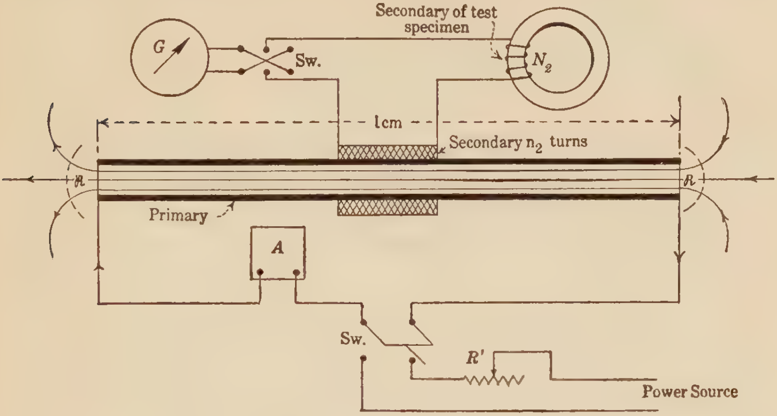


FIG. 183.—Standard solenoid with ring sample and galvanometer.

uniformly with n turns per mean circumferential centimeter and a current of I amperes flows in the winding. The total m.m.f.

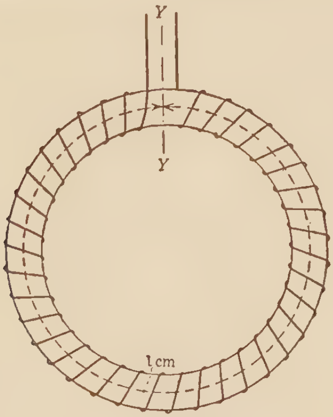


FIG. 184.—Ring solenoid.

F is obviously $0.4\pi nlI$ and if the cross-sectional diameter of the solenoid is small compared with the internal diameter of the ring, the reluctance is l/A.

Hence the flux

$$\phi = \frac{F}{\mathcal{R}} = \frac{0.4\pi nlI}{\frac{l}{A}} = 0.4\pi nIA \text{ maxwells.} \tag{87}$$

and is independent of the length of winding.

The mean flux density

$$B = \frac{\phi}{A} = 0.4\pi nI. \tag{88}$$

That is, the flux density *is equal to the m.m.f. per centimeter length.* and is also independent of the length of winding.

Referring to Eq. (87)

$$\phi = \frac{0.4\pi nI}{\frac{(1)}{(A)}}.$$

$1/A$ is the reluctance *per centimeter* length of the solenoid and $0.4\pi nI$ is the m.m.f. *per centimeter* length of the solenoid. Hence, it may be said that in such a solenoid the m.m.f. in each unit length is entirely utilized in sending the flux through that length.

If the ring solenoid (Fig. 184) be cut along the plane YY and straightened out, a long solenoid like that in Fig. 183 is obtained. This long solenoid must also have a length of l cm. and n turns per centimeter length. The flux, which is shown as acting from right to left within the solenoid, leaves the left-hand end of the solenoid and spreads, theoretically, over an infinite area and returns to enter the right-hand end. If there were no reluctance exterior to the solenoid, all the m.m.f. within the solenoid would be utilized in overcoming the reluctance of the solenoid, as in the ring-solenoid (Fig. 184). Hence the flux in this solenoid could be obtained from Eq. (87). Since, in the exterior return path, the flux spreads theoretically over an infinite area, the reluctance is small except for the parts $\mathcal{R}$ near the two ends of the solenoid, where the area is not infinite.

If, however, the reluctance of the solenoid itself l/A, is large compared with $2\mathcal{R}$, these end reluctances, the total m.m.f. of the solenoid may be considered as being utilized in overcoming its own reluctance. Hence, the flux, near the center of the solenoid,

$$\phi = \frac{0.4\pi nlI}{\frac{l}{A}} = 0.4\pi nIA \tag{89}$$

the same as Eq. (87). Therefore, the flux, ϕ, in such a solenoid is equal to the *m.m.f. per centimeter* multiplied by the cross-section of the solenoid in square centimeters. It is a simple matter to make such a solenoid having a known diameter and a known number of turns per centimeter. If the length of the solenoid is ten times the diameter, Eq. (89) is correct within one-half of 1 per cent., and if the length is twenty times the diameter, the error is only 0.1 of one per cent.

167. Calibration of the Galvanometer.—To calibrate the galvanometer, connections are made as in Fig. 183. The current in the turns of the solenoid is obtained from some steady direct-current source. Its value is controlled by the rheostat R' and read with the ammeter A. A secondary of a known number of turns, n_2, is wound about the solenoid near its center. This secondary is connected in series with the secondary of the test specimen and the galvanometer. Both secondaries are always in circuit in order to keep the value of R in Eq. (85), page 224, constant. A reversing switch is also connected in the galvanometer circuit. To calibrate, open switch Sw and close switch S, which energizes the solenoid without causing the galvanometer to deflect. Close switch Sw and read the current I with ammeter A. Then open S, noting the ballistic throw D_1 of the galvanometer. Several check readings should be taken.

When the switch is opened the total change of flux $\Delta\phi$ is $0.4\pi nIA$ and the quantity Q_1 is discharged through the galvanometer, Eq. (85).

$$Q_1 = \frac{n_2\phi}{R} = n_2\frac{0.4\pi nIA}{R} \tag{90}$$

When the flux ϕ_2 in the specimen is changed by an amount $\Delta\phi_2$ or from ϕ_2 to ϕ_1, from Eq. (85), the quantity,

$$Q_2 = N_2\frac{\Delta\phi_2}{R} = N_2\frac{\phi_2 - \phi_1}{R} \tag{91}$$

where N_2 is the number of secondary turns on the test specimen. Dividing Eq. (91) by Eq. (90)

$$\frac{Q_2}{Q_1} = \frac{\dfrac{N_2(\phi_2 - \phi_1)}{R}}{n_2\dfrac{0.4\pi nIA}{R}} = \frac{N_2(\phi_2 - \phi_1)}{0.4\pi nn_2IA}$$

$$\phi_2 - \phi_1 = \frac{Q_2}{Q_1}\left(\frac{0.4\pi nn_2IA}{N_2}\right)$$

Since the galvanometer deflections are proportional to the quantities Q_2 and Q_1,

$$\phi_2 - \phi_1 = \left(\frac{0.4\pi n n_2 A I}{D_1 N_2}\right) D_2 \qquad (92)$$

The quantity in parenthesis is the galvanometer constant K in Eq. (86) page 224.

Example.—An air solenoid 48 in. long has an inside diameter of 1.75 in., a primary winding of 960 turns and a secondary winding of 2,000 turns. There are 40 turns in the secondary of the test specimen. The galvanometer is connected in series with the secondaries of both the standard solenoid and the test specimen (see Fig. 183). When the current in the primary of the standard solenoid has become steady at 3.82 amp., the circuit is opened and the resulting ballistic deflection of the galvanometer is 12.4 cm. Determine: (*a*) the flux density (gausses) in the standard solenoid; (*b*) the total flux in the standard solenoid; (*c*) the galvanometer constant; (*d*) the flux change in the test specimen when the ballistic throw of the galvanometer is 8.9 cm.

The turns per centimeter length in the standard solenoid,

$$n = \frac{960}{48 \times 2.54} = 7.88.$$

The cross-section of the standard solenoid, $A = \pi/4(1.75)^2 \times (2.54)^2 = 15.5$ sq. cm.

(*a*) From Eq. (88); $B = 0.4\pi \times 7.88 \times 3.82 = 37.8$ gausses. *Ans.*

(*b*) $\phi = BA = 37.8 \times 15.5 = 586$ maxwells. *Ans.*

(*c*) Using Eq. (92)

$$K = \frac{0.4\pi \times 7.88 \times 2{,}000 \times 15.5 \times 3.82}{12.4 \times 40} = 2{,}360$$

(*d*) $\phi_2 - \phi_1 = \Delta\phi_2 = KD_2 = 2{,}360 \times 8.9 = 21{,}000$ maxwells. *Ans.*

168. The Yoke Method.—In the yoke or divided-bar method, which is due to Hopkinson, the test specimen is a cylindrical rod of from ¼ to ½ in. (0.6 to 1.2 cm.) diameter and about 20 in. (50 cm.) long. The rod is in two sections, A and B (Fig. 185 (*a*)), the two ends butting at e. These two ends should have accurately ground flat surfaces so that they make good magnetic contact. The rod fits tightly at both ends into a massive iron yoke Y whose reluctance is negligible in comparison with that of the rod. The effective length of the rod is taken as l cm. the distance between the inside surfaces of the yoke. The exciting coil PP is in two sections and has a total of N turns. The flux is determined by means of a small secondary coil S, through which the rod A passes. By means of a spring, the coil S springs out of the yoke when the part A of the rod is withdrawn.

The galvanometer is connected in series with the coil S and the secondary S' of the standard solenoid (Fig. 185 (a)). The current to the exciting coil is supplied by a battery, or by any steady source of current, through a rheostat R and a reversing switch Sw. A drop-wire often gives better adjustment than a series resistance. The m.m.f. in gilberts per centimeter,

$$H = \frac{0.4\pi NI}{l}$$

To determine some point a on the normal induction curve, the current is increased to a value which gives a m.m.f. ob (Fig. 185 (b)). The end A of the rod is withdrawn allowing S to spring out and the ballistic throw of the galvanometer is read. Since

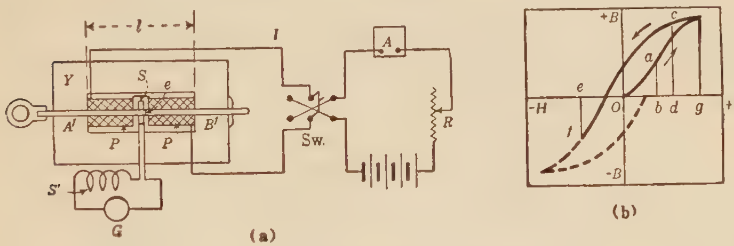


Fig. 185.—The yoke method.

the flux linking S becomes nearly zero when S thus springs out the galvanometer deflection must be proportional to the ordinate ba. To determine point c on the hysteresis loop, the excitation is increased to og and then decreased to od. The end A of the rod is again withdrawn and the galvanometer deflection which is proportional to cd is read. To determine the point f on the hysteresis loop the excitation is increased to og, is reduced to zero at o, the reversing switch is thrown, and the excitation is increased negatively to e. The end A of the rod is again withdrawn and the resulting galvanometer deflection, which is proportional to the ordinate ef, is read.

The galvanometer is calibrated by the method given in Par. 167. The flux density B is readily determined, knowing the flux ϕ and the cross-section A of the rod.

This method is subject to slight errors, due to leakage from the rod to the yoke, imperfect magnetic contacts, and to the hysteresis effects in the yoke.

169. The Ring Method.—In the ring method of magnetic testing, which was devised by Rowland, the form of the sample is such as to give a closed magnetic circuit and usually consists of a ring (Figs. 183 and 186), although if the sample cannot conveniently be made into ring form, it may be in the form of a square. For example, a square is readily made from long, rectangular laminations which are made alternately to butt and lap at the joints. If properly made, the joints in this latter case introduce slight error. The connections in the ring method (Fig. 186 (*a*)) are the same as those for the yoke method (Fig. 185 (*a*)). The principal difference lies in the procedure in carrying the sample through the magnetic cycle.

The primary is wound uniformly on the ring (Fig. 186 (*a*)) or along the four sides, if the sample be in the form of a square. The value of H is given approximately by dividing the total

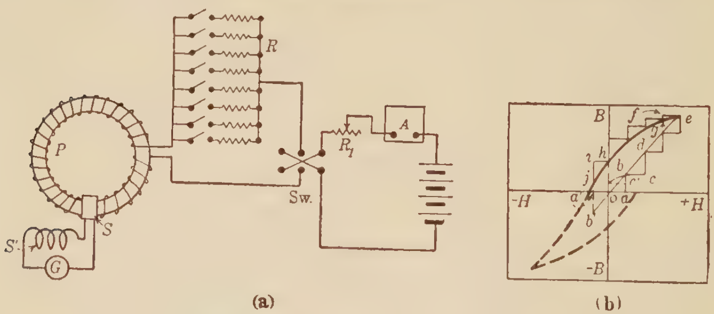


FIG. 186.—The ring method.

m.m.f. by the mean length of sample. The secondary is wound about the specimen in the manner shown in Fig. 183 and Fig. 186 (*a*). The success of the experiment depends on the current being quickly changed by definite amounts. Hence, the parallel arrangement of resistances R (Fig. 186 (*a*)), each of which is provided with a switch, is preferable to a sliding contact resistance. The auxiliary rheostat R_1 is also useful, in that it may be set to correspond to different values of maximum current. A single series resistance, having several switches which short-circuit definite amounts, may also be used to change the current.

Since the sample is a closed iron circuit, there may be considerable residual magnetism or remanence. It is necessary to

reduce this residual magnetism to a very small value. This is done by reversing the exciting current more or less rapidly and at the same time reducing its value slowly to zero. This carries the iron through hysteresis loops of diminishing amplitude and, if properly done, the residual magnetism or remanence may be reduced to a very small value.

The normal curve may be obtained by reversing the magnetizing force each time. For example in Fig. 186 (*b*), let it be desired to obtain the point *b* on the normal curve, corresponding to a flux ϕ_1. If the magnetizing force is carried to *a* and the circuit then opened, the change of flux will not be $ab = \phi_1$ but bc', due to the remanence. However, if the switch *Sw* is reversed, the total change of flux will be $ba + a'b' = 2\phi_1$, where $oa' = oa$. Each point on the curve may thus be determined by obtaining the desired magnetizing force, reversing it, and observing the galvanometer deflection. The galvanometer deflection corresponds to twice the value of flux.

The normal curve may also be obtained by an increment method. The hysteresis loop must be obtained in this manner.

When the magnetizing force is increased from *o* to *a* (Fig. 186 (*b*)), by throwing in a single switch (Fig 186 (*a*)), it causes a change of flux density *ab*, the value of which is determined by the ballistic deflection of the galvanometer. A second switch is then closed increasing *H* by the amount *bc* and the flux density by the amount *dc*, the value of which is determined by the ballistic deflection of the galvanometer. The flux density is thus increased in *increments* to *e*, the total flux density at *e* being $ba + cd$, etc. The galvanometer then measures the *increment* (or *decrement*) of flux between successive magnetizing forces. To obtain points on the hysteresis loop, the magnetizing force is then decreased by an amount *ef* by opening a single switch (Fig. 186 (*a*)), and the decrement of flux density *fg* is read with the galvanometer, etc. When the remanence *oh* is reached, the switch *Sw* is reversed and a single switch (Fig. 186 (*a*)) is closed. This gives a decrement of magnetizing force *hi* and a corresponding flux-density decrement *ij*. The sample is thus carried around the hysteresis loop.

Since the flux density at any point is the sum of successive flux increments, a single error vitiates the entire data. It is,

therefore, advisable to practice carrying the flux density through the cycle represented by the loop once or twice before actually taking data.

Although the ring method gives more precise results than the yoke method, it is more difficult to manipulate and the samples are not of such convenient form. Moreover the entire data may be rendered worthless by a single error, such as failing to read the galvanometer correctly, inadvertently closing a switch, etc.

170. Koepsel Permeameter.—In the preceding two methods of magnetic testing, the measurements are indirect, quite laborious, and involve considerable computation. The Koepsel

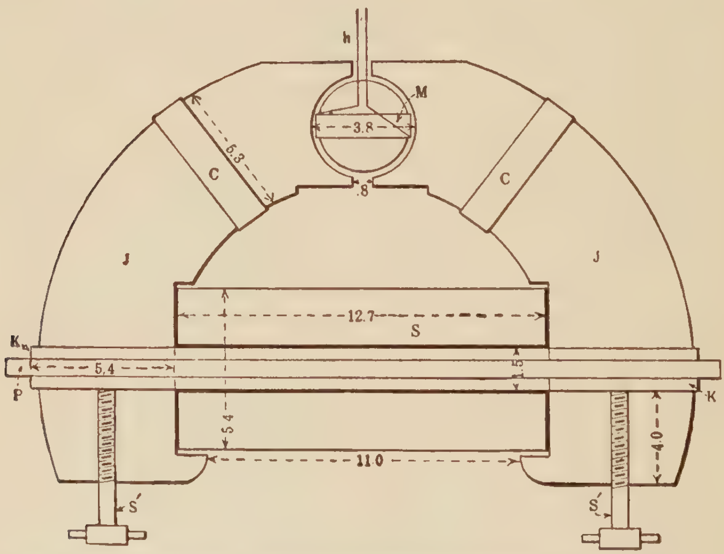


FIG. 187.—Diagram of the Koepsel permeameter (dimensions in cm.).

permeameter is an indicating instrument on whose scale the flux density B in a specimen is read directly, just as current is read on an ammeter scale. The magnetizing force H is also read directly by means of an ammeter. The instrument operates on the principle of the Weston ammeter, (Par. 122) that is, if a coil carries current in a magnetic field, it tends to assume a position perpendicular to the field. In ordinary Weston instruments, the flux is

constant and the deflections are practically proportional to the current in the moving coil; in the Koepsel permeameter the current in the moving coil is constant and the deflections are practically proportional to the flux in the gap.

The instrument (Fig. 187) consists of two massive soft-iron yokes JJ, a moving coil M which turns in a very short air-gap, two compensating coils CC, and the exciting coil S. The specimen P to be tested consists of an iron rod P held into the yokes by iron clamps KK and screws $S'S'$. The test specimen may be square or it may be round, the clamps ordinarily being adapted to hold specimens 6 mm. (0.236 in.) square or 6 mm. diameter.

When the current in the moving coil M is equal to $0.005/A$, where A is the cross-section of the test specimen in square centimeters, the instrument reads the flux density B in the specimen directly. The current for the moving coil is supplied by a 4-volt source, such as three or four dry cells in series, and is controlled by a suitable rheostat. This current, when once adjusted, should remain constant. The exciting coil S is wound with 79.6 $\left(= \frac{100}{0.4\pi}\right)$ turns per centimeter, and the m.m.f. per centimeter,

$$H = 0.4\pi \left(\frac{100}{0.4\pi}\right) I = 100I \text{ gilberts where } I \text{ is the exciting}$$

current. Therefore, the m.m.f. per centimeter may be readily obtained by multiplying the ammeter reading by 100. Ordinarily, the same ammeter is used to measure the moving-coil current and the exciting current by connecting different shunts in circuit by means of plugs.

With current in the exciting coil S, the instrument would normally deflect even if no specimen were in place, since the m.m.f. of the exciting coil is sufficient to produce considerable flux in the yokes. To prevent such deflection, two compensating coils, CC, placed on the yoke near the gap, are connected in series with and opposing S. They are so adjusted that there is no deflection when current flows in S. To minimize deflections of the coil caused by the earth's field, the instrument should be so placed that the axis of the coil is in the plane of the magnetic meridian. A view of the instrument is given in Fig. 188. Although this instrument is simple in its operation and repeats

itself very closely, it is subject to some error, particularly at high flux densities. It is assumed that a fixed percentage of the flux in the rod reaches the gap. This fixed percentage may vary as much as two per cent. depending on the flux density. Also because of the reluctance of the air-gap, leakage, etc. the error in

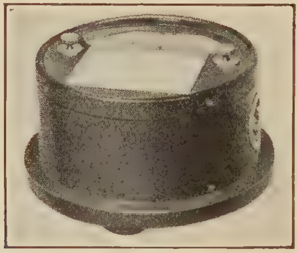


FIG. 188.—Koepsel permeameter.

H under extreme conditions may be as large as 10 per cent. Correction curves usually accompany the instrument so that the effect of such errors may be reduced. The fact that ordinarily the errors are not large makes the instrument particularly useful in comparing magnetic materials and in detecting non-uniformity.

CHAPTER IX

ELECTROSTATICS: CAPACITANCE

Thus far, the electric current, or electricity in motion, only has been considered. Electricity may, however, be stationary or at rest.

Electricity, though at rest has important effects on the media surrounding it. For example, the design and the applications of insulating materials, particularly for high voltages, depend on a knowledge of the laws governing stationary electric charges. It is the purpose of this chapter to consider the laws governing the behavior of electricity while it is at rest, and to determine its effects on surounding media.

171. Dynamic and Static Electricity.[1]—According to modern theory, which has been substantiated by the experimental results of many investigators, the atoms of all matter seem to consist of a positively charged nucleus, around which infinitesimal negative charges appear to rotate with high angular velocity. The individual negative charges, which are called *electrons*, are found to be identical for all matter. In conductors, some of these electrons appear to be free to pass from atom to atom when a difference of potential is impressed across the ends of the conductor. The movement of these electrons constitutes the electric current. Hence, the electric current may be considered as electricity in motion, and is called *dynamic electricity.* Since electrons are negative charges, the direction of their motion is opposite to the conventional direction of current flow.

When the total charge of the electrons in an atom is just equal to the positive charge of the nucleus or proton, the atom is said to be in the uncharged or neutral state. If, however, one or more electrons be removed, the atom is said to be positively charged. If the electrons so removed associate themselves with other neutral matter, this matter is said to become negatively charged.

[1] See Vol. II (revised) Chap. XIV.

In the dynamic circuit, the potential difference withdraws electrons from any given group of atoms, causing these electrons to pass on to the next group. Their places are immediately taken, however, by electrons coming from the preceding adjacent group of atoms which are being acted upon by the same potential difference. Hence, the same number of electrons is always associated with the same atom. If, however, the conducting circuit is not closed electrons are withdrawn from the positive terminus of the circuit and transferred to the negative terminus. Since the conducting circuit is open there is not opportunity for the electrons so displaced at the positive terminus being replaced by electrons from adjacent atoms.

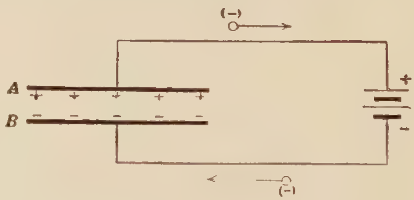


Fig. 189.—Transfer of electrons in condenser plates.

If this transfer of electrons is accomplished by the action of a steady source of potential, the positive and negative charges appear to be at rest. For example, if the insulated parallel discs A and B (Fig. 189) are connected to the positive and negative terminals of a battery or of an influence machine, the action of the applied potential will withdraw some of the free electrons from A and transfer them to B. Hence, plate A appears to be positively charged, since negative charges have been withdrawn from it, and plate B has become negatively charged. Under these conditions, the charge on the plates is frequently called *static electricity*.

From the preceding brief discussion, it appears that dynamic and static electricity are identical in their natures. With dynamic electricity, there is a movement of electrons between the atoms of the material. With static electricity, free electrons have been displaced from the positive to the negative plate and are so maintained by the action of the electric field. As a whole, this displacement has the effect of producing a single negative

charge on the negative plate. Likewise, the positive plate appears to act as if it contained a single positive charge.

The displacement of charges at the positive and negative plates, as in Fig 189, is frequently accompanied by high voltage, particularly if a high-voltage source, such as an influence machine, is used. The high voltage and small quantity under these conditions sometimes convey the impression that static and dynamic electricity are different in nature.

172. Electrostatic Charges.—If the terminals of an electrostatic induction machine be connected to two equal ellipsoids, which are conducting and are insulated (Fig. 190) the ellipsoid

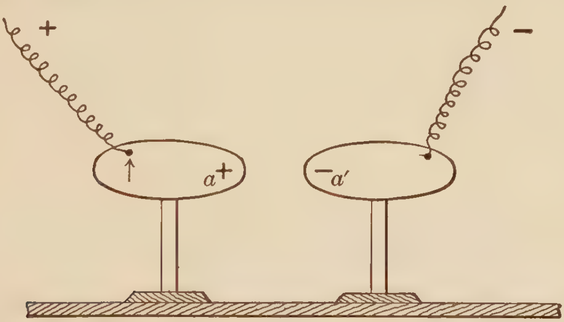


Fig. 190.—Electrostatic charges on insulated ellipsoids.

connected to the positive terminal will be charged with positive electricity and that connected to the negative terminal will be charged with an equal amount of negative electricity. The charges will distribute themselves over the entire surface of the ellipsoids, but the density of the charges will be greatest on the ends of the ellipsoids which are adjacent. This is due to the fact that the positive and negative charges attract each other.

If the two wires from the electrostatic machine be disconnected the two charges will not be sensibly affected. In time they will leak away through the insulating supports.

If the two ellipsoids were free to move they would come together. If they were connected together with a wire a spark would be observed at the instant that contact was made, showing that current flows for an instant from one ellipsoid to the other. Both of the above effects are due to the fact that the positive and negative charges attract each other.

173. Electrostatic Induction.—If a positively charged ellipsoid A (Fig. 191 (a)) be brought near another insulated ellipsoid B, which initially had no charge, a minus charge, b, will be found on the end of B nearest A. As B did not hold any charge initially, and it is assumed to be perfectly insulated, no electricity can have gone out from B and none can have reached it from external sources, so that the net charge on B must still be zero. Therefore, a positive charge b' must also appear on B at the outer end farthest from A. This charge must be equal to b,

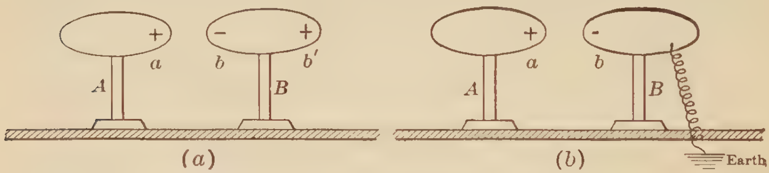


Fig. 191.—Electrostatic induction.

and as the two are of opposite sign the net charge on B is still zero. It will be noted that the minus charge b is as near as possible to the positive inducing charge a, whereas the positive charge b' is as far away as possible from the positive charge a. This is due to the fact that the unlike charges attract each other and that like charges repel each other.

Also charges a and b are called *bound charges*, and charge b' is a free charge. This may be proved by connecting B to ground (Fig. 191 (b)). The charge b' will be found to have escaped to ground, whereas the two charges a and b remain. Charge b' will seek a position as far away from a as possible.

If a were a negative charge, b would be a positive charge.

The above experiments are all illustrative of the following laws of electrostatics.

Charges of unlike sign attract each other and charges of like sign repel each other.

A positive charge will induce a negative charge on a body near it, or

A negative charge will induce a positive charge on a body near it.

This is similar to magnetic induction, where a north pole induces a south pole, etc. (see Par. 16).

174. Unit Charge and Coulomb's Law.—In the fundamental relationships of electrostatics, the electrostatic system of units (see p. 52) is employed. Conversion of these units into those of the practical system is readily made when necessary (see Appendix B, p. 475).

A unit electrostatic charge (e.s.u.) is defined as that charge which when placed 1 *cm. in a vacuum from an equal charge, repels it with a force of* 1 *dyne.* The unit charge is frequently called the *statcoulomb.*

Coulomb's Law.—By performing a number of experiments with charged spheres, Coulomb, between the years 1785 and 1789, proved the following law. *The force acting between two charged*

FIG. 192.—Force between electrostatic charges.

bodies in air is proportional to the product of the charges and inversely proportional to the square of the distance between them.

That is,

$$f = \frac{qq'}{r^2} \text{ dynes (see Fig. 192).} \tag{93}$$

If the charges are in a medium whose dielectric constant is κ, Eq. (93) becomes

$$f = \frac{qq'}{\kappa r^2} \text{ dynes,} \tag{94}$$

where q and q' are the charges in statcoulombs and r the distance between the charges in centimeters. It is assumed that the charges are concentrated at points.

Example.—Two small spheres in air, spaced 14 cm. between centers, are charged respectively with two positive and five negative e.s.u. What is the force in dynes acting between the spheres?

From Eq. (93),

$$f = \frac{2 \times 5}{14^2} = \frac{10}{196} = 0.0510 \text{ dyne.} \quad \textit{Ans.}$$

175. The Dielectric (or Electrostatic) Field.—It has already been shown (Chap. I), that the medium in the neighborhood of a

magnet appears to be in a stressed condition, and that a force acts on any north or south pole placed in such a magnetic field. Likewise, a condition of stress appears to exist in the neighborhood of electric charges, and force acts on any positive or negative charge placed in this field. This condition of stress manifests itself if the charges become sufficiently large in magnitude. The stress may become so great as to cause complete mechanical rupture of the intervening medium, followed by an arc discharge (see Par. 178).

The region in which the condition of stress exists is called a *dielectric* (*or electrostatic*) *field.* As with the magnetic field, the condition of stress may be represented by lines. A field in which there exists one line per square centimeter, taken normal to the direction of the field, exerts a force of 1 dyne on a unit charge.

It follows that 4π lines must emanate from each unit charge (see p. 8, Par. 13). Hence $4\pi q$ lines must emanate from a charge of q units.

These lines of force in the dielectric have properties similar to those of magnetic lines of force (*not* magnetic lines of induction).

1. *Every line originates on some positive charge and terminates on some negative charge.*

2. *A unit positive charge, if placed at the surface of the positive electrode, will be urged along the lines of force to the negative electrode with a force in dynes at each point equal to the number of lines per square centimeter at that point, the area being taken normal to the direction of the lines.*

3. *The dielectric field tends so to conform itself that the number of lines is a maximum.* The lines behave like stretched rubber bands.

A dielectric field between two irregular electrodes is illustrated in Fig. 193. It will be noted that each line originates at a positive charge and terminates at a negative charge. That is, the lines are not closed. Any positive unit charge is urged along a line of force from the positive to the negative electrode. The lines have the appearance of elastic bands tending to contract, and therefore tend to reduce the dielectric reluctance of the field to a minimum. They distribute themselves exactly as the flow or stream lines of an electric current do or as the lines of force in the magnetic circuit distribute themselves.

A dielectric line of force must always be normal to a conducting surface where it leaves or enters that surface. If this were not true, there would be a component of the force tangential to the conductor at its surface, and a resulting potential difference which would cause a flow of current. Current cannot flow, since the *static* condition is assumed. Hence, there cannot be a component of force tangential to the surface of the conductor, and every line must therefore be normal to the surface at the point where it leaves or enters the conductor.

Dielectric flux is denoted by the symbol ψ.

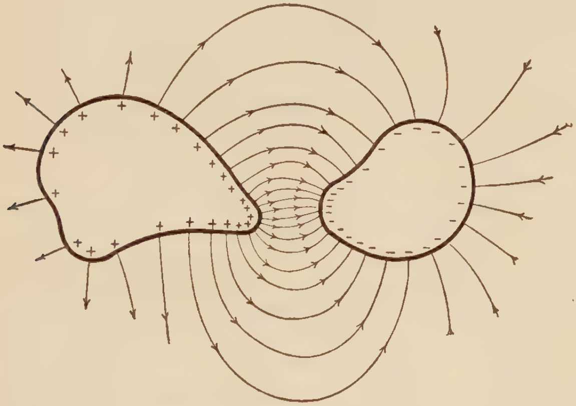


FIG. 193.—Electrostatic field between charged electrodes.

176. Charged Spheres.—Figure 194 shows an isolated sphere having a positive charge.[1] *This* charge must lie entirely on the surface of the sphere. As the individual positive charges repel one another, they will separate as far as possible from one another, and must all take positions on the surface of the sphere.

If the corresponding negative charge is sufficiently far removed, or if it exists on an outer concentric spherical shell, the positive charge from symmetry must be uniformly distributed on the surface of the sphere. Again, from symmetry, the dielectric lines must all enamate radially and uniformly from the surface of the sphere. If these lines were continued inwards, they would therefore meet at the center of the sphere. Hence, so far as

[1] For every positive charge, there must be an equal negative charge. The corresponding negative charge may exist on the walls of the room, on the surface of the earth, or at some remote place.

points in space outside the sphere are concerned, the charge has the same effect as if it were concentrated at the center of the sphere.

If there are q unit charges on the sphere, then from Par. 175, $4\pi q$ lines emanate from the sphere. If the sphere has a radius of

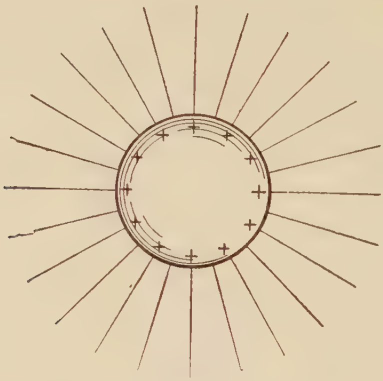


FIG. 194.—Isolated charged sphere.

r cm., the area of its surface is $4\pi r^2$ sq. cm. Therefore, the density of the lines of force at its surface must be $\frac{4\pi q}{4\pi r^2}$ or $\frac{q}{r^2}$ lines per square centimeter. Hence, the force at the surface of a sphere of radius r, charged with q e.s.u., is q/r^2 which, from Eq. (93)

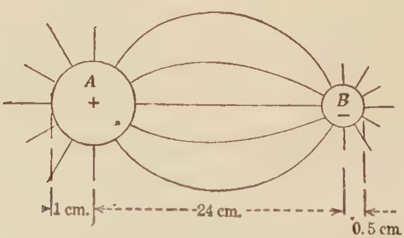


FIG. 195.—Charged spheres in air.

(p. 239), is the same as if the entire charge were concentrated at the center of the sphere.

Example.—Two spheres A and B (Fig. 195) having radii of 1 cm. and 0.5 cm., respectively, are spaced 24 cm. between centers in air. If the first sphere is charged positively and the second negatively with 20 e.s.u. each,

determine: (*a*) the number of dielectric lines emanating from each sphere; (*b*) the dielectric flux density at the surface of each sphere, due to its own charge; (*c*) the maximum force at the surface of each sphere; (*d*) the intensity of the dielectric field at a distance of 10 cm. from the center of sphere *A* and on the line joining the centers of the two spheres; (*e*) the force between the spheres and its direction.

(*a*) $\psi = 4\pi 20 = 251$ lines from each sphere. *Ans.*

(*b*) $D_A = \dfrac{4\pi 20}{4\pi(1)^2} = 20$ lines per square centimeter. *Ans.*

$D_B = \dfrac{4\pi 20}{4\pi(0.5)^2} = 80$ lines per square centimeter. *Ans.*

(*c*) If each sphere were isolated, the force at the surface of each would be given by (*b*), but if the spheres are not isolated, the charge on each sphere exerts a force at the surface of the other. This force will be a maximum at that point on either sphere which is nearest to the other, since the force varies inversely as square of the distance.

This force at the surface of sphere *B* due to sphere *A*, and on the line that joins the centers of the spheres, acts toward the center of *B*, since the positive charge on *A* repels a unit positive charge at *B*. The force on this same positive unit charge, due to the charge on *B*, acts toward the center of *B* since the negative charge on *B* attracts this charge. Hence both forces act in the same direction at this point. Likewise, the charges on both spheres *A* and *B* act in the same direction at the point on the surface of sphere *A* nearest *B*. Therefore, the force at the surface of each sphere is a maximum at the point where a line joining their centers intersects their surfaces. Hence,

$$f_A = 20 + \frac{20}{(23)^2} = 20 + 0.038 = 20.04 \text{ dynes. } \quad \textit{Ans.}$$

$$f_B = 80 + \frac{20}{(23.5)^2} = 80 + 0.036 = 80.04 \text{ dynes. } \quad \textit{Ans.}$$

(*d*) The intensity of the field at any point is equal to the force exerted on a unit positive charge placed at that point.

$$f = \frac{20}{(10)^2} + \frac{20}{(14)^2} = 0.20 + 0.102 = 0.302 \text{ dyne. } \quad \textit{Ans.}$$

(*e*) From Eq. (93),

$$f = \frac{20 \times 20}{(24)^2} = 0.695 \text{ dyne and is a force of attraction. } \quad \textit{Ans.}$$

It follows from the foregoing that, with an isolated sphere, the density of the dielectric lines of force varies *inversely as the square of the distance* from the center of the sphere.

177. Dielectrics.—If electrostatic or dielectric phenomena are being considered, the medium between two conductors is called a *dielectric*. The dielectric properties of a medium are determined by the relationship between dielectric lines and potential. On the other hand, the insulation properties of this same medium

relate to the relationship between current flow and potential. For example, air is not a particularly good dielectric, its dielectric strength being only about 75,000 volts to the inch, but it is one of the best insulators known.

Thin samples of rubber will withstand voltages as high as 450,000 volts to the inch, but they will permit a much greater leakage current than air will. That is, rubber is a better dielectric but a poorer insulator than air.

It was stated in Par. 175 that the dielectric field resembled both the electric circuit and the magnetic field in that dielectric lines distributed themselves as lines of current flow and magnetic lines do. No matter how much current, however, flows in a conductor, the conductor is not injured mechanically, provided it can be kept cool. Neither is a magnetic conductor injured, no matter how many magnetic lines exist in it. Dielectric media, however, have the property of permitting only a limited number of dielectric lines per unit of area without rupture occurring. When the dielectric flux density exceeds this limiting value, puncture occurs and may be followed by a dynamic arc, which burns and chars the dielectric. At the present time, the mechanism of the break-down of solid and liquid dielectrics is little understood although several rational theories have been advanced.

The ability of a substance to resist dielectric break-down is called its *dielectric strength*. This is expressed in volts per unit thickness when the substance is placed between flat electrodes having rounded edges. For example, the dielectric strength of air is approximately 3,000 volts per millimeter. Rubber and varnished cambric have a much greater dielectric strength than air, an average value for rubber being 16,000 volts per millimeter, or 400,000 volts per inch, and for cambric being about twice as great as the value for rubber.

The volts per unit thickness impressed across a dielectric is called the *voltage gradient*. For example, if 24,000 volts is impressed across 30 mils of insulation, the gradient is 24,000/30 or 800 volts per mil. With insulating substances, both the insulation properties and the dielectric properties must be considered.

178. Ionization of Air; Corona.—When solid dielectrics are subjected to sufficiently high dielectric stress, they rupture, and

the dynamic arc which follows chars and disintegrates the dielectric. Although the exact mechanism of the rupture is not understood, undoubtedly there first occurs a destruction of the molecular structure of the medium in some one spot, due to a separation of the electrons from their atomic nuclei, as a result of the high potential gradient.

With gaseous dielectrics, such as air, the mechanism of dielectric rupture is much better understood. This is due in part to the fact that the molecular structure of gases is much simpler than that of most other substances. Also the gas particles are not destroyed by the rupture, and since they are mobile they can be collected and analyzed even after they have been subjected to rupturing stresses. A gas atom consists of a positive nucleus with one or more minute electric charges or electrons moving in orbits about the nucleus. A hydrogen atom has but one electron, while a helium atom has two electrons.

In a dielectric field the electrons of a gas, being negative charges, are attracted to the positive electrode; the positive nuclei or ions are likewise attracted to the negative electrode. Hence, the dielectric field tends to separate the electrons from their nuclei. The dielectric forces holding the electrons to their nuclei are so great, however, that a prohibitively high voltage gradient would be necessary to separate them by this effect alone.

There are always some free ions and electrons in any gaseous medium. Under the action of the electric field the ions and, more particularly, the electrons will accelerate. While moving toward the electrodes they will collide with the neutral atoms that happen to be in their paths. These collisions result in other electrons being knocked away from their nuclei, thus producing more free ions and electrons. Hence the process tends to be a cumulative one. When the potential gradient becomes sufficiently great (approximately[1] 30,000 volts per centimeter for air at 760 mm. pressure and at 20°C.), the velocities acquired by the ions become sufficiently high to produce ions by collision more rapidly than the ions are withdrawn from the field. The gas is then said to be *ionized*. Ionized gas conducts current but has, notwithstanding, a *very high* resistance. This conduction of current through an ionized gas is a convection phenomenon not unlike the conduc-

[1] This corresponds to 100 e.s. lines per square centimeter.

tion of electricity through an electrolyte. Thus the dielectric strength of the ionized gas has become pratically nil. If a source of potential having considerable power behind it is used, rupture of the gas occurs and a dynamic arc follows.

A partial ionization of a gas without complete rupture may be accomplished, if an arrangement somewhat like that shown in Fig. 196 is employed. One electrode, the upper in Fig. 196, is sharp and may conveniently be a needle. The lower one is a flat plate. With the application of potential, dielectric lines form between the needle point and the plate. Owing to the sharpness of the needle point, the flux density near it is very great, whereas the density near the plate is much smaller in comparison, because of its larger area. Hence, the air in the vicinity of the needle point first becomes ionized, the ionized region extending at first to line *aa*. Beyond *aa* the dielectric field is not sufficiently intense to permit further ionization and the remainder of the air therefore prevents any dynamic current flowing. With further increase in potential the ionized zone may extend to *bb* without complete rupture following. Ultimately, however, the potential may reach a value which is sufficient to rupture the non-ionized region and a dynamic arc follows.

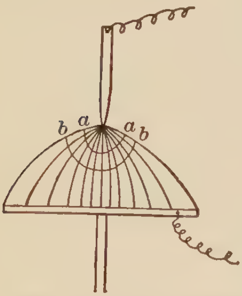


FIG. 196.—Dielectric stress lines between a needle point and a plate.

Ionized air at the surfaces of conductors appears as small tufts and streamers, giving a blueish-reddish light which makes them easily visible in the dark. Such ionized air is called *corona* because of its resemblance to solar corona. Corona is also accompanied by a hissing sound, and the odor of ozone is noticed. In the presence of moisture, nitrous acid is formed, and this phenomenon is the basis of one of the methods of manufacturing, by electrical means, nitrates from the nitrogen of the atmosphere. Corona forms on high-voltage apparatus and transmission lines (see Vol. II, Chap. XIII).

179. Capacitance.—Two conductors separated by a dielectric constitute a *condenser*.

Figure 197 shows two conducting plates connected to a battery, the plates being separated by a dielectric. There is also a single-

pole, double-throw (S.-P.D.-T.) switch S and a galvanometer G in the circuit. If the switch S be closed to the left, the galvanometer will deflect momentarily, and then come back to zero. This indicates that when the switch is closed, a quantity of electricity passes through the galvanometer, but that the current ceases to flow almost immediately. This current flows for a time sufficient to charge the condenser. After the condenser has

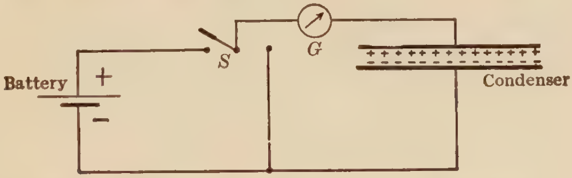


Fig. 197.—Charging and discharging a condenser.

become fully charged, the current ceases because the e.m.f. of the condenser is equal and opposite to that of the battery. As this condenser e.m.f. opposes the current entering the condenser it may be considered as a back e.m.f. Any current which may flow after the condenser has become fully charged is a leakage current flowing through the insulation. If the switch S be opened for a short time, and then closed again, no deflection of the galvanometer will be noted unless there is leakage through the insulation.

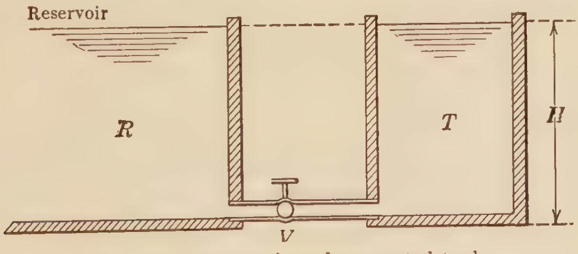


Fig. 198.—Reservoir and connected tank.

This phenomenon of charging a condenser from a battery is not unlike the filling of a tank T from a reservoir R (Fig. 198). When the valve V is first opened, water will rush through the connecting pipe and will continue to flow at a diminishing rate until the level H, of the water in the tank T, is equal to the level of the water in the reservoir. If the tank does not leak, no water

flows through the pipe after the water levels have become equal. In the same way the condenser (Fig. 197) takes current until its potential is equal to that of the battery, after which current ceases to flow. Again, if tank T does not leak, no further flow of water occurs when V is closed and then opened.

To prove that electricity has actually been stored in the condenser (Fig. 197) the switch S may be closed to the right. This short-circuits the condenser through the galvanometer. The galvanometer now deflects momentarily in a direction opposite to that on charge, showing that the current now flows *out* of the positive plate. The condenser now becomes completely discharged, as is shown by there being no longer any deflection of the galvanometer.

If the voltage of the battery (Fig. 197) be increased, the galvanometer deflection on charge and on discharge will increase also. This is due to the fact that the charge given to the condenser is proportional to the voltage across its terminals, just as the amount of water in the tank will be proportional to its height H (Fig. 198). The relation between the voltage, and the charge in a condenser may be expressed by the equation

$$Q = CE. \tag{95}$$

That is, the quantity of electricity in a condenser is equal to the voltage multiplied by a constant C. This constant C is called the *capacitance* of the condenser. The practical unit of capacitance is the *farad*. If C is in farads and E in volts, Q is in coulombs or ampere-seconds.

The farad is too large a unit for practical purposes, as a condenser having a capacitance of 1 farad would be prohibitively large. The capacitance of the earth as an isolated sphere is less than one-thousandth of a farad. The *microfarad* (μf), equal to one millionth of a farad, is the unit of capacitance ordinarily used.

In radio telegraph and radio telephone work, where the capacitances are very small, the microfarad is too large a unit, and the micro-microfarad ($\mu\mu f = 10^{-12}$ farad) is used.

By transposition, Eq. (95) may be written as follows:

$$C = \frac{Q}{E}, \tag{96}$$

$$E = \frac{Q}{C}. \tag{97}$$

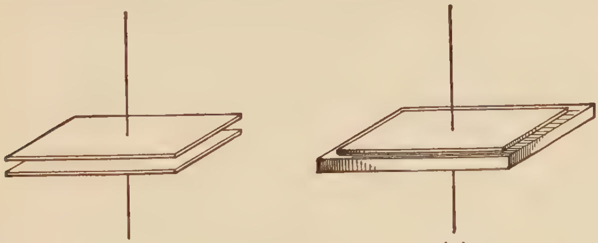
As an example of the use of the above relations, consider the following problem:

A condenser has a capacitance of 200 microfarads and is connected across 600-volt mains. If the current is maintained constant at 0.1 amp., how long must it flow before the condenser is fully charged?

The quantity in the condenser, when fully charged, is $Q = 0.000200 \times 600 = 0.12$ coulomb or ampere-second.

$$0.12 = 0.1t,$$
$$t = 1.2 \text{ sec.} \quad \textit{Ans.}$$

180. Specific Inductive Capacity or Dielectric Constant.—A parallel plate condenser (Fig. 199 (*a*)), with air as a dielectric, has a measured capacitance C_1. If a slab of glass or of hard rubber



(*a*) (*b*)

FIG. 199.—Plate condenser having air and then glass as a dielectric.

be inserted between the plates so as to fill the intervening space completely (Fig. 199 (*b*)), and the capacitance of the condenser again be measured, it will be found to be greater than its previous value. Let this new value be C_2. The increase in capacitance obviously must be due to the presence of the glass or rubber.

The ratio $C_2/C_1 = \kappa$ is called the *specific inductive capacity*, or *dielectric constant* or *permittivity*, of the material between the condenser plates. The specific inductive capacity of air is assumed to be unity, just as the magnetic permeability of air is likewise assumed to be unity.

In the table are given the specific inductive capacities of some of the more common dielectrics:

Bakelite[1]	4.1 to 8.8	Paraffin	1.9 to 2.3
Glass	5.5 to 10	Rubber compounds	3 to 6
Ice	86.4	Hard rubber	1.5 to 3.5
Mica	2.5 to 5.5	Transformer oils	2.3 to 2.6
Paper	1.7 to 2.6		

[1] For more complete data see "Standard Handbook," 5th Edition, Sec. 4, Par. 238, *et seq.*

181. Equivalent Capacitance of Condensers in Parallel.—Let it be required to determine the capacitance, C, of a number of condensers in parallel, the condensers having respective capacitances of C_1, C_2, C_3. This arrangement of condensers is shown in Fig. 200. Let the common voltage across the condensers be E and the total resulting charge Q. Obviously,

$$Q = CE$$

and

$$Q_1 = C_1E, \quad Q_2 = C_2E, \quad Q_3 = C_3E.$$

FIG. 200.—Capacitances in parallel.

The total charge

$$Q = Q_1 + Q_2 + Q_3 = CE.$$
$$CE = C_1E + C_2E + C_3E.$$
$$CE = E(C_1 + C_2 + C_3).$$
$$\therefore C = C_1 + C_2 + C_3. \tag{98}$$

That is, if condensers are connected in parallel, the resulting capacitance is the sum of the individual capacitances.

This is analogous to the grouping of conductances in parallel in the electric circuit.

Example.—Three condensers, having capacitances of 5, 10, and 12 microfarads, respectively, are connected across 600-volt mains. (*a*) What single condenser would replace the combination? (*b*) What is the charge on each condenser?

(*a*) $C = 5 + 10 + 12 = 27$ microfarads. *Ans.*

(*b*) $Q_1 = 5 \times 600 = 3{,}000$ microcoulombs

$Q_2 = 10 \times 600 = 6{,}000$ microcoulombs

$Q_3 = 12 \times 600 = 7{,}200$ microcoulombs. *Ans.*

Total charge $= 16{,}200$ microcoulombs $= 27 \times 600$ m.c. (check).

182. Equivalent Capacitance of Condensers in Series.—In Fig. 201, three condensers, having capacitances of C_1, C_2, and C_3 respectively, are connected in series across the voltage E. It is desired to determine the capacitance of an equivalent single condenser. Let E_1, E_2, and E_3 be the potential differences

across the condensers C_1, C_2, and C_3, respectively. After the voltage E is applied to the system, there will be $+Q$ units of charge on the positive plate of C_1, and by the law of electrostatic induction $-Q$ units must be induced on its negative plate.

Now consider the region a which consists of the negative plate of C_1, the positive plate of C_2, and the connecting lead. This system is insulated from all external potentials, since it is assumed that the condensers have perfect insulation. Before the voltage was applied to the system of condensers, no charge existed in the region a. After the application of the voltage, the net charge in this region must still be zero, as no charge can flow through the insulation (see p. 238, Par. 173). Therefore, $+Q$ units must come into existence in order that the net charge in the region a may remain zero. $(+Q + (-Q)) = 0$. This charge of $+Q$ units will go the plate of C_2 since it is repelled by the $+$ charge on C_1 just as the charge b' (Fig. 191 (a)) took a position on the end of the ellipsoid as far as possible from the positive inducing charge a. The same reasoning holds for the region b, between C_2 and C_3. Therefore, each of the three condensers in series has the same charge Q. (This is analagous to resistances in series, each of which must carry the same current if no leakage exists.)

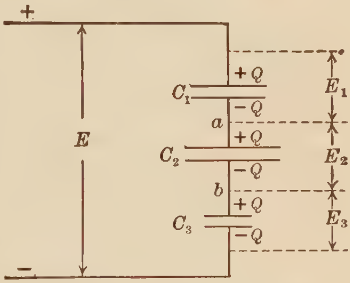


FIG. 201.—Capacitances in series.

Consider the voltages E_1, E_2, E_3.

$$E_1 = \frac{Q}{C_1}, \quad E_2 = \frac{Q}{C_2}, \quad E_3 = \frac{Q}{C_3} \text{ from Eq. (97), page 248.}$$

The sum of the three condenser voltages must equal the line voltage:

$$E_1 + E_2 + E_3 = E.$$

$$E = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Also $E = Q/C$, as by definition the equivalent condenser C must have a charge Q.

Substituting this value for E,

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{99}$$

That is, the reciprocal of the equivalent capacitance of a number of condensers in series is equal to the sum of the reciprocals of the capacitances of the individual condensers.

In assuming for condensers connected in series that with direct current the potential across each condenser is inversely proportional to its capacitance, the factor of leakage is absolutely neglected. If the condensers are even slightly leaky, however, a current flows through the series and eventually the potential distributes itself according to Ohm's law.

$$E_1 = IR_1,\ E_2 = IR_2, \text{ and } E_3 = IR_3,$$

where I is the leakage current, and R_1, R_2, and R_3 are the respective ohmic resistances of the three condensers.

Example of condensers connected in series:

Consider that the three condensers of Par. 181, having capacitances of 5, 10, and 12 microfarads respectively, are connected in series across 600-volt mains. Determine (*a*) the equivalent capacitance of the combination; (*b*) the charge on each condenser; (*c*) the potential across each condenser, assuming no leakage.

(*a*)
$$\frac{1}{C} = \frac{1}{5} + \frac{1}{10} + \frac{1}{12} = 0.383.$$

$$C = \frac{1}{0.383} = 2.61 \text{ microfarads.} \quad \textit{Ans.}$$

(*b*) $Q = 2.61 \times 600 = 1566$ microcoulombs, on each condenser. *Ans.*

(*c*)
$$E_1 = \frac{1{,}566 \times 10^{-6}}{5 \times 10^{-6}} = 313 \text{ volts.}$$

$$E_2 = \frac{1{,}566 \times 10^{-6}}{10 \times 10^{-6}} = 157 \text{ volts.}$$

$$E_3 = \frac{1{,}566 \times 10^{-6}}{12 \times 10^{-6}} = 130 \text{ volts.} \quad \textit{Ans.}$$

183. Energy Stored in Condensers.—As a certain quantity of electricity is stored in a condenser and a difference of potential exists between the positive and negative plates, energy must be stored in the condenser. The existence of this energy is shown by the spark resulting from short-circuiting the condenser plates. The energy in joules or watt-seconds is

$$W = \frac{1}{2} QE. \tag{100}$$

This may also be written:

$$W = \frac{1}{2}CE^2, \tag{101}$$

$$W = \frac{1}{2}\frac{Q^2}{C}. \tag{102}$$

The similarity in form of Eq. (101) to the equation for the energy stored in the magnetic field should be noted (see Eq. (77) p. 217, Par. 162). The energy stored in the dielectric field is proportional to the square of the *voltage*, whereas the energy stored in the electromagnetic field is proportional to the square of the *current*.

Example.—Determine the stored energy in each of the condensers in series of Par. 182 and the total stored energy.

Using equation (102),

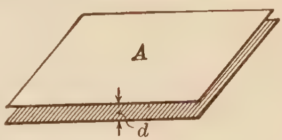
$$W_1 = \frac{1}{2}\frac{(1{,}566 \times 10^{-6})^2}{5 \times 10^{-6}} = 0.2453 \text{ joule.} \quad \textit{Ans.}$$

$$W_2 = \frac{1}{2}\frac{(1{,}566 \times 10^{-6})^2}{10 \times 10^{-6}} = 0.1225 \text{ joule.} \quad \textit{Ans.}$$

$$W_3 = \frac{1}{2}\frac{(1{,}566 \times 10^{-6})^2}{12 \times 10^{-6}} = 0.1020 \text{ joule.} \quad \textit{Ans.}$$

The total energy $W = \frac{1}{2}(1{,}566 \times 10^{-6} \times 600) = 0.4698$ joule. *Ans.*

184. Capacitance of Parallel-plate Condensers.—As a rule it is impossible to calculate the capacitance of a condenser, or the mutual capacitance of conducting bodies, because of their complex geometry and also because the dielectric constants of the intervening media are not always accurately known. There are some simple cases, however, where accurate calculations are possible.



$$C = \frac{\kappa A}{4\pi d \times 9 \times 10^5}$$

FIG. 202.—Capacitance of a plate condenser.

The plate condenser (Fig. 202) is the simplest form of condenser. Let A be the area of one side of each plate in square centimeters, d the distance between plates in centimeters, and κ, the dielectric constant of the medium between the plates. The capacitance is

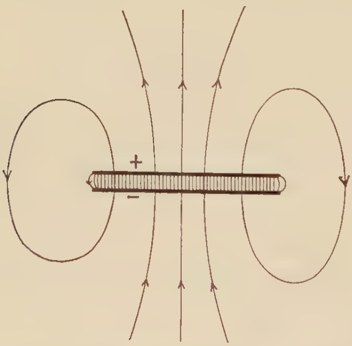
$$C_s = \frac{\kappa A}{4\pi d} \text{ statfarads.} \tag{103}$$

Since the microfarad is 9×10^5 times as great as the statfarad, it is necessary to divide Eq. (103) by 9×10^5 in order to obtain the capacitance in microfarads. Hence,

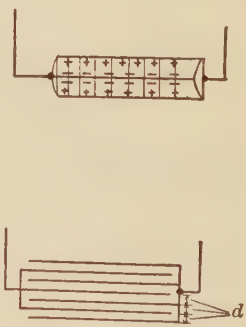
$$C = \frac{\kappa A}{4\pi d \times 9 \times 10^5} \text{ microfarads.} \tag{104}$$

In this equation it is assumed that the electrostatic lines between the two plates are parallel.

The total capacitance of a simple plate condenser of this type cannot be accurately calculated for the following reason: All of the dielectric lines do not lie between the plates as certain lines pass from the back of the positive plate to the back of the negative as shown in Fig. 203 (*a*). This results in the actual capacitance being greater than the value as just calculated. This error may be avoided by using one more plate in one group than



(*a*) Electrostatic leakage lines of a plate condenser.



(*b*) Multi-plate condensers.

FIG. 203.

in the other (Fig. 203 (*b*)). In this case the area A (Eq. (104)) includes both sides of all the plates with the exception of the two outside ones. As the charge on both outer plates is of the same sign and the plates have the same potential, no dielectric lines can pass between them. An error due to the bulging or "fringing" of the lines near the edges of the plates may occur unless the plate area is large compared with the distance between plates.

Example of Condenser Design.—It is desired to construct a plate condenser having a total capacitance of 8 microfarads. The plates are of tin-foil

6 × 8 in. and 1 mil thick. The dielectric is of paper 7 × 9 in. and 2 mils thick and having a dielectric constant of 3. How many sheets of paper and of tin-foil are necessary? What will be the dimensions of the condenser?

The area of each plate is:

$$6 \times 8 \times (2.54)^2 = 309.6 \text{ sq. cm.}$$

The distance between plates:

$$d = 0.002 \times 2.54 = 0.00508 \text{ cm.}$$

The capacitance between two plates (from Eq. 104):

$$C = \frac{3 \times 309.6}{4\pi \times 0.00508 \times 9 \times 10^5} = 0.01616\ \mu f.$$

Therefore:

$$\frac{8}{0.01616} = 495 \text{ sections are needed.}$$

These sections are indicated at d, Fig. 203 (b). This means that 496 plates and 495 sheets of paper are necessary.

Thickness:

$$\begin{aligned} \text{Tin-foil} &= 496 \times 0.001 = 0.496 \text{ in.} \\ \text{Paper} &= 495 \times 0.002 = \underline{0.990} \text{ in.} \\ & \qquad\qquad\qquad\quad 1.486 \text{ in.} \end{aligned}$$

Volume of condenser proper = 7 in. × 9 in. × 1.49 in. *Ans.*

Of course additional outside insulation and a protective covering are necessary.

185. Capacitance of Cylindrical and Spherical Condensers.—A coaxial cylindrical condenser is shown in Fig. 204. The inner

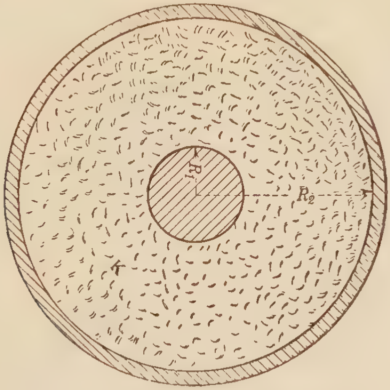


FIG. 204.—Cylindrical condenser.

cylinder has a radius of R_1 cm. and the outer cylinder has an inner radius of R_2 cm. The dielectric medium has a dielectric

constant κ. The capacitance per centimeter axial length of this type of condenser is

$$C_s = \frac{\kappa}{2 \times 2.303 \log_{10}\frac{R_2}{R_1}} \text{ statfarads.} \tag{105}$$

The capacitance of a mile of such a condenser is

$$C = \frac{0.0388\kappa}{\log_{10}\frac{R_2}{R_1}} \text{ microfarads.} \tag{106}$$

Since single-conductor concentric cables are condensers of this type, Eq. (106) has wide practical uses.

Example.—A 1,200-foot length of No. 4 A.W.G., single-conductor, rubber-insulated underground cable has a $\frac{5}{32}$-inch wall of insulation. The diameter of the conductor is 204.3 mils. The capacitance of the cable is measured and found to be 0.105 microfarad. What is the dielectric constant κ of the rubber?

$$R_1 = \frac{0.2043}{2} = 0.1022 \text{ in.}$$

$$\tfrac{5}{32} \text{ in.} = 0.1563 \text{ in.}$$

$$R_2 = \frac{0.1563 + 0.1563 + 0.2043}{2} = 0.2585 \text{ in.}$$

The capacitance of a mile length

$$C = \frac{5{,}280}{1{,}200} 0.105 = 0.462\ \mu f$$

Using Eq. (106),

$$\kappa = \frac{0.462 \log \frac{0.2585}{0.1022}}{0.0388} = \frac{0.462 \times 0.4031}{0.0388} = 4.8. \quad \textit{Ans.}$$

The capacitance of concentric spherical condensers is

$$C_s = \frac{\kappa}{\frac{1}{R_1} - \frac{1}{R_2}} = \frac{\kappa R_2 R_1}{R_2 - R_1} \quad \text{statfarads,}$$

where κ is the dielectric constant, R_2 the radius of the inner wall of the outer sphere, and R_1 the radius of the inner sphere.

If the radius of the outer sphere becomes infinite, $1/R_2$ becomes zero and the capacitance beomes κR_1. In air, therefore, the capacitance in statfarads of an isolated sphere is equal to its radius in centimeters. It follows that the capacitance of the earth as an isolated sphere is approximately 720 μf.

186. Measurement of Capacitance.—There are two common methods of measuring capacitance, the direct-current or ballistic method and the alternating-current or bridge method.

The direct-current method employs a galvanometer which is used ballistically. It can be shown that if the moving coil of the ordinary galvanometer have considerable inertia and be properly damped, its maximum throw, due to the impluse produced by the sudden passage of a current through the coil, is proportional to the total quantity of electricity passing through the galvanometer. This assumes that the entire charge passes through the coil before the coil begins to move (see p. 223, Par. 165). Let D be the maximum galvanometer throw in centimeters. Then:

$$Q = KD, \tag{107}$$

where Q is the quantity and K the galvanometer constant.

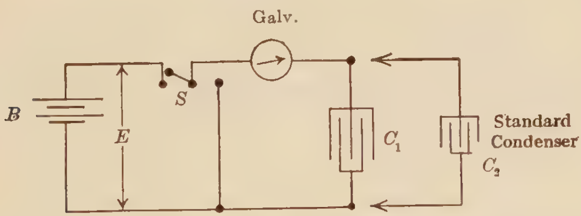


Fig. 205.—Ballistic method of measuring capacitance.

To make the measurement, the apparatus is connected as shown in Fig. 205. A battery B supplies the current for the apparatus. The measurement may be made on either the charge or the discharge of the condenser or check measurements may be made using both the charge and the discharge. If the condenser is at all leaky, the discharge method is preferable.

When the switch S is closed to the left the condenser C_1 is charged through the galvanometer and the maximum throw of the galvanometer is read. Several check readings should be taken. The galvanometer should return immediately to zero. If it shows a steady deflection it indicates a leaky condenser. In a corresponding manner the ballistic throw of the galvanometer may be read on discharge by closing switch S to the right after charging. Let D_1 be the deflection of the galvanometer when C_1

is connected, Q_1 the quantity going into the condenser, and E the voltage across the condenser. Then by Eq. (107),

$$Q_1 = KD_1.$$

Also
$$Q_1 = C_1E,$$

where C_1 is the unknown capacitance.

$$\therefore C_1E = KD_1. \qquad (a)$$

If now the standard capacitance C_2 be substituted for the unknown condenser and another set of readings taken,

$$Q_2 = KD_2,$$

or
$$C_2E = KD_2. \qquad (b)$$

Dividing (a) by (b),

$$\frac{C_1E}{C_2E} = \frac{KD_1}{KD_2}.$$

$$C_1 = C_2\frac{D_1}{D_2}.$$

$\frac{C_2}{D_2}$ is the galvanometer constant.

It is often desirable to use an Ayrton shunt in such measurements as it gives the apparatus greater range. When such a

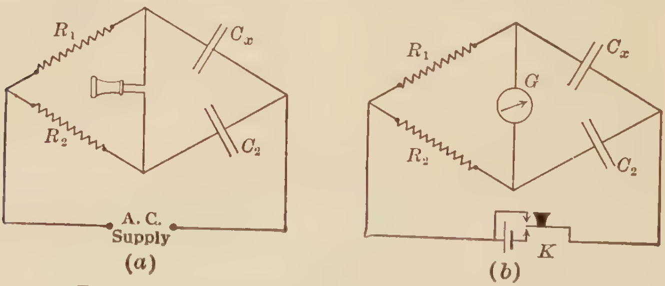


FIG. 206.—Bridge methods of measuring capacitance.

shunt is used, proper correction must be made for its multiplying power.

In the bridge method two capacitances form adjacent arms of a Wheatstone Bridge and two resistances form the other two arms (Fig. 206 (a)). An alternating-current supply is preferable. The secondary of an induction coil may be used as the source of power or a battery with a key may be made to charge and discharge the system as shown in Fig. 206 (b). A telephone is used as a detector except in (b). Let C_x be the unknown capacitance and

C_2 a standard which may or may not be adjustable. R_1 and R_2 are two known resistances, one of which should be adjustable unless C_2 is so.

Either C_2 or one of the resistances is adjusted, until there is no sound in the telephone, showing that the bridge is in balance. Under these conditions:

$$\frac{C_x}{C_2} = \frac{R_2}{R_1},$$

$$C_x = C_2 \frac{R_2}{R_1}. \tag{108}$$

When a battery is used, a double contact key K is necessary. K is pressed and released, and until the bridge is balanced, the galvanometer will deflect both upon the charge of the system, when the key is pressed, and upon the discharge, when the key is released. The bridge is balanced when the galvanometer does not deflect on either charge or discharge. Equation (108) is then applicable.

In the above measurements, it is assumed that there is little if any leakage through the condensers.

187. Cable Testing—Location of a Total Disconnection.—In Chap. VII, it was shown that a grounded fault in a cable could

FIG. 207.—Locating an open in a cable.

be located by suitable resistance measurements, such as the Murray and Varley loop tests. If a cable be totally disconnected and its broken ends remain insulated these loop tests are impossible. The distance to the fault may now be determined by capacitance measurements. The connections are shown in Fig. 207. The capacitance C_1 of the length, x, to the fault is first measured by the ballistic method. If a *similar* perfect cable parallels the faulty cable, the two are looped at the far end and the capacitance C_2 of a length l of the perfect cable plus the length $l - x = 2l - x$ of the faulty cable is measured.

Let c be the capacitance per foot of each cable, assumed to be the same for each:

$$C_1 = xc = KD_1,$$

where K is the galvanometer constant and D_1 the deflection corresponding to C_1.

Likewise,

$$C_2 = (2l - x)c = KD_2.$$

Dividing one equation by the other,

$$\frac{x}{2l - x} = \frac{D_1}{D_2},$$

$$x = l\frac{2D_1}{D_1 + D_2}. \tag{109}$$

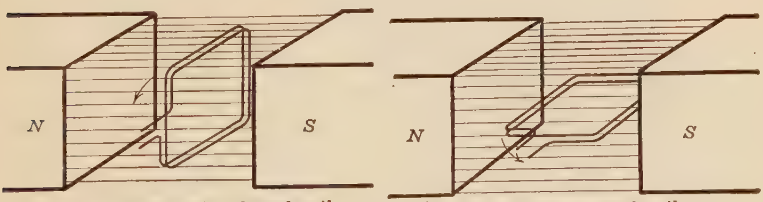
The capacitance per unit length and the total capacitance do not enter into the equation, so that it is not necessary to use a standard condenser for the calibration of the galvanometer. The capacitances of the various lengths are proportional to the galvanometer deflections when corrected for the setting of the Ayrton shunt.

CHAPTER X

THE GENERATOR

Definition.—A generator is a machine which converts mechanical energy into electrical energy. This is accomplished by means of an armature carrying conductors upon its surface, acting in conjunction with a magnetic field. Electrical power is generated by the relative motion of the armature conductors and the magnetic field.

In the direct-current generator the field is usually stationary and the armature rotates. In most types of alternating-current generators the armature is stationary and the field rotates. Either the armature or the field is driven by mechanical power applied to its shaft.



(*a*) Maximum lines passing through coil (*b*) No lines passing through coil

Fig. 208.—Simple coil rotating in a magnetic field.

188. Generated Electromotive Force.—It was shown in Chap. VIII that if the flux linking a coil is varied in any manner, an electromotive force is *induced* in the turns of the coil. The action of the generator is based on this principle. The flux linking the armature coils is varied by the relative motion of the armature and field.

In Fig. 208 a coil revolves in a uniform magnetic field produced by a north and a south pole. In (*a*) the coil is perpendicular to the magnetic field and in this position the maximum possible flux links the coil. Let this flux be ϕ.

If the coil be rotated counterclockwise a quarter of a revolution, it will lie in the position shown in (*b*). As the plane of the coil is parallel to the flux no lines link the coil in this position. Therefore, in a quarter revolution the flux which links the coil has been decreased by ϕ lines. The average voltage induced in the coil during this period is, therefore,

$$e = N\frac{\phi}{t}10^{-8} \text{(Chap. VIII, Eq. 74)},$$

where N is the number of turns in the coil and t the time required for a quarter revolution. But $t = \frac{1}{4R}$ where R = the *revolutions per second*. Therefore, the average voltage during a quarter revolution is

$$e = 4NR\phi 10^{-8} \text{ volts.}$$ Φ maximal

The generation of electromotive force in a moving coil of this type, which is similar to those used in dynamos, may also be analyzed by considering the total electromotive force as being due to the sum of the electromotive forces generated in each side of the coil. The electromotive force of one turn is the sum of the electromotive forces in each conductor forming the sides of the turn, since these conductors are connected in series by the end connections of the turn. The individual electromotive forces are then considered as being generated in the *conductor* rather than induced in the coil. This in no way conflicts with the fact that the induced electromotive force is also due to the change of flux linked with the coil. The same total e.m.f. is obtained under either assumption.

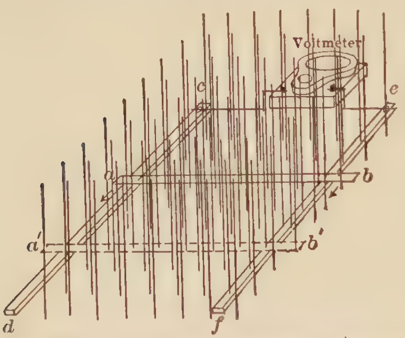


FIG. 209.—Conductor cutting a uniform magnetic field.

Consider the conductor *ab* (Fig. 209) free to slide along the two metal rails *cd* and *ef*. The rails are connected at one end *ce* by a voltmeter. A magnetic field having a density of B lines per sq. cm. passes perpendicularly through the plane of the rails and conductor.

Let the conductor ab move at a uniform velocity to the position $a'b'$. While this movement is taking place, the voltmeter will indicate a certain voltage. This voltage may be attributed to either of two causes.

1. As conductor ab moves to position $a'b'$ the flux linking the conducting loop formed by ce, the rails and ab, is increased, because of the increasing area of this loop.

2. An electromotive force is generated in the conductor ab since it cuts the magnetic field.

Similarly, the electromotive force developed by the coil in Fig. 208 may be attributed to the e.m.fs. generated in the conductors on opposite sides of the coil through their cutting of magnetic lines. These conductors are connected in series by the end conductors, or connectors, which in themselves generate no electromotive force. The direction of the electromotive forces developed in the coil-sides are such that these e.m.fs. are additive.

The electromotive force in volts generated by a single conductor which cuts a magnetic filed is

$$e = Blv10^{-8}, \tag{110}$$

where B, l, and v are mutually perpendicular.

B is the flux density of the field in gausses, l the length of conductor in *centimeters*, and v the velocity of the conductor in centimeters per second.

That the electromotive force induced by a change of the flux linked with a coil is the same as that obtained by considering the e.m.fs. generated by the cutting of magnetic lines by the conductor which make up the coil may be illustrated by a concrete example. Let the flux have a density of 100 lines per square centimeter (Fig. 209). The distance ab is 30 cm. and aa' is 20 cm. The conductor ab moves at a uniform velocity to position $a'b'$ in 0.1 sec. What is the electromotive force across ce?

The change of flux linking the coil is:

$$\phi = 30 \times 20 \times 100 = 60{,}000 \text{ lines.}$$

This change occurs in 0.1 sec.

Then by Eq. (74), page 211,

$$e = 1\,\frac{60{,}000}{0.1}\,10^{-8} = 0.006 \text{ volt.}$$

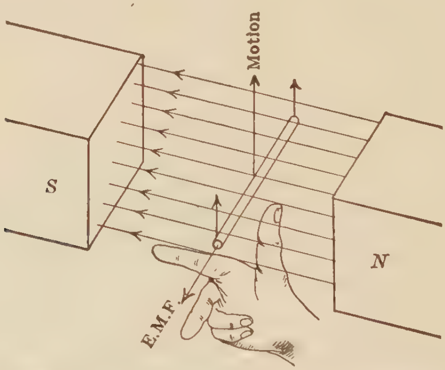
Applying Eq. (110),

$$v = \frac{20}{0.1} = 200 \text{ cm./sec.}$$

$$e = 100 \times 30 \times 200 \times 10^{-8} = 0.006 \text{ volt.}$$

It will be seen that the same result is obtained whether the electromotive force is considered as being generated by the conductor itself cutting the field or whether it is considered as being induced by the change in flux linking the coil.

189. Direction of Induced Electromotive Force. Fleming's Right-hand Rule.—A definite relation exists among the direction of the flux, the direction of motion of the conductor and the direction of the electromotive force induced in the conductor just as a definite relation exists between the direction of current and the flux which it produces.



Fore finger along lines of force. Thumb in direction of motion. Middle finger gives direction of induced emf.

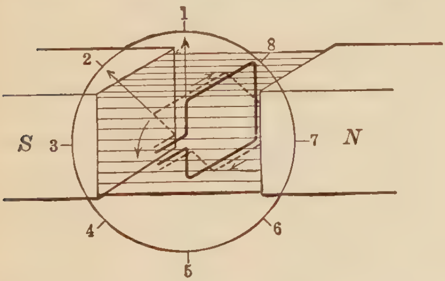
FIG. 210.—Fleming's right-hand rule.

A very convenient method for determining this relation is the *Fleming right-hand rule.* In this rule the fingers of the *right* hand are utilized as follows:

Set the forefinger, the thumb, and the middle finger of the right hand at right angles to one another (Fig. 210). If the *fore finger points along the lines of flux and the thumb in the direction of motion of the conductor, the middle finger will point in the direction of the induced electromotive force.*

This rule is illustrated by Fig. 210.

190. Voltage Generated by the Revolution of a Coil.—A coil of a single turn is shown in Fig. 211 (*a*). The coil rotates in a counterclockwise direction at a uniform speed in a uniform magnetic field. As the coil assumes successive positions, the electromotive force induced in it changes. When it is in position 1 the electromotive force generated is zero, for in this position neither conductor is *cutting* magnetic lines, but rather is moving parallel to these lines. When the coil reaches position 2 (shown dotted) its conductors are cutting across the lines obliquely and the electromotive force has a value indicated at 2 in Fig. 211 (*b*). When the coil reaches position 3, the conductors are cutting the lines *perpendicularly* and are therefore cutting at the maximum possible rate. Hence the electromotive force is a maximum when



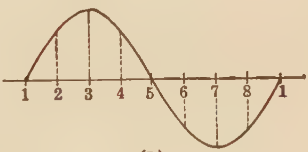


FIG. 211.—Emf. induced in a coil rotating at constant speed in a uniform magnetic field.

the coil is in this position. At position 4 the electromotive force is less, due to a lesser rate of cutting. At position 5 no lines are being cut and as in 1 there is no electromotive force. In position 6 the direction of the electromotive force in the conductors will have reversed, as each conductor is under a pole of opposite sign to that for positions 1 to 5. The electromotive force increases to a negative maximum at 7 and then decreases until the coil again reaches position 1. After this the coil merely repeats the cycle.

This induced electromotive force is alternating and an e.m.f. varying in the manner shown is called a *sine wave* of electromotive force. This alternating electromotive force may be impressed on an external circuit by means of two *slip-rings* (Fig. 212). Each ring is continuous and insulated from the other

ring and from the shaft. A metal or a carbon brush rests on each ring and conducts the current from the coil to the external circuit. (See Vol. II, Chap. I.)

If a *direct current* is desired, that is, one whose direction is always the same, such rings cannot be used. A direct current must always flow into the external circuit in the *same direction.* As the coil current must necessarily be alternating, since the

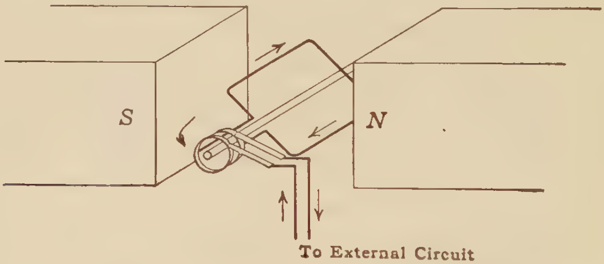


FIG. 212.—Current taken from rotating coil by means of slip-rings.

e.m.f. which produces it is alternating as has just been shown, this current must be rectified before it is allowed to enter the external circuit. This rectification can be accomplished by using a split ring such as is shown in Fig. 213. Instead of using two rings, as in Fig. 212, one ring only is used. This is split by saw cuts at two points diametrically opposite each other.

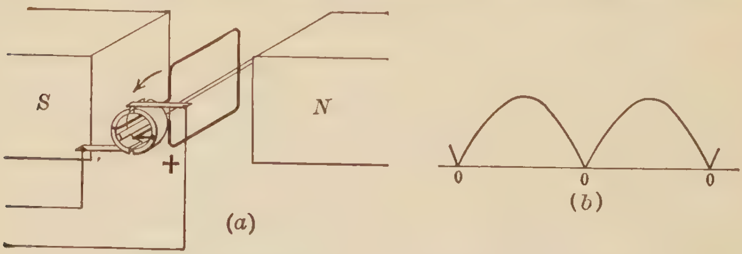


FIG. 213.—Rectifying effect of a split ring or commutator.

The two ends of the coil are connected one to each of the sections or segments so produced.

A careful consideration of Fig. 213 will show that, as the direction of the current in the coil reverses, its connections to the external circuit are simultaneously reversed. Therefore, the direction of flow of the current to the external circuit is not

changed. The brushes pass over the cuts in the ring when the coil is perpendicular to the magnetic field or when it is in the so-called neutral plane and is generating no voltage, as shown in Fig. 211. These neutral points are marked 0-0-0 in Fig. 213 (*b*).

By comparing Fig. 211 (*b*) with Fig. 213 (*b*) it will be seen that the negative half of the wave has been reversed and so made positive.

A voltage with a zero value twice in each cycle, as shown in Fig. 213, could not be used commercially for direct-current

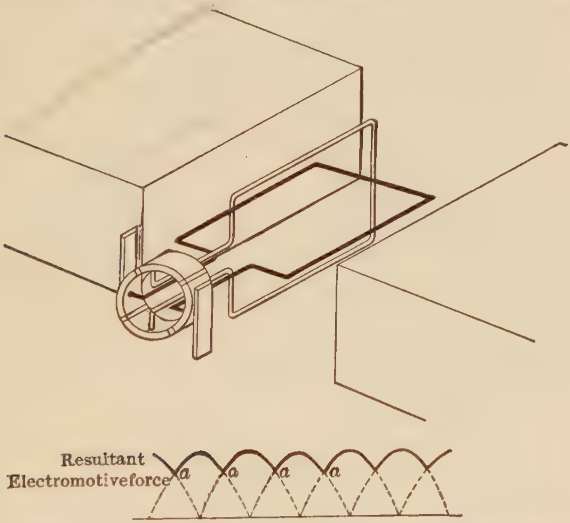


FIG. 214.—Effect of two coils and four commutator segments upon the electromotive force wave.

service. Also a single-coil machine would have a small output for its size and weight. The electromotive force wave of Fig. 213 may be made less pulsating by the use of two coils and four commutator segments (Fig. 214). This gives an *open-circuit* type of winding, since it is impossible to start at any one commutator segment and return to this segment again by following through the entire winding. In this particular arrangement the full electromotive force generated in each coil is not utilized, as one coil passes out of contact with the brushes at points *a*, *a*, *a*, (Fig. 214 (*b*)), and the voltage shown by the dotted lines is not utilized.

191. Gramme-ring Winding.—This type of winding in diagrammatic form (Fig. 215) consists of insulated wire wound

spirally around a ring (or hollow cylinder of iron) with taps taken from the wire at regular intervals and connected to commutator segments. This winding is simple, and has the advantage that a single winding is adapted to any number of poles, if the voltage limitations do not prevent. The portions of the conductors which lie inside the ring cut no flux and act merely as connectors for the active portions of the conductors. Because of the small proportion of active conductors a relatively large amount of copper is required in such a winding. In small machines, there is not sufficient room to carry these inactive conductors back through the armature core. In a gramme-ring winding formed coils cannot be used and this makes the winding

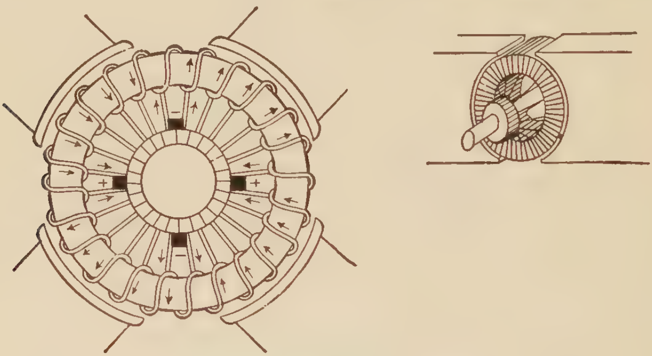


FIG. 215.—Gramme-ring winding.

expensive. This type of winding has a high inductance, which renders good commutation difficult.

It will be noted that the electromotive force between brushes in a gramme-ring winding is the sum of the electromotive forces of all the coils that lie between brushes. When one coil passes a brush another moves forward to take its place. Figure 216 shows the electromotive force between brushes due to four coils, it being assumed that the voltage curve for each is a sine wave. The electromotive force of each coil is plotted separately. These electromotive forces do not all have their zero value at the same time nor do they reach their maximum value at the same time owing to the positions of the individual coils. The resultant electromotive force at any point is the sum at this point of these individual electromotive forces. This voltage should be

compared with the electromotive force obtained with the open-coil winding shown in Fig. 214, in which the resultant electromotive force does not equal the sum of the individual electromotive forces but is made up of the successive tops of the individual waves. It will be noted that a fairly smooth resultant electromotive force is obtained with four coils, the "ripples" being noticeable but comparatively small in magnitude.

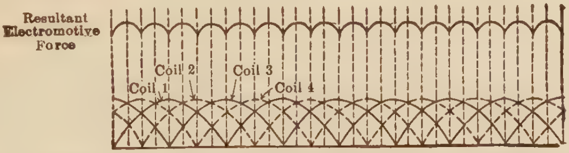


FIG. 216.—Resultant electromotive force due to four series-connected coils between brushes.

A gramme-ring winding is called a *closed* winding, since it is possible to start at any one point in the winding and return to the same point again by passing continuously through the winding.

192. Drum Winding.—The objections to the ring winding are overcome by the use of the drum winding. The conductors of this winding all lie upon the surface of the armature and are

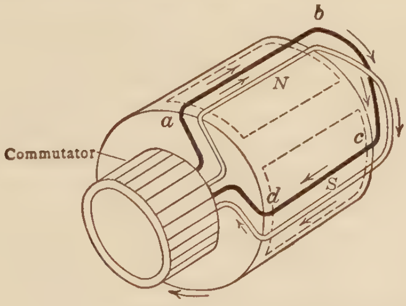


FIG. 217.—Two coils in place on a 4-pole, drum-wound armature.

connected to one another by front and back connections or coil ends (*ad* and *bc*, Fig. 217, are coil ends). With the exception of these end connections, all the armature copper is "active," that is, it cuts flux and so is active in generating electromotive force.

The sides of each coil should be about one pole-pitch (the distance between centers of adjacent poles) apart. If one con-

ductor is under a north pole the other is then under a south pole, and as both move in the same direction, but under different poles, the electromotive forces of these two conductors will be in opposite directions (Fig. 217). Due to the manner in which these conductors are connected at their ends, the electromotive forces in the individual coils are additive.

In most gramme-ring windings, and in the earlier drum-wound machines, the surface of the armature core was smooth. The conductors were held in position partly by projecting pins, and were prevented by binding wires from flying out under the action of centrifugal force. The smooth core construction has been superseded by the "iron-clad" type where the conductors are embedded in slots as indicated in Fig. 220. The slots are lined with insulation and the conductors are held in firmly by wooden or non-conducting wedges in the larger machines (see Fig. 247), and by binding wires in the smaller types (see Fig. 237). These constructions are much better mechanically than the smooth core armature type and they also permit a much shorter air-gap. On the other hand, as the coils are embedded in iron, they have a high self-inductance. This makes commutation more difficult and the flux pulsations due to the armature teeth give pole-face and tooth losses.

193. Lap Winding.—Direct-current armatures are usually wound with form-made coils (Fig. 218). These coils are usually wound on machines with the necessary number of turns, and are then wound with cotton or mica tape. They are then bent into proper shape by another machine. The two ends are left bare so that later they may be soldered to the commutator bars. The span of the coil, called the coil pitch, should be equal or nearly equal to the pole-pitch, so that when one side of a coil is under a north pole the other is under a south pole.

A saving in copper in the end-connections and slightly better commutation result if the coil span is made less than the pole-pitch. Such a winding is called a *fractional-pitch winding*. With direct-current machines, the pitch may be as low as nine-tenths full pitch. If the pitch is too small, a very appreciable reduction in the induced e.m.f. results.

Usually two coil-sides occupy one slot, one coil-side lying at the top and the other at the bottom of the slot. That is, if the side

of one coil is in the bottom of a slot, its opposite side lies in the top of some other slot. This allows the end connections to be easily made as the coil ends can be bent around one another in a systematic manner, passing from the bottom to the top layer by means of the peculiar twist in the ends of the coils.

The bundle of wires constituting one side *ab* (Figs. 217 and 219) of a coil will be termed a winding element. This may consist of one or of several conductors taped together. Even

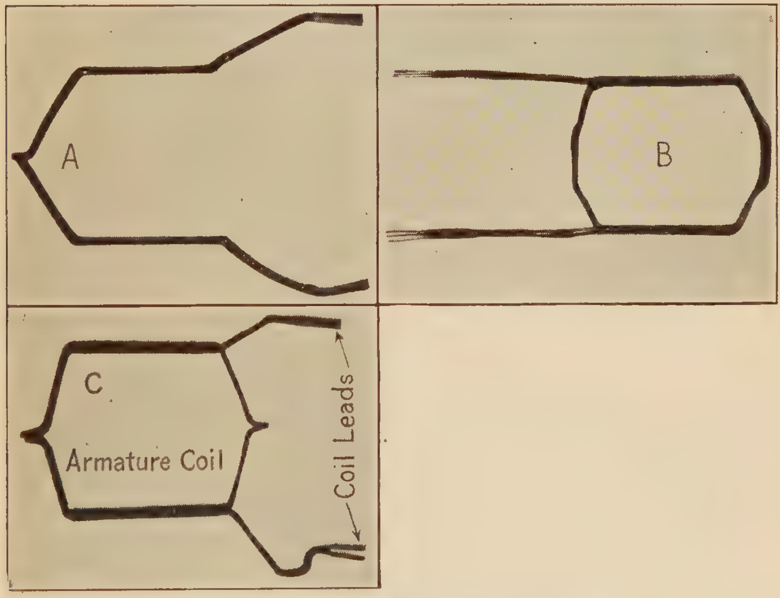


FIG. 218.—Formed armature coils.

when there are several conductors, they will be shown as a single conductor in the wiring diagram, as indicated in Fig. 219. Obviously there will be twice as many of these elements as there are coils. The number of elements that the coil advances on the back of the armature is the *back pitch* of the winding and will be denoted by y_b. This back pitch is determined by the span of the connection *bc* (Fig. 217). The number of elements spanned on the commutator end of the armature is called the *front pitch* and will be designated by y_f. This may be greater or less than the back pitch *but not equal* to it. If it be greater, the winding is

retrogressive; that is, it advances in a counterclockwise direction when viewed from the commutator end. If the front pitch be less than the back pitch, the winding is progressive; that is, it advances in a clockwise direction when viewed from the commutator end. This is illustrated in Figs. 220 and 221. Conductor 1 is connected on the back of the armature to conductor 10. Therefore, the back pitch $y_b = 9$. Conductor 10 is then connected back to 3 on the front of the armature, the connection being made at the commutator segment. Therefore, the front pitch $y_f = 7$. This winding is therefore *progressive*.

As most windings are now made in two layers, only two-layer windings will be considered. The coil-sides or elements lying

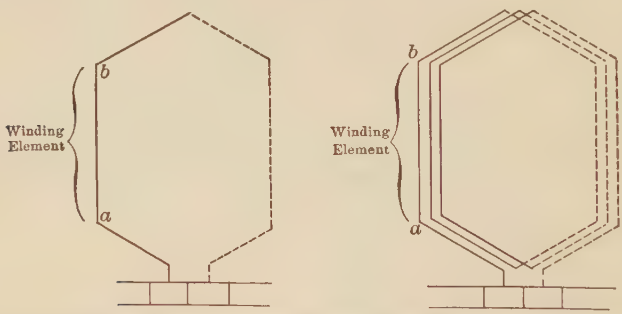


FIG. 219.—Single coil representing a three-turn coil of an armature winding.

in the tops of the slots will be given odd numbers and those in the bottoms of the slots even numbers (Fig. 220). If one side of a coil lies in the bottom of a slot, the other side must lie in the top of some other slot. Hence y_b and y_f must both be odd. If y_b and y_f were both even, all the conductors would lie in *either* the tops of the slots or in the bottoms of the slots. It would be impossible, therefore, for one side of any coil to lie in the bottom of one slot and the other side of the same coil to lie in the top of some other slot. Hence y_b and y_f must both be odd if the winding is to be properly placed on the armature.

It follows that the front and back pitches must differ from each other by 2. That is

$$y_b = y_f \pm 2 \tag{111}$$

The average pitch:

$$y = \frac{y_b + y_f}{2}. \tag{112}$$

The + sign in Eq. (111) indicates that the winding is progressive, that is, progresses in a clockwise direction when viewed from the commutator end. The − sign indicates a retrogressive

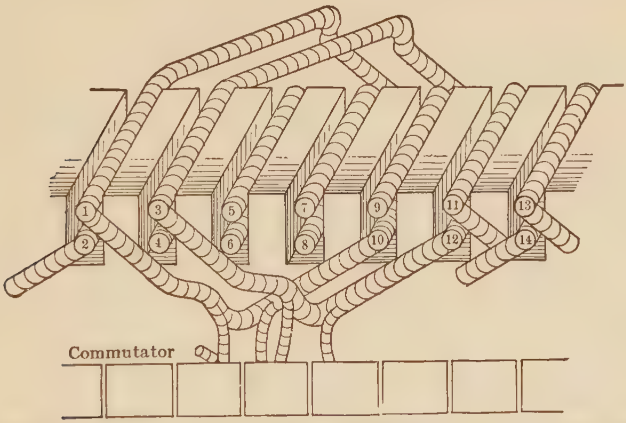


FIG. 220.—Simplex lap winding having back pitch of 9 and front pitch of 7.

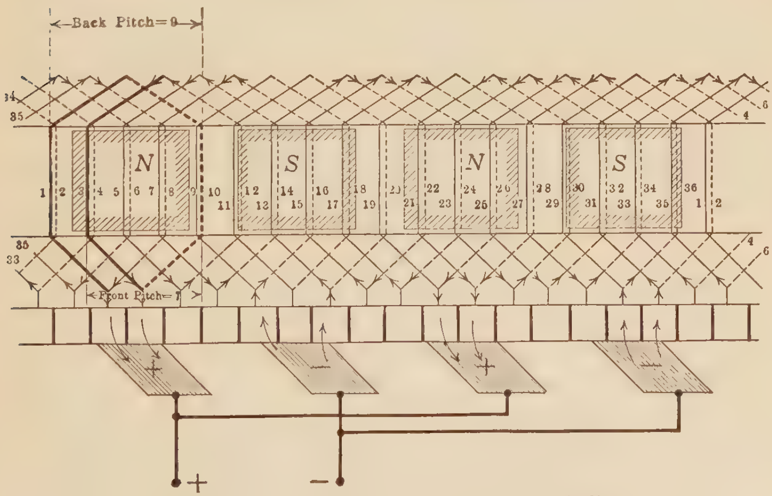


FIG. 221.—Development of a 4-pole lap winding.

winding whose advance is in a counterclockwise direction when viewed from the commutator end.

Commutator Pitch.—It will be seen that for every coil one commutator segment is necessary. Therefore, the number of commutator segments

$$N_c = N = \frac{Z}{2},$$

where Z is the total number of winding elements on the surface of the armature and N is the number of coils.

From Figs. 220 and 221 it will be seen that the winding advances one *commutator segment for each complete* turn. Hence in such a winding the commutator pitch $y_c = 1$.

In designing a winding it is necessary that the opposite sides of each coil lie under different poles so that the two electromotive forces generated in the coil sides may be additive. Hence, the average pitch should be nearly equal to the number of elements per pole.

The three fundamental conditions to be fulfilled by a *lap winding* are:

1. The pitch must be such that the opposite sides of the coil lie under unlike poles.

2. The winding must include each element once and only once.

3. The winding must be re-entrant or must close on itself.

Example.—Assume that the armature of a 4-pole machine has 18 slots. Design a 2-layer lap winding having two elements per slot.

There are 36 elements. The average pitch should be nearly equal to $\frac{36}{4} = 9$. The back pitch can be made equal to 9.

$$y_b = 9 \qquad y_f = 7$$

Starting at 1, the winding will progress as follows:

1–10–3–12–5–14–7–16–9–18–11–20–13–22–15–24–17–26–19
28–21–30–23–32–25–34–27–36–29–2–31–4–33–6–35–8–1.

The above is called a winding table. It is very useful in checking the winding. By proper checking it may be seen that each conductor is included once and only once and that the winding closes at the same conductor, 1 in this case, at which it began. The winding is shown in Fig. 221 as if it were split axially and rolled out flat. It will be noted that the brushes rest on segments to which are connected elements which lie midway between the poles, as 1 and 19, for example. That is, the brushes are con-

nected to conductors in which practically no e.m.f. is being induced.

194. Multiple Coils.—In the larger sizes of machines it is often necessary to place several coil-sides or elements in one slot, usually 4, 6, or 8. More than 8 coil-sides per slot are rarely used. The reason for placing several coil-sides in a slot is as follows: If two elements per slot were used, one in the top layer and one in the bottom layer, a large number of slots would be necessary. This would reduce the size of the slots and make the space factor (ratio of the copper cross-section to the slot cross-section) low. Also the tooth roots would be so narrow that the teeth would be mechanically weak. By placing several elements in each slot the number of slots is reduced and larger slots result. This also reduces the cost of winding.

A coil made up of several individual coils is shown at B in Fig. 218. The three individual coils are taped together and placed in the slots as a unit. A careful examination of the armature of Fig. 237, page 293, shows four wires running from the top of each slot to the commutator, indicating a quadruple coil.

The numbering and connections of the conductors are in no way different from those already described in the case of but two coil-sides per slot.

The selection of the pitch, where several coil-sides per slot are used, is more restricted than it is with two elements per slot.

Assume that a 6-pole machine has 72 slots and 6 elements per slot. The total number of elements on the armature surface:

$$Z = 72 \times 6 = 432.$$

The pitch should be approximately

$$y = \frac{432}{6} = 72.$$

Let

$$y_b = 71,$$
$$y_f = 69.$$

If this back pitch is used, a coil must reach from coil-side 1 to coil-side 72 (Fig. 222). Then the next coil-side, which is taped to coil-side 1, will obviously reach to coil-side 74. These two coils, therefore, span different distances on the armature, since coil-sides 72 and 74 lie in different slots. Hence, although the left-hand sides of these two coils and coil-side 5 are taped together as a unit, the right-hand coil-sides cannot be so taped together,

since they lie in different slots. This can be seen from a study of Fig. 222. In practice, it is obviously desirable that if any number of coil-sides are to be placed together in the top of any one slot, their opposite sides should all be placed together in the bottom of some other slot. This makes it possible to tape all these coils together and place them in both slots as a unit.

Therefore, if in the above case $y_b = 73$ and $y_f = 71$, the coil containing coil-side 1 will connect from the upper *left-hand* side of slot A to the lower *left-hand* side of slot B, that is, from coil-side 1 to coil-side 74. Coil-side 3 will connect from the center and top of slot A to the center and bottom of slot B, and coil-side 5

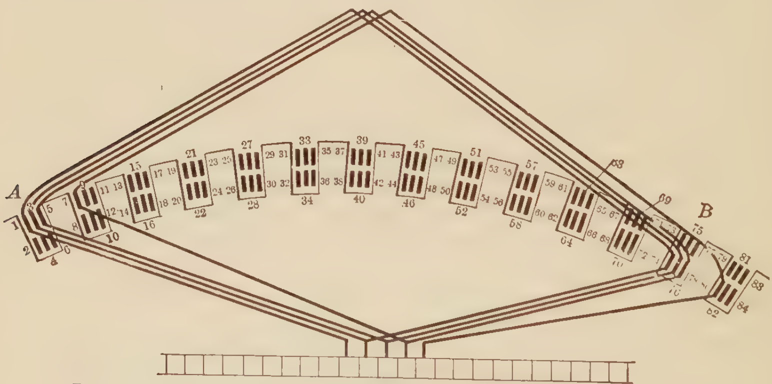


FIG. 222.—Method of connecting the conductors of a triple coil.

will connect from the upper right-hand side of slot A to the lower right-hand side of slot B. As all three coils now span the same distance on the armature, they will be equal in size, form, etc. Moreover, the three single coils can be taped together to form a *triple coil* and placed in the two slots as a unit. Therefore, if three coils have their adjacent sides in the top of one slot, their other sides should lie together in the bottom of some other slot. This condition is obtained by making the back pitch one greater than a multiple of the number of coil-sides or elements per slot. For example, in the illustration just given, y_b is equal to 73, one greater than 72, 72 being a multiple of 6.

Coils taped together and placed in the slots in this manner are called *multiple coils*.

195. Paths through an Armature.—If four batteries, each having an electromotive force of 2 volts and a current capacity of 10 amp. be connected in parallel (Fig. 223 (*a*)) there will be *four* paths for the current to follow in going through the batteries. The voltage of the combination will be 2 and the ampere capacity 40, making a total power capacity of 80 watts. If now these same batteries be arranged in two groups of two in series (Fig. 223 (*b*)) there result but two paths for the current to follow, but the voltage is now 4 volts. The current capacity is now 20 amp., and the power capacity is $4 \times 20 = 80$ watts, its previous value.

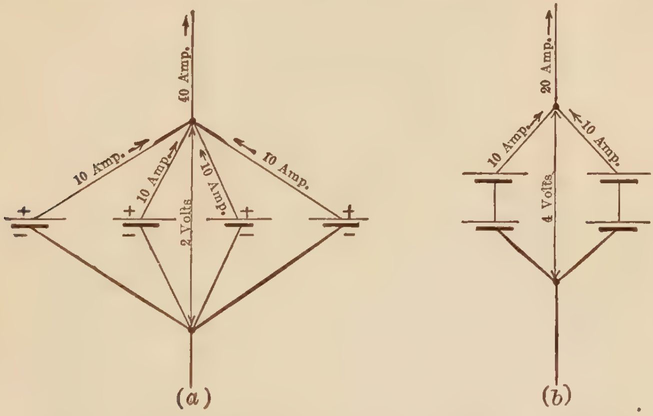


FIG. 223.—Parallel and series-parallel arrangement of batteries.

Similarly the conductors in an armature may be so connected that certain groups of conductors are in series. These groups may then be so connected that there are two or more paths in parallel. To determine the number of such parallel paths, start at one of the machine terminals, as for example, the negative, and see how many different paths through the armature it is possible to follow in order to reach the positive terminal.

The simplest arrangement of conductors occurs in the gramme-ring winding. Figure 224 (*a*) shows such a winding for a 4-pole machine.

Starting at the (−) terminal, one path may be followed by going to brush (*a*), through the winding at (1) to brush (*d*) and then to the (+) terminal.

A second path is obtained by going to brush (*a*), then through path (2) to brush (*b*) and then to the (+) terminal.

A third path is obtained by going to brush (*c*), through path (3), then through brush (*b*) to the (+) terminal.

A fourth path is obtained by going to brush (*c*), through path (4) to brush (*d*) and then to the (+) terminal.

This makes four separate paths between the (−) and (+) terminals, these paths being in parallel.

Assume that there are 10 amp. per path and 20 volts between brushes. The armature may be considered as being equiva-

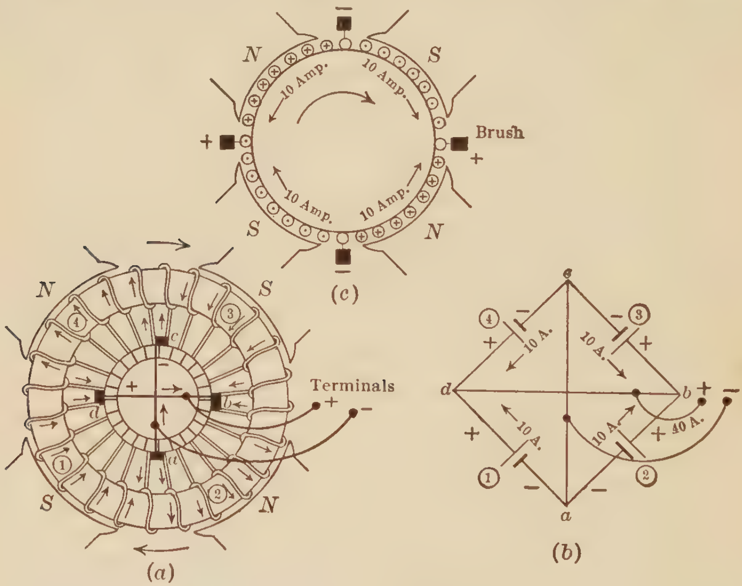


FIG. 224.—Four paths in parallel through an armature.

lent to four batteries connected as shown in Fig. 224 (*b*), each battery delivering 10 amperes at 20 volts. Battery 1 corresponds to path 1, battery 2 to path 2, etc.

It will be seen that the four batteries are connected in parallel because their four positive terminals and their four negative terminals are respectively connected together. The total current delivered will be 40 amp. at 20 volts. In a similar manner each path in the ring winding will deliver 10 amp., making 20 amp. per brush or 40 amp. per terminal. The potential difference between brushes will be 20 volts.

The paths through a drum winding are not as easy to follow as those through a ring winding. Figure 225 shows the 18-slot drum winding of Fig. 221 developed in circular form. For the sake of simplicity two paths are shown with heavy lines, one from brush *a* to brush *b*, and the other from brush *c* to brush *d*. These constitute two paths. By tracing through the lighter lines, two more paths may be found, one between brushes *c* and *b* and the other between brushes *a* and *d*, making four paths in all.

In all simplex lap windings there are as many paths through the armature as there are poles.

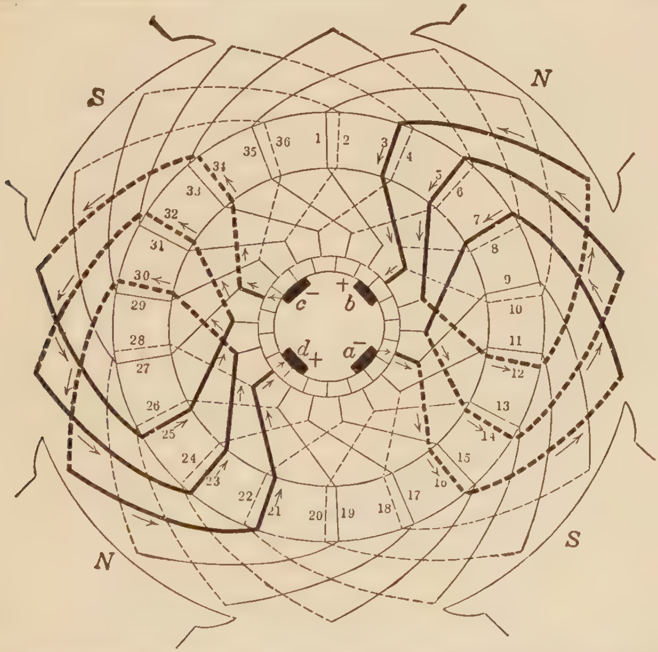


FIG. 225.—Heavy lines show two of the four parallel paths of a lap winding.

196. Multiplex Windings.—Figure 226 shows a 36-slot, 4-pole winding, in which every alternate slot is filled. There are two coil-sides per slot. The back pitch, y_b, is 17, and coil-side 1 connects to coil-side 18 on the back of the armature. Coil-side 18 then connects to 5 on the front of the armature, making the front pitch $y_f = 13$. Instead of returning to the coil-side differing by 2 from the initial coil-side, the return is made to a coil-side

differing by 4 from the initial coil-side. Coil-sides 3 and 4 are not connected to this winding. Furthermore, only alternate commutator segments are utilized. It will be seen that this winding closes on itself after going once around the armature; that is, this winding is re-entrant and is in itself complete in the same manner as any simplex 18-slot winding (see Fig. 225).

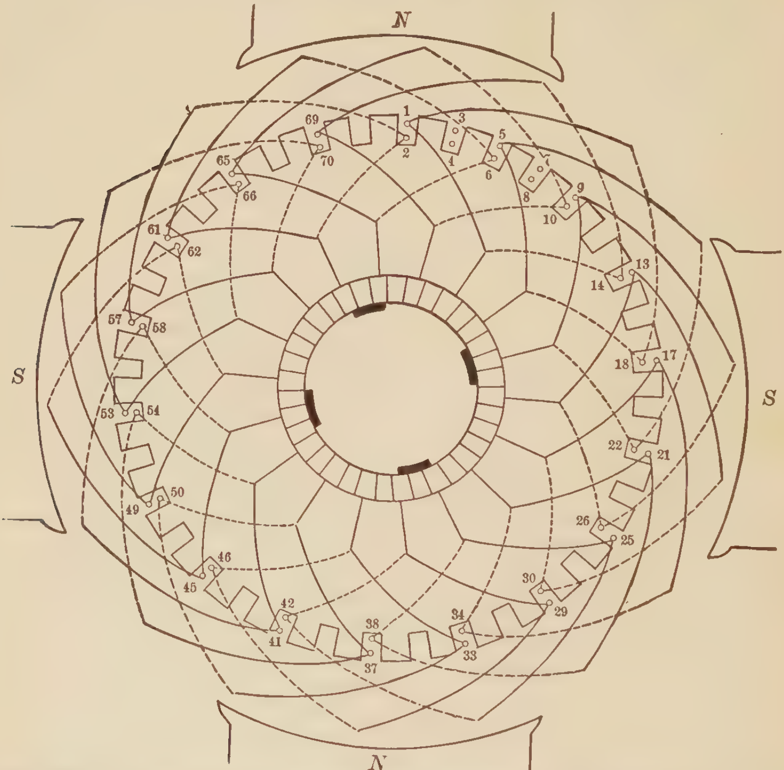
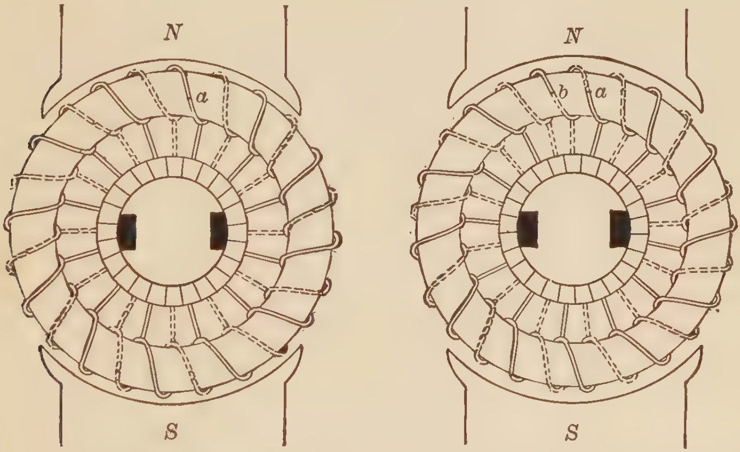


Fig. 226.—Duplex doubly re-entrant lap winding—one winding only being shown.

As this winding uses only alternate slots and alternate commutator segments, another winding, the duplicate of this one, can be placed in the vacant slots, this new winding having the same front and back pitch as the other, and being connected to the commutator segments not utilized by the other. This winding will also close on itself, and is, therefore, re-entrant.

These two windings are separate and are insulated from each other on the armature, but are connected together electrically by the span of the carbon brushes on the commutator. This condition is perhaps more clearly shown in the simple gramme-ring winding of Fig. 227 (*a*), where one winding is in solid lines and the other in dotted lines. These two windings are in parallel, so that the number of paths is now twice what it would be in a simplex lap winding. As each of the two windings closes on itself, the winding is said to be *doubly re-entrant.* It is necessary with this type of winding that the brush span at least two commutator segments.



(*a*) Duplex doubly re-entrant gramme-ring winding. (*b*) Duplex singly re-entrant gramme-ring winding.

FIG. 227.

When there are two such windings in parallel, the winding is said to be *duplex.* Therefore, this is a *doubly re-entrant duplex winding.* Obviously, three or more such windings can be placed on an armature, making the winding triplex, quadruplex, etc., the number of such windings being called the multiplicity of the winding.

Let m = the multiplicity of the winding.

The number of paths p' in a lap winding is

$$p' = mp, \tag{113}$$

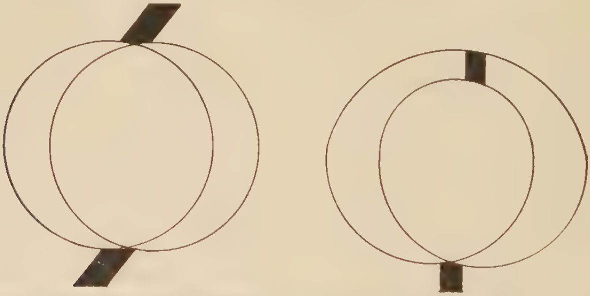
where p is the number of poles.

The relation of the back and the front pitch becomes

$$y_b = y_f \pm 2m. \qquad (114)$$

This should be compared with Eq. (111) where $m = 1$.

If the number of coils (Fig. 226) be odd, that is, if there are 35 or 37 coils and commutator segments, the winding will not close after having gone once around the armature, but will return one slot, or two conductors, to the right or to the left of the one at which it started. (If there are more than four elements per slot the winding may return to the same slot at which it started, but removed by two coil-sides from the coil-side at which it started.) Therefore, this winding does not close or become re-entrant, after having passed once around the armature, but must pass around once again before closing. This is illustrated in Fig. 227 (*b*). The initial winding starts at *a*. After



(*a*) Duplex doubly re-entrant winding. (*b*) Duplex singly re-entrant winding.

FIG. 228.—Duplex windings in diagrammatic form.

passing once around the ring armature it does not close at *a* as does the winding in Fig. 227 (*a*), but terminates at *b*, one conductor removed from *a*. The second winding, shown dotted, starts at *b* and after passing once around the armature, closes at *a*. Although this winding passes around the armature twice, it closes only once, so is said to be *singly* re-entrant. Therefore, this constitutes a *singly re-entrant duplex* winding. The two windings are the same electrically. Their difference is best illustrated by the two simple diagrams of Fig. 228.

From the foregoing it is seen that with duplex lap windings, the commutator pitch $y_c = 2$, with triplex windings $y_c = 3$, etc.

197. Equalizing Connections in Lap Windings.—Lap windings may consist of several paths in parallel, the parallel connections

being made through the brushes. If several batteries are connected in parallel and their e.m.fs. are not equal, currents circulate among the batteries, even when no external load is being supplied. This means a constant loss of energy which heats the batteries.

This same condition exists in generator armatures. Because of very slight inequalities in the air-gap, due to the wearing of the bearings, lack of mechanical alignment, etc., there may be slight differences of electromotive force in the different paths through the armature. These differences of e.m.f. will cause currents to flow between different points in the armature,

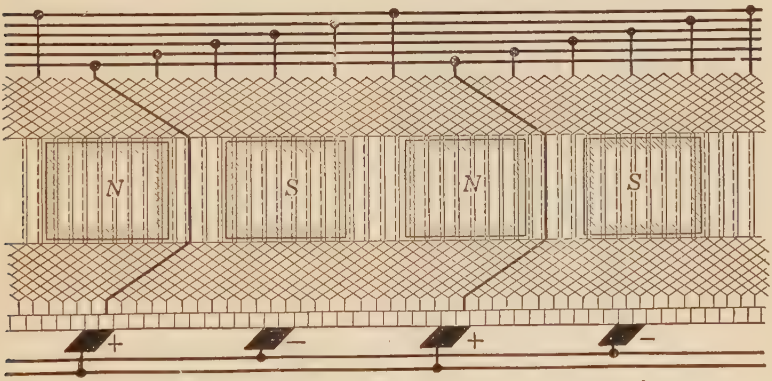


FIG. 229.—Simplex lap winding with equalizing connections.

and these currents must flow through the brushes even when no current is being delivered by the generator. To relieve the brushes of this extra current, several points in the armature which are simultaneously at equal potentials are connected together by heavy copper bars. This allows these circulating currents to flow from one point in the armature to another without passing through the brushes.

The action of the equalizer connections, however, differs from that of the brushes. The equalizing currents through the brushes are direct currents, which bring the different paths to equality of voltage by the *IR* drops which they produce in the different armature paths. The currents through the equalizing connections are alternating currents. These alternating currents produce m.m.fs. which react on the poles of the machine in such a

manner that they tend to bring the *induced* e.m.fs. in every path to equality. Hence, they accomplish the equalization of voltage without the heating which the brush currents would produce.[1]

To make these equalizer connections, the number of coils should be a multiple of the number of poles, and the coils per pole should be divisible by some small number as 2 or 3. As an example, assume an 8-pole generator having 12 slots per pole and two coil-sides per slot. There will be 96 slots and 192 coil-sides. The number of coil-sides per pole will be 24. Let $y_b = 25$ and $y_f = 23$. A portion of this winding is shown in Fig. 229. It will be noted that every fourth coil is connected to an equalizing connection. The coils that are connected to the same equalizing connection occupy the same positions relative to the poles. (See the two half-coils drawn with heavy lines.) This is necessary as such coils should be generating the *same* voltage at every instant. It will be noted in Fig. 229 that the two segments under the two positive brushes are connected together by an equalizing connection.

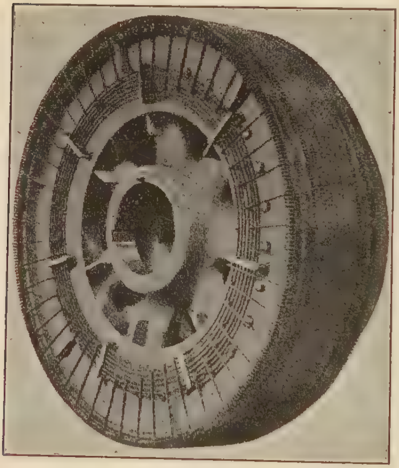


FIG. 230.—General Electric Company direct-current armature with equalizer rings.

Theoretically, every coil should be connected to an equalizing connection, but as this would require an undue number of such connections, it is sufficient, practically, to connect every third or fourth coil. This is the reason that the number of coils per pole should be divisible by a small number as 2, 3, or 4. Figure 230 shows a large direct-current armature with the equalizer connections at the back of the armature.

198. Wave Winding.—It has been shown that in the case of the *lap* winding a conductor under one pole is connected directly to

[1] See "Theory of the Action of Equalizer Connections in Lap Windings," by A. D. MOORE, *Elec. Journ.*, p. 624 (Dec. 1926).

a conductor which occupies a nearly corresponding position under the next pole. This second conductor is then connected *back* again to a conductor under the *original* pole, but removed two or more conductors from the initial conductor. This is shown in Fig. 231 (*a*), where conductor *ab* under a north pole is connected to conductor *cd* having a corresponding position under the next south pole. Conductor *cd* is then connected to *ef* which is adjacent to *ab* under the original north pole. Obviously it would make no difference so far as the direction and magnitude of the induced e.m.f. in the winding is concerned if the connection, instead of returning back to the same north pole, advanced *forward* to the next north pole, as shown in Fig. 231 (*b*). When the connection is so made, the winding passes successively every north and south pole before it returns again to the original

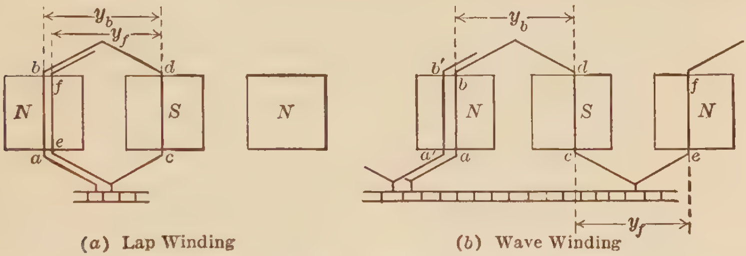


Fig. 231.—Lap and wave windings.

pole, as shown at $a'b'$ in Fig. 231 (*b*). The winding after passing once around the armature reaches conductor $a'b'$ lying under the same pole as the initial conductor *ab*. When a winding advances from pole to pole in this manner, it is called a *wave winding*. The number of units spanned by the end connections on the back of the armature is called the *back* pitch and is denoted by y_b in Fig. 231 (*b*). This is similar to the corresponding term in the lap winding shown in Fig. 231 (*a*). The number of elements which the end connections span on the commutator end of the armature is the *front* pitch and is denoted by y_f. This should also be compared with Fig. 231 (*a*). As in the lap winding, y_f and y_b must both be odd in order that one side of a coil may lie in the top of a slot and the other side in the bottom of a slot. Unlike the lap winding, y_f may equal y_b in the wave winding.

The above is illustrated as follows:

A certain wave winding may have a back pitch of 23 and a front pitch of 19. The average pitch,

$$y = \frac{23 + 19}{2} = 21.$$

Likewise both the front and the back pitch may each be 21 making the average pitch 21.

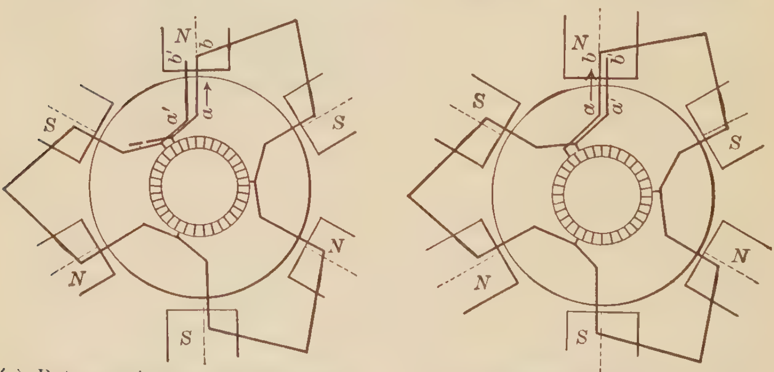
In any event, the average pitch,

$$y = \frac{y_b + y_f}{2}. \qquad (115)$$

y may be either even or odd.

When the winding viewed from the commutator end falls in a slot to the left of its starting point as $a'b'$ (Figs. 231 (*b*) and 232 (*a*)) after passing once around the armature, the winding is *retrogressive*. If, on the other hand, it falls to the right of its starting point, as shown in Fig. 232 (*b*), it is *progressive*.

The wave winding is much more restricted in its relation to the number of slots and coils than is the lap winding, for the following



(*a*) Retrogressive wave winding—34 commutator segments.

(*b*) Progressive wave winding—32 commutator segments.

Fig. 232.

reason. In a simplex wave winding, after having passed once around the armature, the winding must fall *two* conductors either to the right or to the left of the conductor at which it started. Thus in Fig. 232 (*a*), if there are two conductors per slot and conductor ab lies in the bottom of one slot, conductor $a'b'$ must lie in the bottom of the slot next to ab. As there are two coil-

sides in each slot, this means that conductors ab and $a'b'$ will differ from each other by 2.

Let y be the average pitch. Assume that the winding closes after passing once around the armature, which, of course, it should not do as this would constitute a short-circuit. Then:

$$py = Z,$$

where p is the number of poles and Z the number of coil-sides or elements. But the winding must *not* close after passing once around. In fact, it must not close until every slot is filled. Therefore, after passing once around the armature, the product py cannot equal Z but must be $Z \pm 2$. That is:

$$py = Z \pm 2,$$

or

$$y = \frac{Z \pm 2}{p}. \tag{116}$$

The $+$ sign indicates a *progressive* winding and the $-$ sign a *retrogressive* winding.

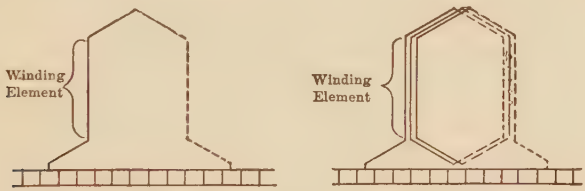


Fig. 233.—Single-turn coil representing a 3-turn coil for winding diagram.

As an illustration, assume that a 4-pole armature has 63 slots and four conductors per slot, making 252 winding elements. Let the average pitch be 63, the front and back pitch both being 63. As in the lap-winding diagrams, a single-turn coil will be used to represent a coil having several turns, as indicated in Fig. 233. Starting at element 1, the winding will advance as follows:

1–64–127–190–(253 or 1)

That is, the winding will close on itself after going once around the armature, which condition constitutes a short-circuit and makes the winding impossible. (The method by which a winding may be placed in these slots will be shown later.) Therefore, a wave winding is impossible in a 4-pole machine if 252 winding elements are to be included.

Let N_c be the number of commutator segments, which is also the number of coils.

$$N_c = \frac{Z}{2}, \quad Z = 2N_c.$$

Let p_1 = pairs of poles = $p/2$. $p = 2p_1$.
Substituting in equation (116)

$$y = \frac{2N_c \pm 2}{2p_1}.$$

$$N_c = p_1 y \pm 1. \tag{117}$$

If p_1 is odd and y is odd, the product $p_1 y$ is odd, as the product of two odd numbers is always odd. Adding or subtracting unity makes N_c even.

Therefore, with a wave winding whose average pitch is *odd* and having 6, 10, 14 poles, or 3, 5, 7 pairs of poles, the number of commutator segments and coils must each be *even*. If the average pitch is *even* the number of commutator segments and coils must each be *odd*.

On the other hand, if p_1 is *even*, corresponding to 4, 8, or 12 poles, the product $p_1 y$ is even, so that N_c must be *odd*. The application of Eq. (117) is illustrated in Fig. 232. There are 6 poles and the average pitch y is 11. Applying Eq. (117), $N_c = 3 \times 11 + 1 = 34$; $N_c = 3 \times 11 - 1 = 32$. The 34 segments are shown in (*a*), Fig. 232, which gives a retrogressive winding, and the 32 are shown in (*b*), which gives a progressive winding. N_c is even in either case.

Commutator Pitch.—It can be seen from a study of Fig. 232 that the commutator pitch in a wave winding is quite different from that in a lap winding. In both (*a*) and (*b*) the commutator pitch, y_c = 11 segments. The commutator pitch *cannot* be equal to the total number of segments divided by the pairs of poles, since the winding would close on itself after one passage around the armature if this were the case. For example, if the total number of segments in Fig. 232 (*a*) and (*b*) be divided by the pairs of poles, there results $11\frac{1}{3}$ and $10\frac{2}{3}$. The difference between these numbers and 11 represents the creepage of the wave winding. This creepage is necessary in order that the winding shall not close on itself after one passage around the armature.

The foregoing gives another limitation of the wave winding and shows why the 252 element (126–coil) winding just considered is impossible. The number of coils must be *odd* in a 4-pole winding. However, if one coil were omitted, making 250 elements, the winding would progress as follows:

1–64–127–190–(253 or 3)
–66–129–192–5, etc.

That is, the winding would advance by two elements after each passage around the armature, which condition makes the winding possible. This, of course, reduces the number of commutator

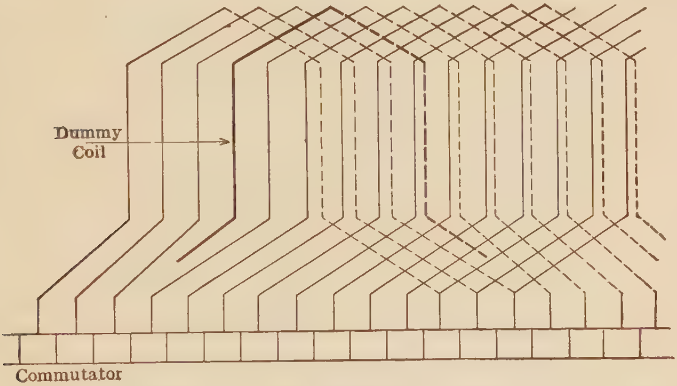


Fig. 234.—Dummy coil and "creeping" in a forced wave winding.

segments and coils from 126 to 125, an odd number. If, in this case, the armature stampings were standard, having 126 slots, the winding would be possible by omitting one coil. This coil would be inserted in the slots just the same as the other coils, except that its ends would not be connected to the commutator segments but would be taped and thus insulated from the main winding. The coil would serve only as a filler, and is called a "dummy coil." In this case there would be a slight "creeping" of the winding with respect to the commutator, as shown in Fig. 234. This is called a *forced* winding.

If the coils used in a wave winding consist of more than one turn, they will have the ends brought out and connected in the manner shown in Fig. 218 and in Fig. 233.

199. Number of Brushes.—Figure 235 shows the beginning of a wave winding, which begins at positive brush *a* and advances

once around the armature. This is a 6-pole winding in a machine having 44 commutator segments. The pitch is found from Eq. (117).

$$44 = 3y \pm 1.$$

$$y = \frac{44 + 1}{3} = 15, \text{ using the } + \text{ sign.}$$

This is also equal to the number of commutator segments by which the winding advances per *pair* of poles, or each time that it is connected to the commutator. Therefore, the segment connections, starting with 1, are 1–16–31–2–17, etc., as shown in Fig. 235. The winding ends one segment beyond the starting point for each complete passage around the armature, showing that the correct pitch has been chosen.

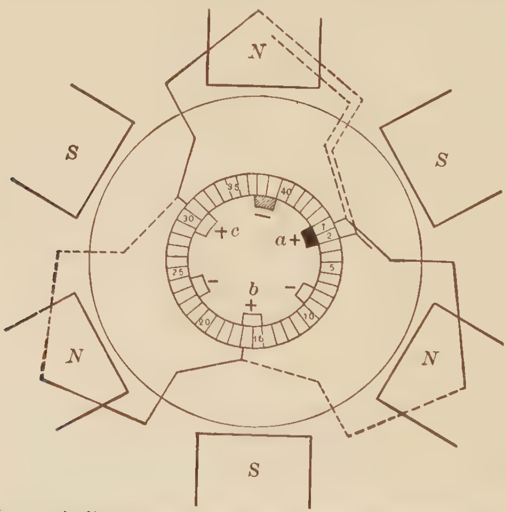


FIG. 235.—Wave winding—three positive brushes connected by the winding itself.

There are three positive brush sets, a, b, and c, and also three negative brush sets, the same number that would be used in a lap winding. It should be noted that the three + brushes, a, b, and c are all connected together *directly* by the winding. Moreover, the conductors which connect these three brushes all lie between the poles in the neutral plane, where they are not cutting any magnetic lines and are for the instant, therefore, dead conductors. Hence, if brushes b and c were removed, the current

could easily pass through the armature to brush *a* and thence to the external circuit. In like manner, two of the negative brushes could be removed without serious disturbance. It is desirable to utilize all six brush sets, as two brush sets would mean a commutator three times as long in order to obtain the necessary brush area.

In a wave winding only two brushes are necessary, regardless of the number of poles, although it is usually desirable to use the same number of brushes as poles.

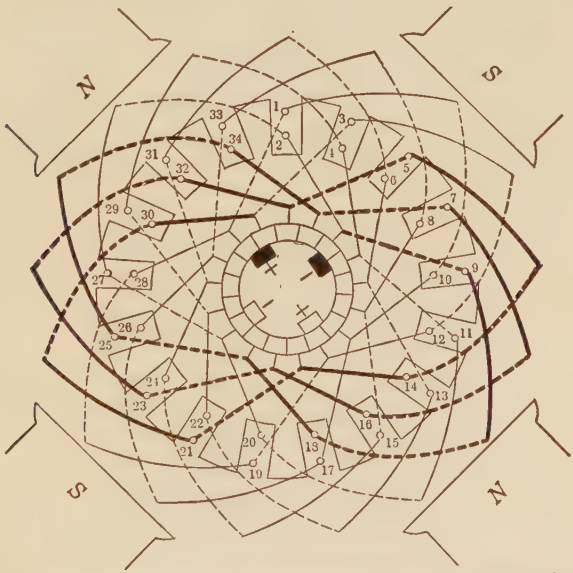


FIG. 236.—17-slot, 4-pole, simplex wave winding; back pitch = 9, front pitch = 7; one of two parallel paths shown heavy.

There are cases, however, where it is desirable to use only two brushes. The best example is in railway motors where it would be difficult to obtain access to four or six brushes. By means of a small hand hole in the motor casing, it is a comparatively simple matter to reach two brushes located on the top of the commutator.

200. Paths through a Wave Winding.—In a simplex wave winding there are always *two* parallel paths, regardless of the number of poles. Figure 236 shows a 4-pole, 17-slot, simplex

wave winding, having two coil-sides per slot. One of the paths is shown by the heavy lines. Approximately half the winding is heavy, the other half constituting the other path. (The coils short-circuited by the brushes are not included.) A wave winding may be duplex, triplex, or have any degree of multiplicity just as the lap winding may.

The paths through the armature depend only on the degree of multiplicity and not on the number of poles. A simplex wave winding always has two paths, a duplex winding four paths, etc.

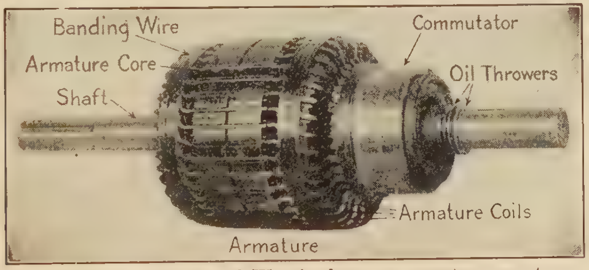
It is interesting to compare the current and voltage of an armature for the various ways of connection. Consider a 6-pole machine. When connected as a simplex lap winding let its e.m.f. be 300 volts and the armature current per terminal be 120 amp. The following table gives the values of current and e.m.f. obtainable when the winding is changed, the total number of armature conductors remaining fixed.

	Paths	Volts	Amperes	Kilowatts
Simplex lap	6	300	120	36
Duplex lap	12	150	240	36
Triplex lap	18	100	360	36
Simplex wave	2	900	40	36
Duplex wave	4	450	80	36
Triplex wave	6	300	120	36

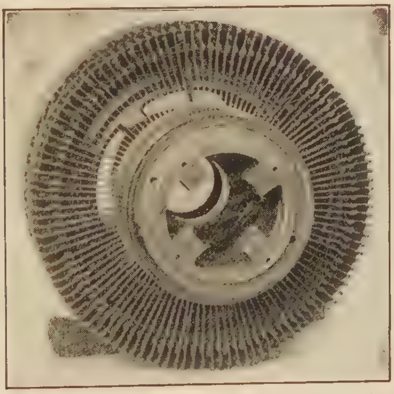
It will be noted that in this particular machine the triplex wave winding gives the same result as the simplex lap winding. The kilowatt capacity is not affected by the connection used. The above relations should be kept in mind when it is desired to change a machine from one current and voltage rating to another. This may often be done merely by changing the commutator connections.

201. Uses of the Two Types of Winding.—A wave winding has an advantage in that it gives a higher voltage with a given number of poles and armature conductors. It is used, therefore, in small machines, especially those designed for 600-volt circuits. In this case a lap winding would result in a very large number of small conductors. This in turn means a higher winding cost and less efficient utilization of the space in the slots.

The wave winding has the additional advantage that the electromotive force in each path is produced by series-connected conductors, which lie under successive north and south poles. Any magnetic unbalancing, therefore, due to such causes as air-gap variation and difference in pole strength, does not produce



(a) 25-hp. wave-wound Westinghouse generator armature.



(b) End view of an armature showing open construction—Westinghouse commutating-pole d.-c. motor.

Fig. 237.

cross currents, because the corresponding conductors of each and every path are moving by the same poles and the effect of such unbalancing will be the same in each path. Hence no equalizer connections are necessary.

The possibility of using only two brushes with a wave winding, and the corresponding advantage in railway motors, have already been mentioned.

When large currents are required, the lap winding is more satisfactory, since it gives a large number of paths. As 200 amp. per path is practically the limit, a large number of paths

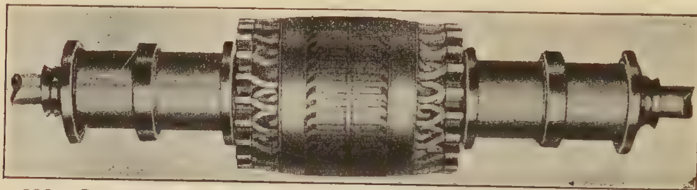


FIG. 238.—Low-voltage, high-speed G. E. armature for electrolytic work. (Note double commutator and shrink rings.)

must be used where heavy current output is desired. This is particularly true of large engine-driven multipolar generators.

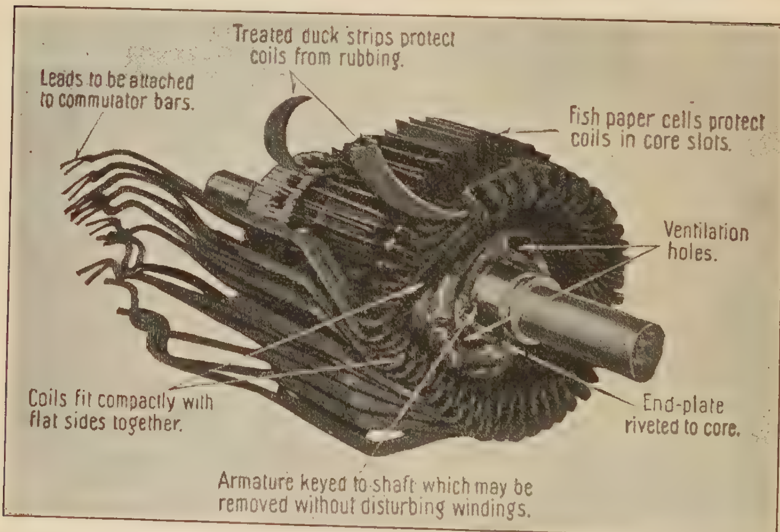


FIG. 239.—Partly wound armature showing method of assembling coils (Westinghouse).

Figs. 237 and 238 show two different types of armature and Fig. 239 shows an armature in the process of being wound.

DYNAMO CONSTRUCTION

202. Frame and Cores.—The frame or yoke of a dynamo has two functions. It is a portion of the magnetic circuit (see Figs.

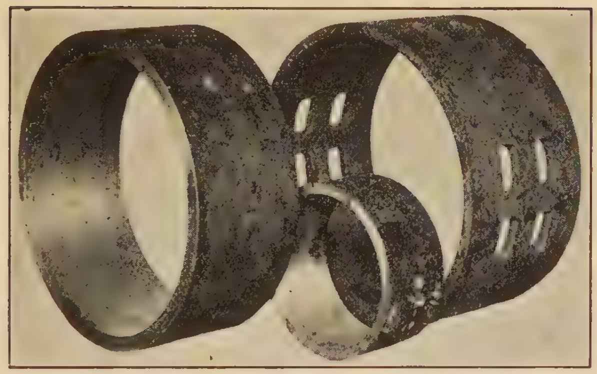


FIG. 240.—Frame rings—Westinghouse type S. K. motor.

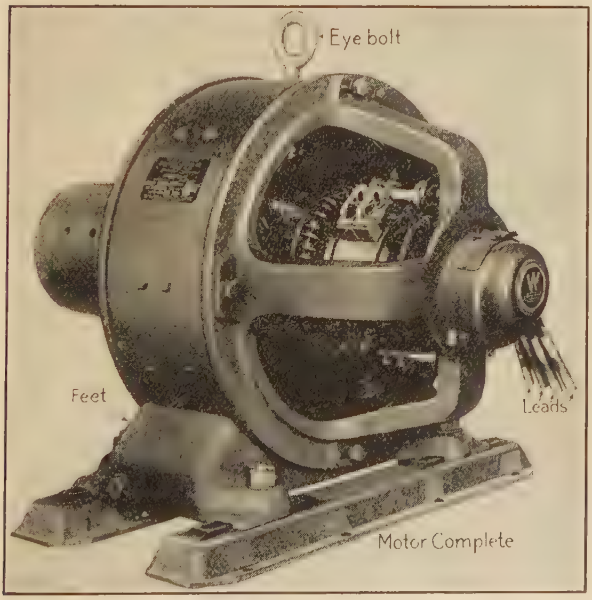


FIG. 241.—Westinghouse 230-volt, 35-hp., 850-r.p.m., shunt motor.

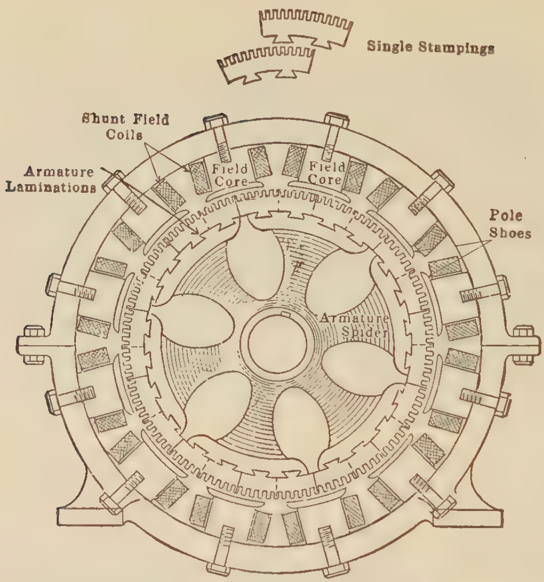


FIG. 242.—Construction of a 12-pole, direct-connected, engine-driven generator.

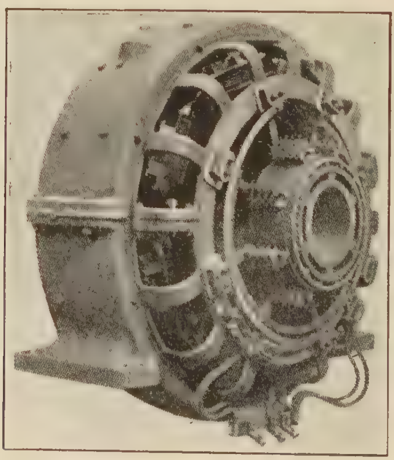


FIG. 243.—View of a 300-kw., 600-volt machine, commutator end showing brush gear and terminals.

39, 40, and 41) and it acts as a mechanical support for the machine as a whole. In small machines, where weight is of little importance, the yoke is often made of cast iron. The feet almost always form a part of the casting. In another type of construction a steel plate is rolled around a cylindrical mandrel and then welded (Fig. 240). The feet in this case are made of steel stampings and are riveted on (Fig. 241). In larger machines the yoke is made of cast steel and is usually more or less oval in cross-section, Figs. 242 and 243. The feet are a part of the yoke casting. The yoke for the larger machine is usually cast in two pieces which are bolted together. This facilitates the shipment of large machines and allows the armature to be removed easily.

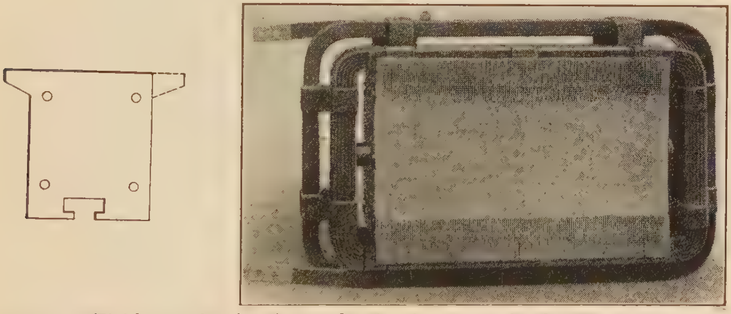


Fig. 244.—Field core, lamination and pole-piece assembled—Westinghouse d. c. motor.

203. Field Cores and Shoes.—The field cores are made of forged steel, cast steel and steel laminations. When made of cast or forged steel they are usually circular in cross-section, as such a section allows the minimum length of turn for a given core section. These cores are held to the yoke by bolts (Figs. 242 and 243). The laminated cores are built of sheet steel stampings (Fig. 244). They are stacked so that the pole tip comes alternately on one side and the other. This results in there being but half the iron in a pole-tip cross-section and so producing a saturated pole tip, which assists commutation. When stacked to the proper thickness, they are riveted together and dove tailed to the yoke. In this case a separate pole shoe is not necessary. A laminated or a solid steel pole shoe may be bolted to the solid

cores, the laminated type being used on the larger machines to reduce pole-face losses. (See Par. 257, page 415.)

204. The Armature.—The armature is made of sheet steel discs (14 to 25 mils thick) punched out by a die. The slots may be cut by the die or they may be cut out afterward with a slotting machine. In small motors these stampings are keyed directly to the shaft (Fig. 245). After every 2 or 3 in. of laminations a suitable spacer is inserted to form a ventilating duct (Fig. 246). The laminations are clamped together by end plates (Fig. 245), which are in turn held by nuts on the shaft or by bolts passing through the laminations. The laminations are perforated to allow air to pass through the armature axially and out radially

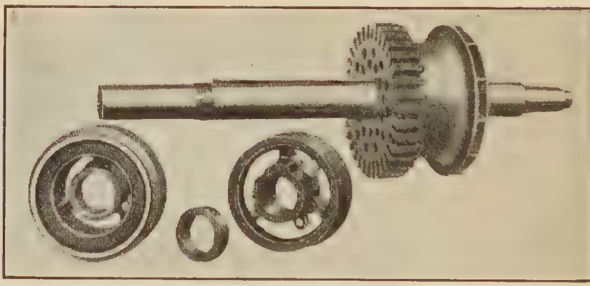


Fig. 245.—Armature construction of a small motor.

through the ducts. Frequently a blower is attached to the end plate (Fig. 245) to facilitate ventilation.

In machines of medium size, the stampings are assembled and keyed to an armature spider, which is in turn keyed to the shaft (Fig. 246). This reduces the amount of sheet steel necessary and at the same time permits a free passage of air through the center of the armature. This air is then thrown out through the ventilating ducts by centrifugal action, as indicated by the arrows. The stampings in Fig. 246 are clamped together by end plates held by through bolts. These end plates may also serve as supports for the overhang of the armature coils.

When the armature becomes greater than 30 in. in diameter, it is not economical to stamp out a complete ring. Such armatures are made up of segments similar to those shown in Fig. 242. These are dovetailed to the armature spider, each segment lapping the joint in the next layer.

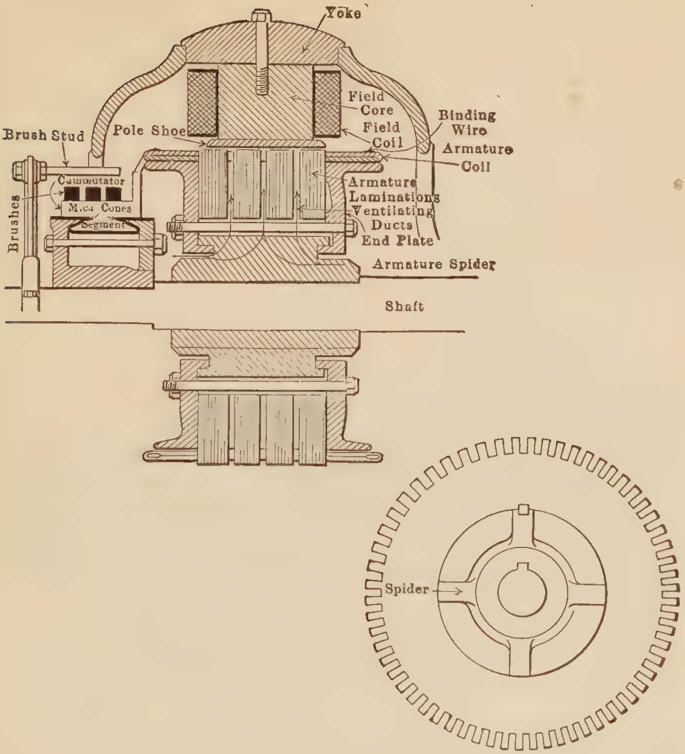
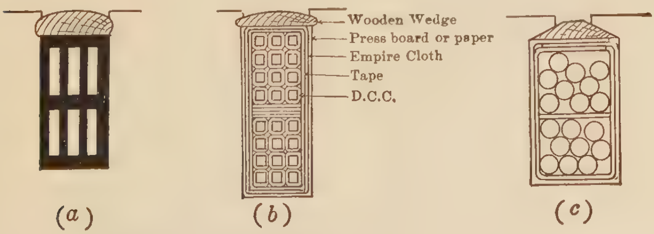


FIG. 246.—Cross-section of a moderate size generator; armature stamping.

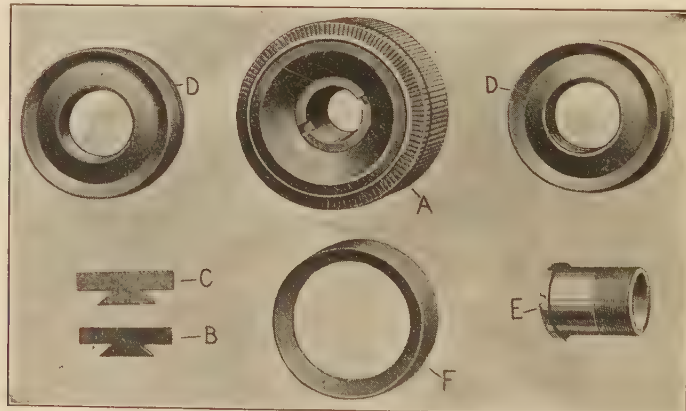


(a) Open slot containing triple coil.
(b) Open slot containing two coil sides, 12 turns per coil.
(c) Semi-closed slot and "mush" winding.

FIG. 247.—Types of slot.

The slots may be straight sided (Fig. 246) in which case the conductors are held in the slots by binding wires. In the larger machines the conductors are held in the slots by wooden wedges (Fig. 247). The slots must be well insulated, as grounds are troublesome and are expensive to repair. A layer of a hard substance such as fish paper, fiber or press board should be placed next to the laminations. This in turn should be lined with varnished cambric or empire cloth. The conductors themselves are usually covered with cotton insulation, except in the heavy bar windings. The groups of conductors are bound together in one coil by cotton tape (see Fig. 239).

To reduce the flux irregularities in the air gap, due to the teeth, a semi-closed slot (Fig. 247 (*c*)) is used occasionally. In this case



(*A*) Assembled commutator.
(*B*) Commutator bar.
(*C*) Mica commutator insulating strip.
(*DD*) Clamping flanges.
(*E*) Drawn steel tube.
(*F*) Insulation used between clamping flanges and commutator bars.

Fig. 248.—Crocker-Wheeler commutator and details.

the individual conductors must be placed in the slot one by one, so the coil ends must be taped after the coils are placed in the slots. Such a winding is called a "mush" winding. The expense of winding prevents the general use of this type of slot in direct-current machines.

205. The Commutator.—The commutator is made of wedge-shaped segments of hard-drawn or drop-forged copper, insulated from one another by thin layers of mica. The segments are held

together by clamping flanges (*DD*, Fig. 248), which pull the segments inward when the flanges are drawn together by through-bolts. These flanges are prevented from short-circuiting the segments by two cones of built-up mica (*F*, Fig. 248). This construction is illustrated by the commutator of the machine shown in Fig. 246.

The leads from the armature coils may be soldered into small longitudinal slits in the ends of the segments or the segments may have risers (Fig. 246) to which these leads are soldered (see also Fig. 237 (*b*)).

206. Field Coils.—The field coils are usually wound with double-cotton-covered (d.c.c.) wire. The coils are dried in a vacuum and then impregnated with an insulating compound.

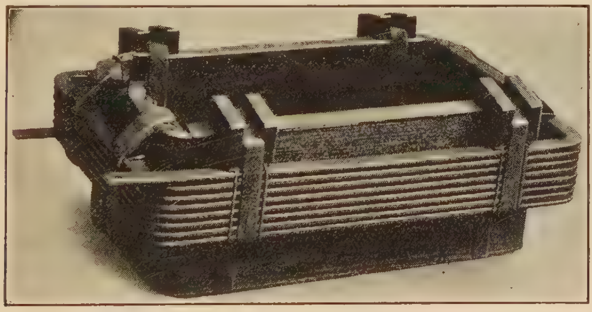
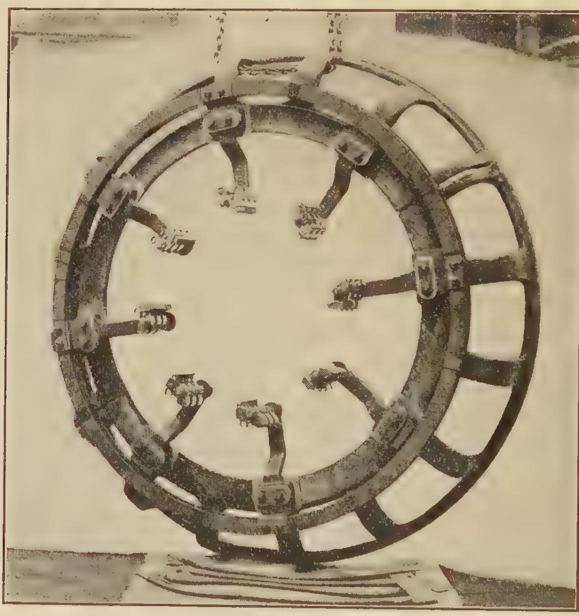


Fig. 249.—Shunt field coil and edgewise series winding.

The outer cotton insulation is often protected by tape or cord on the outside. In the larger machines an air space is often left between layers for ventilating purposes. The coils are also wound on metal spools (Fig. 249). An edgewise series winding, set some distance from the shunt winding, also is shown here.

207. The Brushes.—The function of the brushes is to carry the current from the commutator to the external circuit. They are usually made of carbon, although in very low-voltage machines they may be made of copper gauze or patented metal compounds. The brush holder (Fig. 250), is fastened to the brush stud and holds the brush in its proper position on the commutator. The brush should be free to slide in its holder in order that it may follow any irregularities in the commutator. The brush is made to bear down on the commutator by a spring (Fig. 250). The pres-

sure should be from 1 to 2 lb. per square inch. To decrease the electrical resistance, the upper portion of the brush is copper



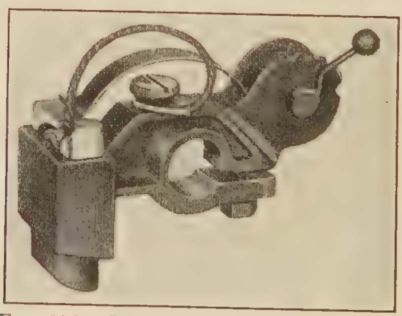


Fig. 250.—Rocker ring and brush holder.

plated and this plating is connected to the brush holder by a pig tail made of copper ribbon. A rocker ring with cross-connections is also shown in Fig. 250.

CHAPTER XI

GENERATOR CHARACTERISTICS

208. Electromotive Forces in an Armature.—The method of determining the average electromotive force in simple single-coil and two-coil generators was discussed at the beginning of Chap. X. In these generators the e.m.f. varied sinusoidally with time. In commercial generators this is not usually true. Commercial generators ordinarily have closed-coil windings and a considerable number of armature inductors. Consider Fig. 251, which shows a portion of an armature under a north pole and moving

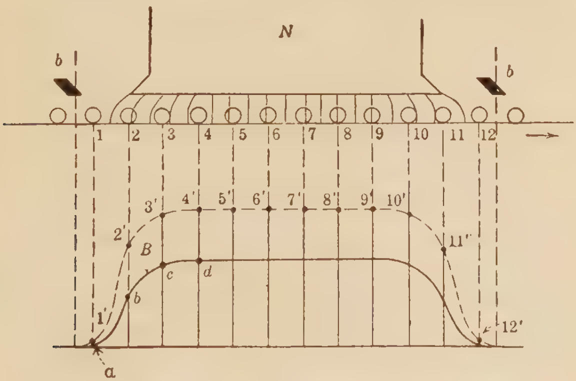


FIG. 251.—Emf. induced in conductors between brushes.

from left to right. The flux entering the armature from this north pole is also shown. Below, the curve B shows the flux density along the armature. That is, the ordinate at each point gives the *flux density* at that point, expressed preferably in lines per square centimeter. The total flux is given by the area under the curve multiplied by the effective axial length of the armature. Midway between poles this flux density is zero and under the pole where the air-gap is practically uniform, the flux density is practically uniform.

In Fig. 251 are also shown twelve conductors on the surface of the armature. At this instant these conductors lie between the two brushes bb. A study of an armature winding shows that these twelve conductors are all connected in series between brushes, as is shown, for example, in Fig. 225, page 279. The total e.m.f. between brushes must therefore be the sum of the e.m.fs. generated in these twelve conductors. The e.m.f. per single conductor at any instant is given by Eq. (110), page 263.

$$e = Blv10^{-8} \text{ volts.}$$

If the speed is constant, the induced e.m.f. in any single conductor is proportional to the flux density B in which that conductor finds itself. For example, conductor 2 finds itself in a field whose density is given by ordinate b. The e.m.f. induced in this conductor will be given by the ordinate $2'$, which is equal to ordinate b multiplied by a constant of proportionality. Likewise, the e.m.f. induced in conductor 3 is given by ordinate $3'$ which is equal to ordinate c multiplied by the same constant of proportionality. Conductor 1 lies in the interpolar space, where the flux density, given by ordinate a, is low and the e.m.f. is given by ordinate $1'$. From similar reasoning it is seen that the total e.m.f. between brushes bb is given by the sum of all the ordinates $1'$ to $12'$, inclusive. It is also seen that if a smooth curve be drawn through points $1'$, $2'$–$12'$, it is identical in shape with the curve B, being equal to B multiplied by the constant of proportionality. The sum of all the ordinates $1'$ to $12'$, inclusive, is equal to the total induced e.m.f. between brushes bb. This total e.m.f. is also equal to the average value of these ordinates multiplied by the number of ordinates, twelve in this case.

The e.m.f. ordinates (Fig. 251) give the e.m.fs. in the various conductors which at the instant shown happen to be lying between the two brushes bb. Since the conductors are all moving, these ordinates likewise give the e.m.f. induced in any single conductor as it takes successive positions 1, 2, etc. between brushes bb. Hence it follows that the total e.m.f. between brushes is given by the average e.m.f. *per conductor* multiplied by the number of conductors.

Further, it follows that the curve giving the variation with time of the e.m.f. in a single conductor, as it cuts a flux at constant speed, is of the same shape as the flux density curve.

In Fig. 252 are shown two north poles and a south pole of a generator. For simplicity, the armature surface is shown as if it were rolled out flat. The flux density curve is given below. The positive ordinates of the distribution curve are north pole flux entering the armature and the negative ordinates are flux leaving the armature and entering a south pole. The total flux leaving a north pole is given by the area under one of the positive parts of the distribution curve multiplied by the axial length of the pole. Similarly, the total flux leaving the armature and entering a south pole is given by the area of one of the negative parts of the distribution curve multiplied by the axial length of the pole. The maximum flux density is given by the ordinate $B_{\text{max.}}$

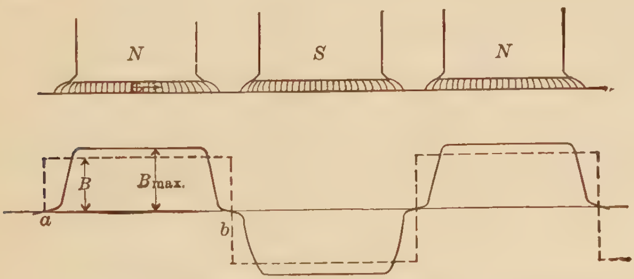


FIG. 252.—Flux distribution at no load of a d.-c. generator.

Each positive part and each negative part of the curve may be replaced by a rectangle having the same area, as shown by the dotted line in Fig. 252. The height of this rectangle will be B maxwells per square centimeter, which is equal to the *average* value of the flux density over an entire pole pitch. Let it be required to determine the average electromotive force induced in a single conductor as it passes through the flux of successive poles.

Let the total flux leaving a north pole or entering a south pole be ϕ maxwells. Let A be the pole-face area in square centimeters, l the active length of the conductor in centimeters, s the speed of the armature in revolutions per *second*, and P the number of poles.

When the conductor passes through the distance ab, or one pole pitch, the average induced e.m.f., by Eq. (110), page 263, and from page 304 is

$$e = Blv10^{-8},$$

where B is the average flux density, l the active length of the conductor in centimeters, and v the velocity of the conductor in centimeters per second.

$$v = \frac{ab}{t},$$

where t is the time required for the conductor to traverse the distance ab.

$$e = \frac{Bl(ab)}{t}10^{-8} = \frac{\phi}{t}10^{-8},$$

since $Bl\ (ab)$ gives the total flux between the points a and b as cut by the conductor and is therefore equal to ϕ.

The time $$t = \frac{1}{sP}.$$

Therefore, the average e.m.f. per conductor is

$$e = \frac{\phi}{1/sP}10^{-8} = \phi sP10^{-8}.$$

If there are Z such conductors and p paths through the armature, there must be Z/p such conductors in series (see page 277).

Hence the total e.m.f. generated between brushes is

$$E = \frac{\phi sPZ}{p10^8}\text{ volts.} \tag{118}$$

Example.—A 900-r.p.m., 6-pole generator has a simplex lap winding. There are 300 conductors on the armature.

The poles are 10 in. square and the average flux density is 50,000 lines per square inch. What is the voltage induced between brushes?

$$\phi = 10 \times 10 \times 50{,}000 = 5{,}000{,}000 \text{ lines},$$
$$s = {}^{900}\!/_{60} = 15 \text{ r.p.s.}$$
$$P = 6,$$
$$p = 6 \text{ (see Par. 195)},$$
$$E = \frac{5{,}000{,}000 \times 15 \times 6 \times 300}{6 \times 10^8} = 225 \text{ volts.} \quad \textit{Ans.}$$

209. The Saturation Curve.—Equation (118) may be written as follows:

$$E = \left(\frac{PZ}{60p10^8}\right)\phi S,$$

where S = revolutions per minute.

The quantity within the parenthesis is constant for a given machine and may be denoted by K.

Therefore:

$$E = K\phi S. \tag{119}$$

The induced e.m.f. in a machine, therefore, is directly proportional to the flux and to the speed.

If the speed be kept constant, the induced voltage is directly proportional to the flux, ϕ.

The flux is produced by the field ampere-turns, and as the turns on the field remain constant, the flux is a function of the field current. It is not directly proportional to the field current because of the varying permeability of the magnetic circuit.

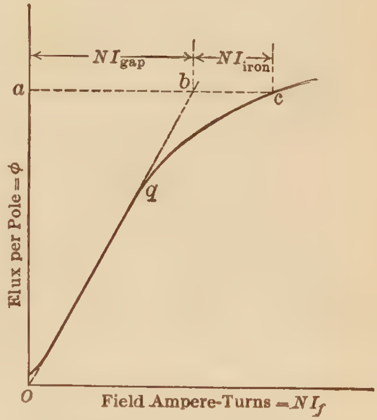


FIG. 253.—Saturation curve.

Figure 253 shows the relation existing between the field ampere-turns and the flux per pole. The flux does not start at zero ordinarily but at some value slightly greater, owing to the residual magnetism in the machine. At first the line is practically straight, since most of the reluctance of the magnetic circuit is in the air-gap. At the point q the iron begins to be saturated and the curve falls away from the straight line.

The number of field ampere-turns for the air-gap and for the iron can be approximately determined for any point on the curve.

Let it be required to determine the ampere-turns for the gap and for the iron at the point c. From the origin draw ob tangent to the saturation curve and also draw the horizontal line ac. The line ob is the magnetization curve of the air-gap, if the reluctance of the iron at low saturation be neglected. Therefore, the ampere-turns required by the gap are equal to ab and those required by the iron are equal to bc.

From Eq. (119) the induced voltage is proportional to the flux, if the speed is maintained constant. Therefore, if the induced voltage be plotted against field current as abscissas, a curve similar to that of Fig. 253 is obtained. This is shown in Fig. 254 and differs from the curve of Fig. 253 only by a constant

quantity (KS). Two curves are shown in Fig. 254, one plotted for 1,200 r.p.m. and the other for 900 r.p.m. The curves are similar, any ordinate of the lower curve being 900/1,200 of the value of the corresponding ordinate of the upper curve. Thus, at ordinate ac,

$$\frac{ab}{ac} = \frac{900}{1,200}.$$

Also at ordinate $a'c'$

$$\frac{a'b'}{a'c'} = \frac{900}{1,200}.$$

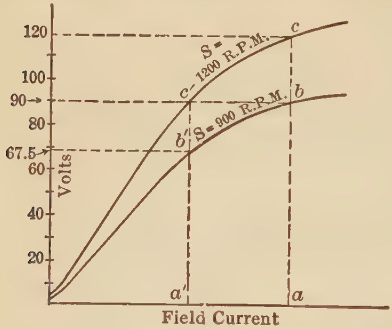


FIG. 254.—Saturation curves for two different speeds.

If the saturation curve of a generator for one speed has been determined, saturation curves for other speeds may be readily found by the method just indicated.

210. Hysteresis.—The saturation curve Oab (Fig. 255 (a)) is determined for *increasing* values of the field current. If when point b is reached the field current be decreased, the curve

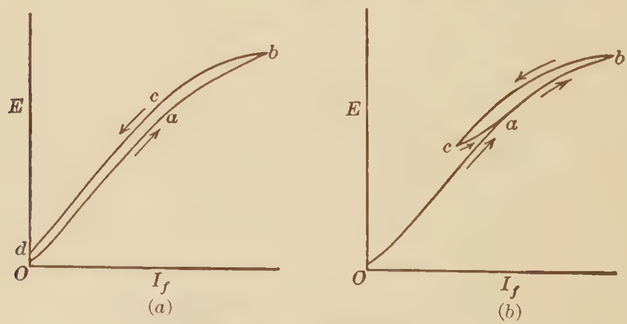


FIG. 255.—Hysteresis loops.

will not retrace its path along the curve baO. For any given field current, the corresponding induced voltage will now be greater than it was for *increasing* field currents. This is shown by the curve bcd. This is due to hysteresis in the iron (see p. 205, Par. 157).

Figure 255 (b) shows the effects obtained when the curve is carried up along the path Oab, back to c, and at c the field current

is again increased, the curve ultimately coming back to *Oab* at the point *a*.

It is evident that for any given value of field current, there is no single value of flux. The value of flux for any given field current depends on whether the field current was *increased* until it reached the value in question or whether it was *decreased*. This characteristic of the magnetic circuit should be carefully borne in mind, for the operating characteristics of both generators and motors are affected to a considerable degree by hysteresis in the magnetic circuit.

211. Determination of the Saturation Curve.—To determine the saturation curve experimentally, connect the field, in series with

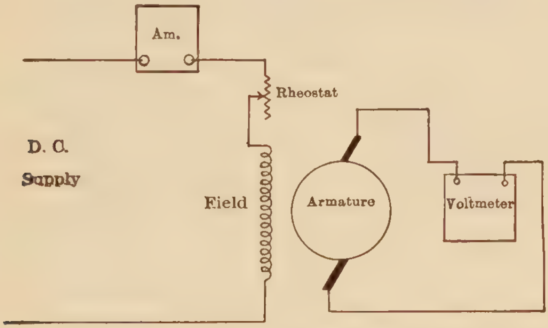


FIG. 256.—Connections for obtaining saturation curve.

an ammeter and rheostat, across a direct-current source of power. A voltmeter should be connected across the armature terminals. Obviously the ammeter measures the field current, values of which are plotted as abscissas; the voltmeter reads the values of induced armature voltage, which are plotted as ordinates. These connections are shown in Fig. 256. As the voltage drop within the armature due to the voltmeter current is negligible, the terminal volts and the induced volts under these conditions are identical. During the experiment the speed should be determined each time that the other readings are taken (see p. 409). If the speed cannot be maintained constant, corrections can be easily made for any variation, by the method described in Par. 209.

When the saturation curve of a shunt generator is determined it may be difficult to obtain a sufficiently high resistance to

reduce the field current to its lower values. A drop wire connection (Fig. 257) allows field currents as low as zero to be obtained without the use of excessive resistance. Such a connection is easily made with the well-known "3-point" type of field rheostat, shown in Fig. 257.

In determining the saturation curve experimentally, the field current should be varied continuously in one direction, either up or down, as shown in Fig. 255 (*a*). Otherwise minor hysteresis loops, such as shown in Fig. 255 (*b*), will be introduced.

The field current in this experiment should be obtained from a supply other than the generator itself, for two reasons: If the generator excited its own field, the voltage and field current would be interdependent and it would be difficult to adjust

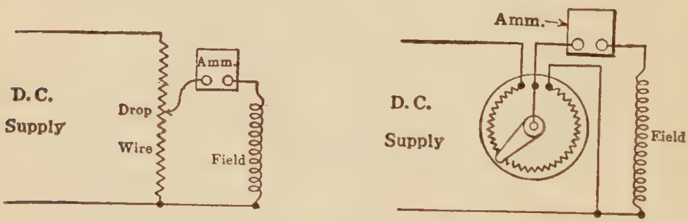


Fig. 257.—Drop-wire connections for obtaining field current.

the field current without the voltage in turn changing this adjustment. Also a voltage drop would exist in the armature due to the field current. The voltmeter would not then be reading the true induced voltage, although the error from this cause would be slight.

212. Field Resistance Line.—By Ohm's law the current in a simple circuit is proportional to the voltage, for a constant resistance. If the current be plotted against volts (Fig. 258) a straight line passing through the origin results. For example, if the resistance of a field circuit be 50 ohms, the current will be 2 amp. when the voltage is 100 volts; 1.5 amp. when the voltage is 75 volts; and 1 amp. when the voltage is 50 volts. This relation is shown in Curve II, Fig. 258. Curve I shows the resistance line for 80 ohms field resistance. It will be noted that at 80 volts the current is 1.0 amp., at 40 volts it is 0.5 amp., etc. Curve III shows the same relation for a field resistance of 40 ohms.

It will be noted that the higher the resistance the greater the slope of the resistance line. In fact the slope of the line is equal to the field resistance in ohms since the tangent of the angle which the line makes with the axis of abscissas is E/I.

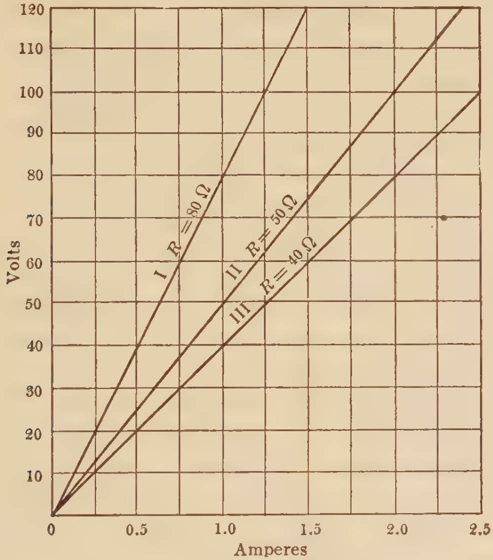


FIG. 258.—Field resistance lines.

213. Types of Generators.—There are three general types of generator in common use, the shunt, the compound and the series. In the shunt type the field circuit is connected across the armature terminals, usually in series with a rheostat (Fig. 259). The shunt field, therefore, must have a comparatively high resistance in order that it may not take too great a proportion of the generator current. The compound generator is similar to the shunt, but has an additional field winding connected in series with the armature or load (Fig. 295, p. 348). The series generator is excited entirely by a winding of comparatively few turns connected in series with the armature and load (see p. 356).

214. The Shunt Generator.—Figure 260 shows the saturation curve of a shunt generator and its shunt-field resistance line drawn on the same plot. This field, it will be observed, has a

resistance of 24 ohms, so that at 120 volts it takes 5 amp.; at 60 volts 2.5 amp.; etc.

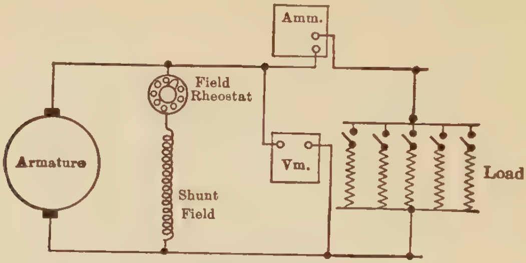


FIG. 259.—Shunt generator connections.

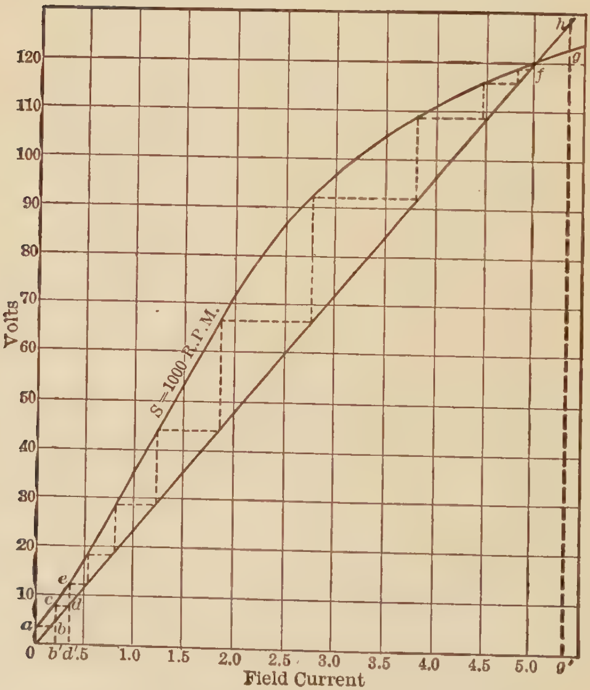


FIG. 260.—Method of shunt generator building up.

At the instant of starting a generator the induced voltage is zero. The generator may come up to voltage in the following manner: As the generator is brought up to speed there will be a

small voltage *o-a*, in this instance about 4 volts, induced in the armature due to the residual magnetism of the machine. This 4 volts also exists across the field, because the field is connected across the armature terminals. The value of field current which flows in virtue of this 4 volts can be obtained by drawing a horizontal line from *a* until it meets the field resistance line at *b*. The current in this particular case is *ob′* or about 0.2 amp. By consulting the saturation curve it will be seen that for this field current the induced voltage, *b′c*, is about 8 volts. The 8 volts produces about 0.33 amp. in the field, as may be seen by projecting across to the field resistance line at *d*. This field current *od′* produces a voltage *d′e*, which in turn produces a higher value of field current. Thus it will be seen that each value of field current produces a voltage in excess of its previous value and this increased voltage in turn increases the field current, that is, the action is cumulative. The machine will continue to build up until point *f* is reached, where the field resistance line crosses the saturation curve. The machine cannot build up beyond this point for the following reasons:

Consider a point *h* on the field resistance line, above *f*. This point represents a field current *og′* of about 5.3 amp. To produce this field current requires a voltage *g′h* of about 128 volts. But this field current of 5.3 amp. produces an induced voltage *g′g* of only 122 volts. If 128 volts are required to produce the field current of 5.3 amp. and the machine can only produce 122 volts at this field current, it is obvious that the machine cannot build up to the point *h*.

It is evident that a machine would build up indefinitely if its iron did not become saturated.

215. Critical Field Resistance.—If the resistance of the field (Fig. 260) be increased to 60 ohms, the field resistance line will be represented by *oa* (Fig. 261). This line crosses the saturation curve at point *a′*, corresponding to about 6 volts. Therefore, with this value of field resistance, the generator will not build up beyond *a′*. If the field resistance be slowly decreased until the field resistance line reaches *ob*, the generator will be observed to start building up rapidly. It will of course stop building up at the point *b′*. The value of the field resistance corresponding to *ob*, which makes the field resistance line tangent to the saturation

curve, is called the *critical field resistance.* In this particular case, the critical field resistance is 120/3.25 or 36.9 ohms. If the field resistance exceeds the critical value, the generator cannot build up.

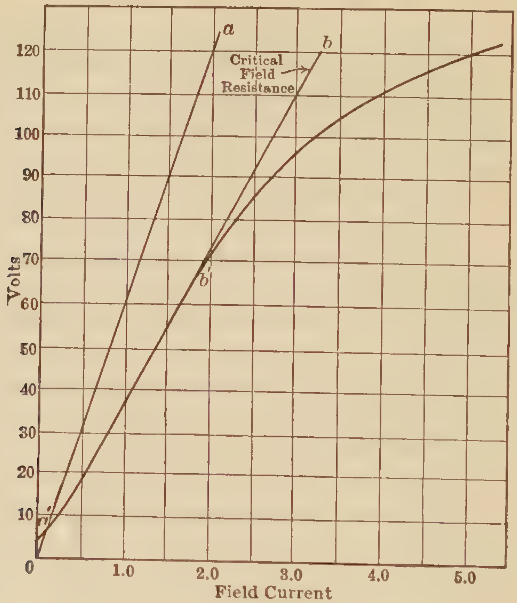


FIG. 261.—Critical field resistance.

216. Generator Fails to Build Up.—There are three common reasons for a generator failing to build up.

1. The shunt field may be connected in such a way that the current sent through it on starting is in such a direction as to "buck" or reduce the residual magnetism, instead of increasing it. Of course under these conditions, the generator cannot build up. To test for this, open the field circuit. If the voltage rises when the field is opened, the current is bucking the residual magnetism and the field should be reversed. If opening and closing the field produces no effect upon the voltmeter it may be assumed that the field circuit is open.

2. The field resistance may be greater than the critical field resistance. In this case, the procedure is to reduce the field resistance until the machine builds up.

3. There may be no residual magnetism in the machine, due to jarring or to too long a period of idleness. If the armature circuit is not open and the voltmeter is known to be all right, the absence of residual magnetism will be indicated by the voltmeter's not reading. To remedy the difficulty, it may be necessary to connect the field terminals temporarily across a separate supply circuit in order to build up the residual magnetism. This is called "flashing" the generator. If the generator has a series field, a convenient method is to connect a low-voltage source, such as a storage battery or even a dry cell, across the series field. This may produce enough magnetism to cause the machine to begin to build up. One or two trials may be necessary in order to secure the proper polarity.

217. Armature Reaction.—Figure 262 (*a*) shows the flux passing from the field poles through an armature when there is no current in the armature conductors. This flux is produced entirely by the ampere-turns of the field. The neutral plane, which is a plane perpendicular to the flux, coincides with the geometrical neutral of the system. At the right is shown a vector F which represents the m.m.f. producing this flux, in magnitude and direction. At right angles to this vector F is the neutral plane.

In Fig. 262 (*b*) there is no current in the field coils, but the armature conductors are shown as carrying current. This current is in the same direction in the armature conductors as it would be were the generator under load. The current obviously flows in the same direction in all the conductors that lie under one pole. The current is shown as flowing into the paper on the left-hand side of the armature. (This current direction may be checked by Fleming's right-hand rule, Par. 189.) These conductors combine their m.m.fs. to send a flux *downward* through the armature, as shown in the diagram, this direction being determined by the corkscrew rule. The conductors on the right-hand side of the armature are shown as carrying current coming out of the paper. They also combine their m.m.fs. to send a flux *downward* through the armature. That is, the conductors on both sides of the armature combine their magnetomotive forces in such a manner as to send flux down through the armature. The direction of this flux is perpendicular to the polar

axis. To the right of the figure the armature m.m.f. is represented in direction and magnitude by the vector F_A.

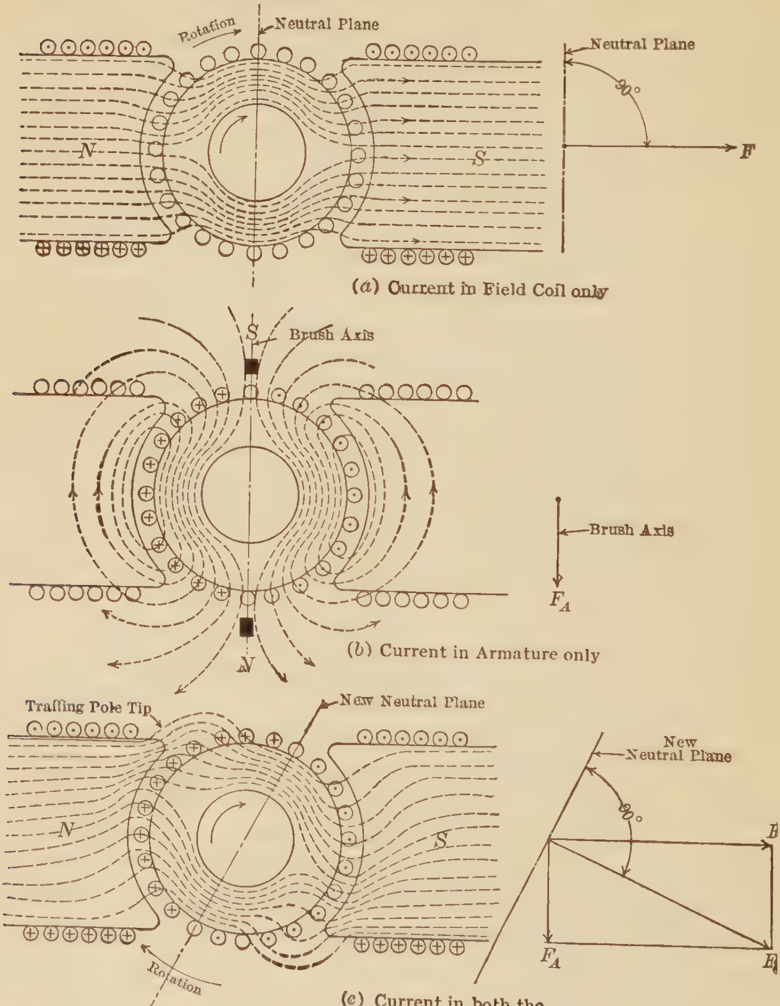


FIG. 262.—Effect of armature reaction upon the field of a generator.

Figure 262 (c) shows the result obtained when the field current and the armature current are acting simultaneously, which occurs when the generator is under load. The armature magnetomotive

force crowds the symmetrical field flux shown in (*a*) into the upper pole tip in the north pole and into the lower pole tip in the south pole. As the generator armature is shown rotating in a clockwise direction, it will be noted that the flux is crowded into the *trailing* pole tip in each case. On the other hand, the flux is weakened in the two *leading* pole tips.

The effect of the armature current is to displace the field in the direction of rotation of the generator. It should be kept firmly in mind that the flux is *not* pulled around by the mechanical rotation of the armature.

To the right of Fig. 262 (*c*) the effect of armature reaction is shown by vectors. The field vector F and the armature vector F_A combine at right angles to form the resultant field vector F_o.

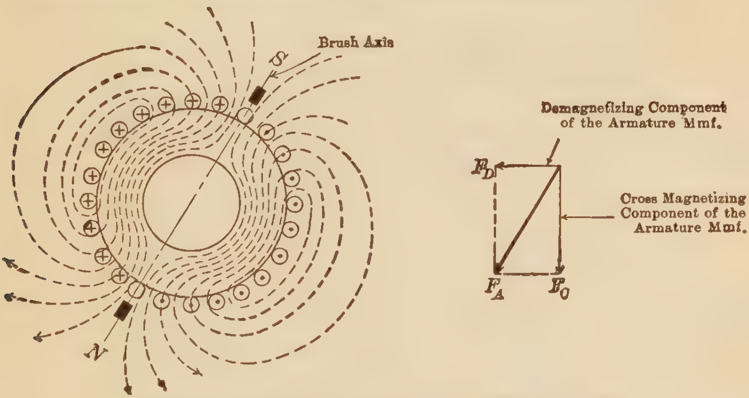


FIG. 263.—Relation of armature field to brush axis.

The direction of F_o is downward and to the right, which corresponds to the general direction of the resultant flux in the drawing. The neutral plane must be at right angles to F_o, provided the direction of the resultant flux is the same as that of the resultant magnetomotive force.

As the neutral plane is perpendicular to the resultant field, it will be observed that it too has been advanced. It was shown in Chap. X that the brushes should be set so that they short-circuit the coil undergoing commutation as it is passing through the neutral plane. When the generator delivers current the brushes should be set a little ahead of this neutral plane, as will be shown later. If the brushes are advanced to correspond to

the advance of the neutral plane, all the conductors to the left of the two brushes must still carry current into the paper, and those to the right must carry current out of the paper. The result is shown in Fig. 263. *The direction of the armature field moves with the brushes.* Its axis always lies along the brush axis. Therefore F_A, instead of pointing vertically downward, now points downward and to the left, as is shown by the vector.

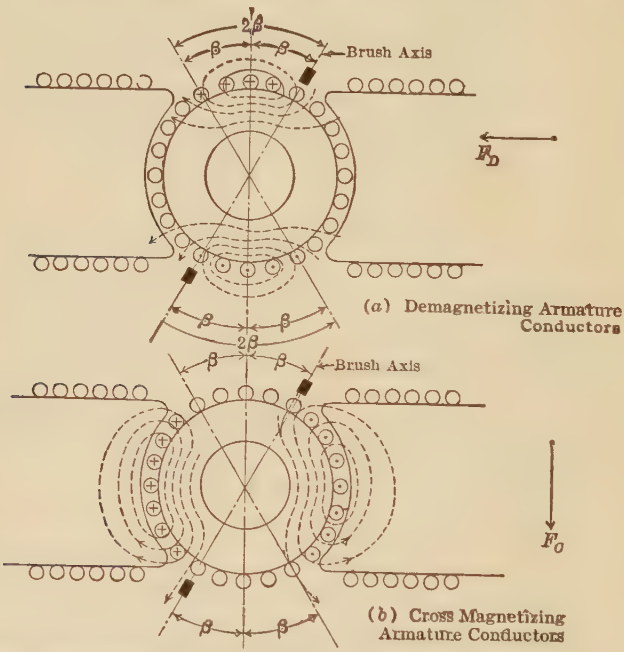


FIG. 264.—Demagnetizing and cross-magnetizing components of armature reaction.

F_A may be resolved into two components, F_D parallel to the polar axis and F_C perpendicular to this axis.

It will be noted that F_D acts in direct opposition to F, the main field (Fig. 262). Therefore, it tends to reduce the total flux and so is called the *demagnetizing* component of armature reaction. F_C acts at right angles to F and produces distortion. Therefore, it is called the *cross-magnetizing* component of armature reaction.

The exact conductors which produce these two effects are shown in Fig. 264. In (*a*) the brushes are shown as advanced

by an angle β to correspond to the advance in the neutral plane. All the conductors within the angle 2β, both at the top and at the bottom of the armature, carry current in such a direction as to send a flux through the armature from right to left. This may be checked by the corkscrew rule. These conductors thus act in direct opposition to the main field and are therefore called the *demagnetizing* armature conductors. Their magnetomotive force is represented by the component F_D (Fig. 263).

Figure 264 (*b*) shows the flux produced by the conductors not included within twice the angle of brush advance. The direction of this flux is downward and perpendicular to the polar axis. These conductors cross-magnetize the field. The m.m.f. producing this flux is represented by the component F_C (Fig. 263). The resultant of F_D and F_C is F_A.

It should be remembered that the demagnetizing and cross-magnetizing *ampere-turns* are equal to one-half the number of the corresponding *ampere-conductors*.

Example.—A 4-pole dynamo has 288 surface conductors. The machine is lap wound and the armature current is 120 amp. The brushes are advanced 15 space degrees. How many demagnetizing and how many cross-magnetizing armature ampere-turns are there?

Twice the angle of brush lead is 30°. There are four brushes, so that the total number of degrees covered by the demagnetizing conductors is 120°. Therefore one-third the conductors on the armature, or 96 conductors, are demagnetizing conductors.

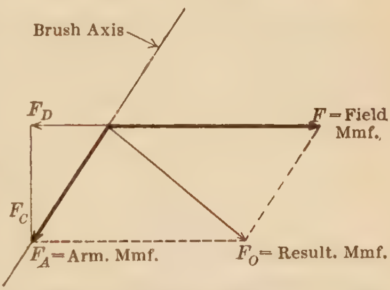


Fig. 265.—Resultant effect of armature and field m.mfs.

As the machine is lap wound there are four paths through the armature. The current per path = 120/4 = 30 amp.

Demagnetizing ampere-conductors = 30 × 96 = 2,880.

Demagnetizing *ampere-turns* = 2,880/2 = 1,440. *Ans.*

The number of cross-magnetizing conductors must be two-thirds of the conductors on the armature. Therefore, the number of cross-magnetizing ampere-turns is

$$\frac{192 \times 30}{2} = 2{,}880. \quad Ans.$$

Figure 265 shows the method of finding the resultant magnetomotive force acting on the armature. F is the field magneto-

motive force and F_A is the armature magnetomotive force, acting along the brush axis after the brushes have been advanced. F_o is the resultant of the two, being less than F due to the demagnetizing component of F_A. F_A can be resolved into two components at right angles to each other, F_D the demagnetizing component of the armature m.m.f. and F_C the cross-magnetizing component of the armature m.m.f.

218. Armature Reaction in Multipolar Machines.—Reactions occur in multipolar machines in the same manner as in the bipolar machines that have just been described. The picture to the eye

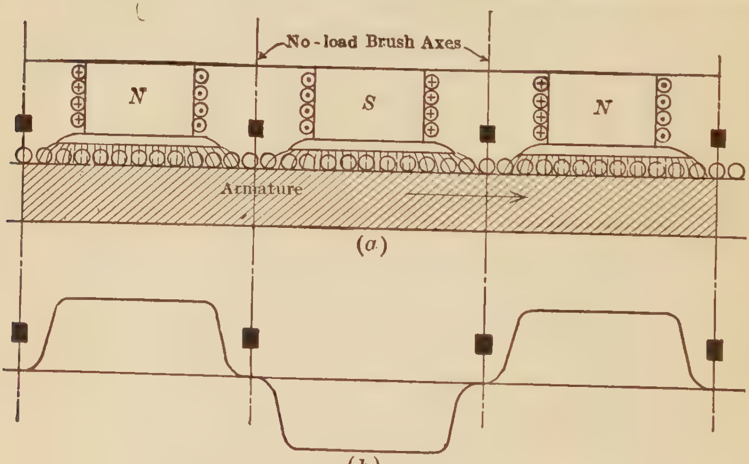


FIG. 266.—Field flux of a multipolar generator.

may be a little different, however. In Fig. 266 the armature and the field poles of a multipolar machine are shown, the armature being shown as a flat surface, for convenience.

In (*a*) are shown the alternate north and south poles, together with the magnetic flux entering the armature. There is no current flowing in the armature conductors. In (*b*) the flux distribution is shown. It will be observed that it is symmetrical about the polar axis. It is substantially constant under the pole shoe and drops off gradually at the edges, due to fringing. It falls to zero and reverses in the interpolar spaces. The neutral plane is the region where the flux is zero and under no-load conditions is midway between the poles.

Figure 267 (*a*) shows the armature conductors carrying current, the field current being zero. These armature conductors produce a flux in a manner similar to that shown in Fig. 262 (*b*). The magnetomotive force of the armature is not uniform, but varies uniformly from zero at the pole axis to a maximum in the center of the interpolar spaces. The armature conductors between the lines *qr* and *st* may be considered as constituting a pancake coil, the current flowing into the paper in the conductors on the left and out of the paper in the conductors on the right. Midway between *qr* and *st* the magnetomotive force will be a maximum, as the magnetomotive forces of all the conductors on

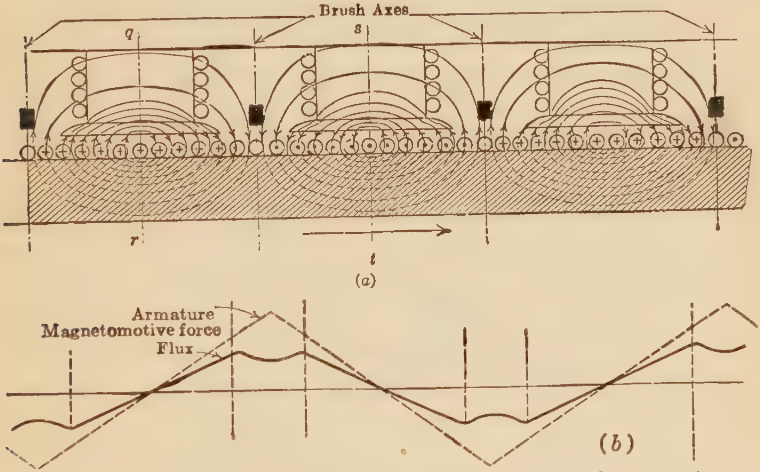


FIG. 267.—Flux due to armature reaction in a multipolar generator.

both sides are acting together at this point. The magnetomotive force directly under the pole centers is zero since for every ampere-conductor on one side of any pole axis there is a symmetrically-spaced ampere-conductor on the other side carrying an equal current on the *same* direction. The net magnetomotive force at the pole-centers due to all such ampere-conductors is obviously zero. The magnetomotive force distribution along the air-gap is shown by the dotted line (Fig. 267 (*b*)). Magnetomotive force tending to send flux into the armature is shown as positive; magnetomotive force tending to send flux out of the armature is shown as negative. If the reluctance of the air-gap were constant, the

flux density curve would be identical in shape with the m.m.f. curve. Owing to the high reluctance of the interpolar spaces, the *flux* density curve has not the same shape as the m.m.f. curve, but droops in the interpolar spaces as shown in Fig. 267 (*b*).

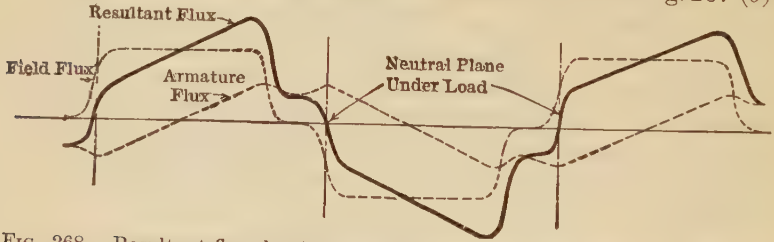


FIG. 268.—Resultant flux density found by combining field flux (Fig. 266) and armature flux densities (Fig. 267)

The resultant flux density at each point is found by adding the two flux density curves of Figs. 266 and 267, as is done in Fig. 268. (This assumes constant permeability in the iron.) It will be noted that the flux peaks on the trailing pole tip (Fig. 268) as in the bipolar generator. Also the neutral plane has advanced

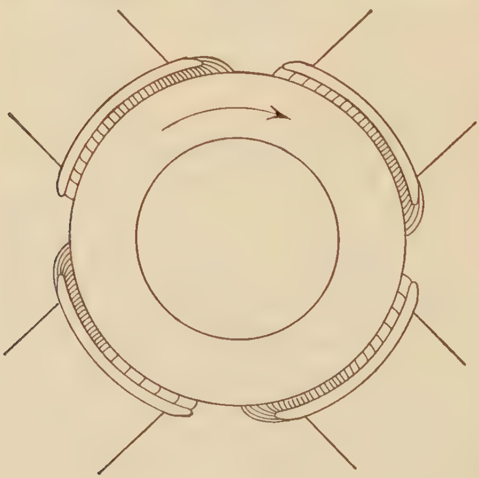


FIG. 269.—Field distortion in a 4-pole generator.

in the direction of rotation. In order to keep the brushes in the neutral plane, they should be advanced as the neutral plane advances. Figure 269 shows the crowding of flux in the trailing pole tips in a 4-pole generator.

219. Compensating Armature Reaction.—As the cross-magnetizing effect of the armature usually necessitates the shifting of the brushes with load, it is desirable to minimize armature reaction if this can be done conveniently. One practical method, when laminated pole cores are used, is to use a stamping having but one pole tip, as shown in Fig. 244, page 297. These are alternately reversed when the core is built up. This leaves spaces between the pole-tip laminations, resulting in the pole tips having but one-half the cross-section of iron along their lengths. Therefore, the pole tip becomes highly saturated and its permeability greatly reduced. This tends to prevent the flux from crowding into the trailing tip.

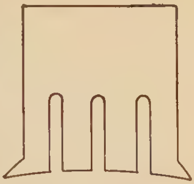
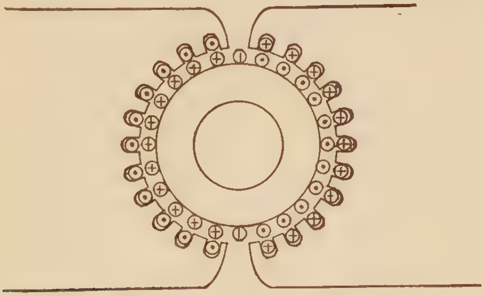
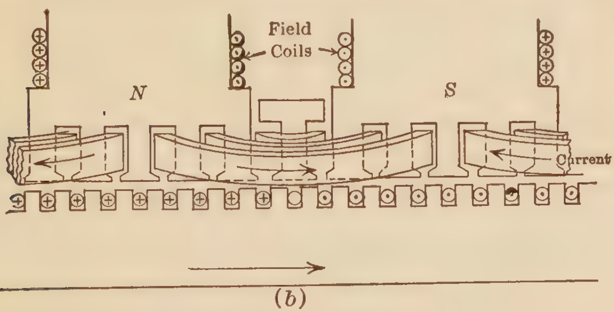


FIG. 270.—Longitudinal slots in pole-face for reducing armature reaction.



(a)

FIG. 271a.—Compensation of armature reaction with pole-face conductors.



(b)

FIG. 271b.—Turns in pole faces to compensate armature reaction.

Another method is to introduce longitudinal slots in the pole faces, as shown in Fig. 270. These slots introduce high reluc-

tance in the path of the armature flux but have little effect on the field flux.

The Thompson-Ryan method is to compensate armature reaction by magnetomotive forces equal and opposite to those of the armature. In order to be effective, these compensating magnetomotive forces should be equal and opposite to those of the armature at every point. This principle is illustrated by Fig. 271 (a), which shows conductors embedded in the pole faces close to the armature. Each conductor carries a current opposite to that of its corresponding armature conductor. This winding is connected in series with the armature so that the magnetomotive forces are opposite and equal at all loads. These windings allow the use of a very short air-gap, with the accompanying reduction in field copper and in field loss.

The Thompson-Ryan principle has been applied to many modern machines where commutation difficulties are unusually great, as in alternating-current series motors (see Vol. II, Chap. X) and in large rolling-mill motors. The conductors are installed in the pole faces in the manner indicated in Fig. 271 (b). This type of construction is used in the Ridgeway dynamo.

The conductors are connected in series with the armature and are so adjusted that their ampere-turns are in almost exact opposition to the armature ampere-turns at each point. They do not as a rule have the same number of conductors as the armature because the armature current at each point is that of one armature path, whereas the current through this compensating winding is the sum of the currents in the various parallel paths through the armature. The small poles between the main poles (Fig. 271 (b)) are saturated for any flux tending to leak between the main poles.

Armature reaction is also reduced by increasing the length of the air-gap, thus offering higher reluctance to the armature flux. A longer air-gap means more field copper and a greater field current, however.

220. Commutation.—It has been shown that the electromotive force induced in any single coil of a direct-current generator is alternating, and in order that the current may flow always in the same direction to the external circuit, a commutator is necessary. Figure 272 shows the changes of current in an armature coil as it

approaches and recedes from the brushes. It is assumed that ideal commutation is being realized. The load is such that 20 amp. flow in each path of the armature, making 40 amp. leaving the machine by this one brush. The current distribution throughout the brush is also assumed to be uniform.

When in positions (1), (2), and (3) each coil (and, therefore, successive positions of any one particular coil) carries 20 amp. As the brush covers four segments and the current distribution is uniform, 10 amp. must flow into the brush from each segment.

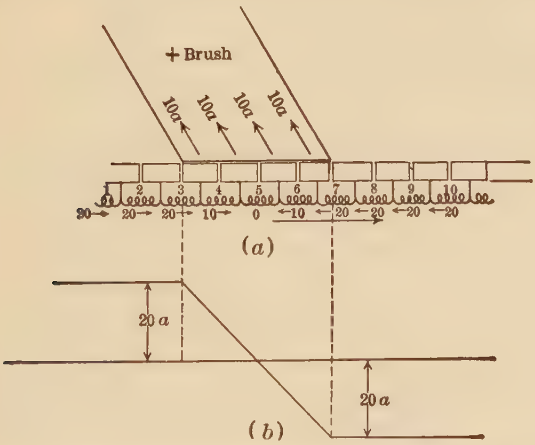


FIG. 272.—Current in coil undergoing commutation—ideal conditions.

ment. Therefore, when passing from position (3) to (4), the coil must lose the 10 amp. which pass from this segment into the brush. Hence, in position (4) the coil carries only 10 amp.

Before reaching position (5) the coil gives up another 10 amperes so that the current is zero when the coil reaches position (5). When the coil reaches position (6) the current flows through the coil in the reverse direction, due to current entering the brush from another armature path. The current reaches 20 amp. in position (7) and remains 20 amp. in the further positions (8), (9), and (10).

Therefore, commutation consists of two parts:

1. Reversing the current in any coil from its full positive value to an equal negative value. This reversal must take place in the

short time interval required for a segment to pass under the brush.

2. The current supplied by the two paths meeting at the brush must be conducted to the external circuit.

Part (1) is illustrated by Fig. 272 (*b*). The current in the coil is +20 amp. until the brush is reached, when it reverses at a uniform rate to a value of −20 amp. This is perfect commutation.

The foregoing ideal commutation is only approximated in practice. There are two causes preventing its realization.

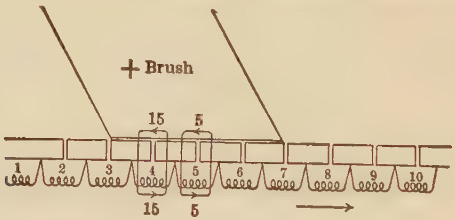


FIG. 273.—Short-circuit currents through brush.

It will be noted that when the coil is in positions (4), (5), and (6) it is short-circuited by the brush. If any voltage is being induced in the coil when it is in these positions, a large current will necessarily flow, since the resistance of the short-circuited coil is very low. This resistance consists merely of the resistance of the coil plus the contact resistance of the brush. This contact resistance constitutes the major portion of the total resistance. Figure 273 shows assumed currents of 15 and 5 amp. flowing in coils (4) and (5) respectively, due to voltages induced in them while they are being short-circuited by the brush.

If the local short-circuit currents of Fig. 273 be superposed upon those of Fig. 272 (*a*) the current distributes itself over the brush in the manner shown in Fig. 274 (*a*). There are now 45 amp. entering the brush and 5 amp. leaving it. Therefore, the brush must carry 50 amp. instead of 40, and in one place there are 20 amp. per segment, or twice that which occurred under the ideal conditions of Fig. 272. This will tend to produce heating and undue sparking under the heel of the brush.

Figure 274 (*b*) shows the manner in which the current in the coil varies under these new conditions. Instead of dropping uni-

formly from 20 amp. it first *rises* to 25 amp. before starting to reverse. It will be noted that the time for reversing from $+20$ amp. to -20 amp. has been reduced from time t to time t_1, which makes commutation more difficult. The curve of Fig. 274 occurs when the brush is too far back of the neutral plane. Voltages are then induced in the coils as they are undergoing commutation.

The curves of Fig. 272 (*b*) and 274 (*b*) are called commutation curves.

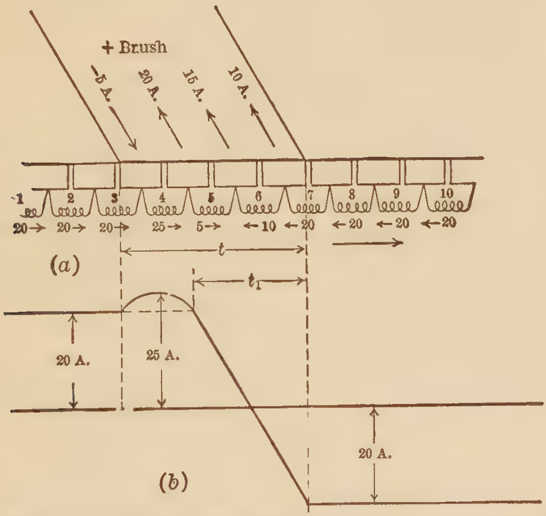


FIG. 274.—Change of current in coil when brushes are too far back of the neutral plane.

If the brushes are placed too far ahead of the neutral plane, short-circuit currents flow under the toe of the brush, resulting in the current distribution and commutation curve of Fig. 275. This condition produces undue sparking under the toe of the brush.

If the brushes are too wide, both the heel and the toe will short-circuit coils in which voltages are induced, resulting in the commutation curve of Fig. 276. Moving the brushes either backward or forward does not assist matters in this case. The only remedy is a narrower brush.

221. The Electromotive Force of Self-induction.—Figure 277 (*a*) shows an armature coil just as it is entering the commu-

tation zone. The slot conductors are embedded in iron and, due to the current flowing in the coil, considerable flux passes through the coil, in this case upward. Let the value of the flux be ϕ_1. In Fig. 277 (*b*) the same coil is shown just after it has left the commutation zone. The current through the coil is the same as its previous value, but it now flows in the reverse direction. The flux is still ϕ_1 but it has been reversed in direction.

Therefore, in the time t seconds required for a segment to pass the brush or commutating zone, the flux has changed by

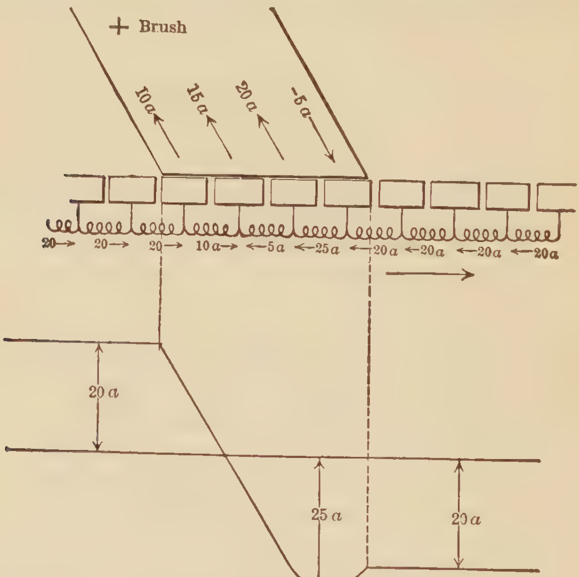


FIG. 275.—Commutation with the brushes too far ahead.

$2\phi_1$ lines. This is shown in Fig. 278, where the ideal commutation curve is assumed. This change of flux will induce a voltage,

$$e = -N\frac{2\phi_1}{t}10^{-8} \text{ volts (from Eq. 74, p. 211).}$$

N being the number of turns in the coil.

This voltage, with its proper direction, is shown in Fig. 278. It is called the electromotive force of self-induction.

Instead of looking upon this as a voltage phenomenon, it may be considered as follows: The armature coil has self-inductance, and mutual inductance with other armature coils. This induc-

tance tends to prevent the current reversing in the same manner that inductance tends to prevent change of current in any circuit.

Therefore, even though the brushes are set exactly in the neutral plane and the coils undergoing commutation are cutting

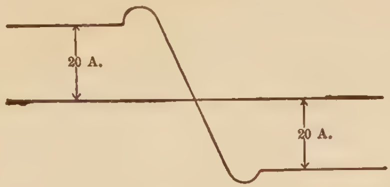


FIG. 276.—Commutation with too wide a brush.

no magnetic lines, there will be a voltage induced in the coil due to its own self-inductance and to mutual inductance. To eliminate this voltage it is necessary to set the brushes *ahead* of

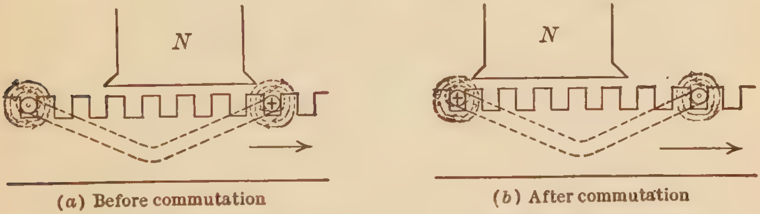


FIG. 277.—Change of flux through a coil undergoing commutation.

the neutral plane in a generator. When the coil is undergoing commutation it finds itself in a field of the same polarity as that which the conductors leaving the commutation zone are about to

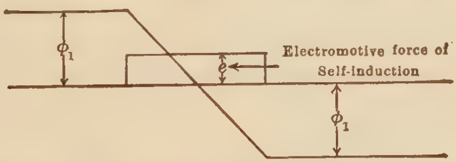


FIG. 278.—Electromotive force of self-induction in a coil undergoing commutation.

enter. Therefore, this field induces a voltage which assists the current to reverse.

Another way of stating it is that the electromotive force induced in the coil due to its cutting flux in the zone ahead of the

neutral plane is in exact opposition to the electromotive force of self- and mutual induction, shown in Fig. 278, and so neutralizes it.

It is therefore necessary that the brushes be kept *ahead* of the neutral plane in a *generator*, in order to obtain satisfactory commutation under load conditions.

222. Sparking at the Commutator.—The voltages induced in a coil due to the shifting of the neutral plane and also due to its own self-inductance are comparatively low in value, being of the order of magnitude of from a few tenths of a volt to perhaps 4 or 5 volts. But they are acting in a circuit having a very low resistance. The resistance of one coil is extremely low so that most of the circuit resistance is at the brush contact. If the brush contact resistance is too low, these short-circuit currents may reach such values as to produce severe sparking at the brushes. On the other hand, a brush with low contact resistance is desirable from the standpoint of carrying the current out to the external circuit with minimum contact loss.

Copper brushes have a very low contact resistance, but the short-circuit currents are excessive when they are used. Therefore, their application is limited to very low-voltage, high-current machines In this case copper gauze is often used. Another disadvantage of using copper brushes is that they "cut" the commutator mechanically.

Carbon brushes have a much higher contact resistance than copper and therefore limit the short-circuit currents, giving much more satisfactory results. In addition, they are more or less graphitic in their composition and so lubricate the commutator to a certain extent. Unusually hard carbon brushes may cut the commutator. Different grades of carbon are required for different machines.

The passage of the current from the commutator to the brush is more of an arc phenomenon than it is one of pure conduction. A careful examination will show myriads of minute arcs existing between the brush surface and the commutator. The voltage drop between the commutator and the brush, instead of being proportional to the current (as it would be with conduction only) is substantially constant and is equal to about 1 volt per brush. Bits of copper may be found in the positive brush due to the

arcing. The voltage drop across the negative brush is different from that across the positive brush, due to the copper being positive in one case and negative in the other. All these facts substantiate the arcing theory.

Another proof is the so-called "high mica." After a machine has been in operation for a considerable time, it often happens that the mica insulation between the commutator segments protrudes above the surface of the commutator, resulting in so-called "high mica" (Fig. 279). It was long supposed that this was due to the mica being harder than the copper, which resulted in the wearing away of the copper more readily than the mica. The fallacy of this supposition is of course evident. Even though the mica is much harder than the copper, the two must always wear evenly, for the brush cannot grind the copper until it comes in contact with it. Hence, the brush must grind down the mica before it can touch the copper, if "high mica" is due to mechanical abrasion alone.

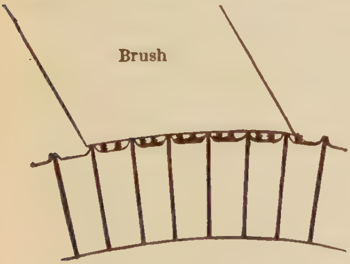


Fig. 279.—Commutator with high mica.

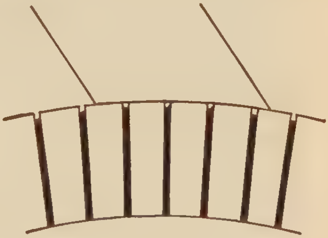


Fig. 280.—Undercut mica commutator.

The rational explanation of high mica is dwelt upon in some detail by B. G. Lamme in a paper presented before the American Institute of Electrical Engineers.[1] The copper is not worn away as generally supposed, but is carried away by the minute arcs that exist between the brush and the commutator, as shown in Fig. 279. This may be proved by running two similar machines for the same periods of time, one of the machines delivering current and the other having no current at all in the brushes and commutator. High mica will ultimately appear on the commutator

[1] "Physical Limitations in D. C. Commutating Machinery," by B. G. Lamme, A. I. E. E. *Trans.*, Vol. XXXIV, Part II, p. 1739, 1915.

which carries current, if conditions warrant, whereas it will be found impossible to produce high mica on the machine which carries no current.

High mica may be reduced by the use of fairly hard brushes which grind the mica down. Amber mica appears to be softer than white mica and is therefore often preferred for segment insulation. In modern practice the mica is under-cut by many manufacturers, that is, the top of the mica is below the commutator surface, as is shown in Fig. 280. There is some disadvantage in this construction, in that small bits of copper, carbon and dirt collect in the groove and may ultimately short-circuit the segments. These grooves can be easily cleaned out, however.

Fig. 281.—Proper method of fitting brushes.

The result of any arcing under the brush is to pit the commutator. As irregularities and depressions in the commutator tend to prevent the brush making intimate contact with the commutator, arcs of increasing magnitude will be formed. The

deeper the depressions, or the higher the mica, the larger and more vigorous these arcs become. Hence, any condition which produces sparking and so roughens the commutator only increases the sparking and roughening, or, these actions are cumulative. If a commutator is sparking badly and the cause of the sparking is not corrected, the commutator will deteriorate very rapidly and soon become inoperative.

The brushes should be fitted very carefully to the commutator surface by grinding with sandpaper in the manner shown in Fig. 281. Carbon on the surface of the commutator should be removed with an oily cloth. Do not use waste. A slightly roughened commutator may be partially smoothed with fine sandpaper. Do not use emery, as the particles of emery are conducting and may short-circuit the commutator bars. If the

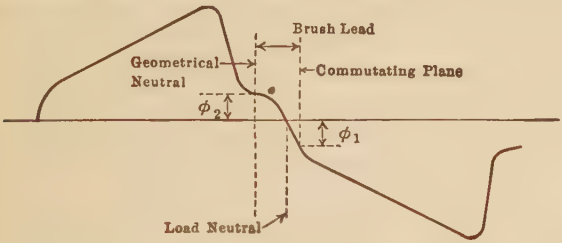


FIG. 282.—Brush advance to proper commutating plane.

commutator is grooved by the brushes, or is otherwise in poor condition, it should be turned down in a lathe.

Other difficulties, such as loose mica and loose segments, are more serious in character. It is often possible to rectify these difficulties by tightening up the commutator clamp bolts.

223. Commutating Poles (Interpoles).—Figure 282 shows the geometrical neutral or no-load neutral plane and the neutral plane under load. It will be noted that this is merely Fig. 268 reproduced. If the brushes remained in the no-load neutral plane, there would be severe sparking under load conditions, because of the very appreciable flux density, ϕ_2, due to armature reaction, now existing in the neutral zone. The brushes will not commutate properly even if advanced to the load neutral plane. This is due to the fact that the electromotive force of self-induction exists in the coils undergoing short-circuit, even if the voltage

due to the pole flux is zero. The brushes must be advanced so that the short-circuited coils are cutting the flux density ϕ_1 of the next pole, as shown in Fig. 282, in order that an electromotive force may be generated which will balance the electromotive force of self-induction. It will be noted that this position is in the fringe of the next pole flux. A very slight movement of the brushes in either direction makes a very marked change in the flux density so it is difficult to obtain good commutation under these conditions. In fact, it may be impossible to obtain satisfactory commutation because of the steepness of the flux-distribution curve. When the best position of the brushes is obtained, the trailing tip of each brush may be in too strong a field and the leading tip in too weak a field.

If a flux density having the same value as ϕ_2, but opposite to it in direction, could be produced in the geometrical neutral, it is obvious that the flux density in the neutral plane could be brought to zero in spite of armature reaction. If a flux density having a value $\phi_2 + \phi_1$ were produced, satisfactory commutation would be obtained without moving the brushes. It is the function of commutating poles to produce just this flux density.

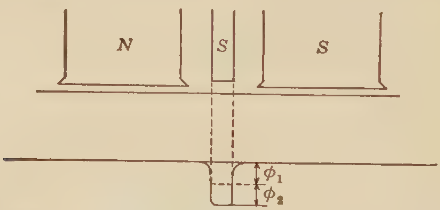


FIG. 283.—Flux produced by commutating pole alone.

Commutating poles consist of narrow poles located between the main poles. They send a flux into the armature which is of the proper magnitude to produce satisfactory commutation. For example, in Fig. 282 the commutating pole must first produce a flux density equal to ϕ_2 so as to neutralize, in the neutral zone, the increase of flux density due to armature reaction. It must also produce an additional flux density ϕ_1 to balance the electromotive force of self-induction in the coil undergoing commutation. This commutating-pole flux is shown in Fig. 283. The pole producing it at this point must be a south pole. Figure 284 shows the resultant flux obtained by combining Figs. 282 and 283.

As the armature reaction and the electromotive force of self-induction in the coils undergoing commutation are both proportional to the armature current, the compensating flux produced by the commutating poles must also be proportional to the

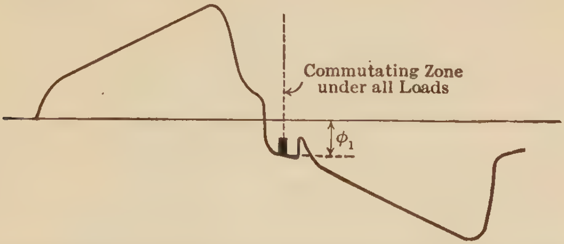


FIG. 284.—Resultant of main flux and commutating-pole flux—machine loaded.

armature current. The commutating poles are wound, therefore, with a few turns of comparatively heavy wire and are connected in series with the armature, as shown in Fig. 285. The air-gap between these poles and the armature is large, so that the commu-

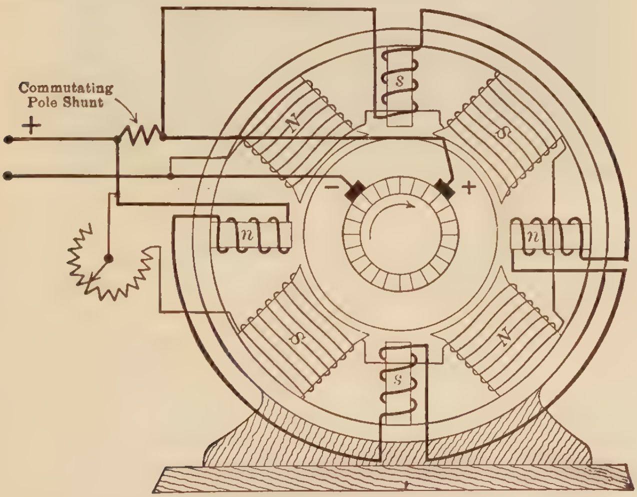


FIG. 285.—Connections of shunt field and commutating poles.

tating-pole flux is nearly proportional to the armature current at all loads.

It should be noted that the sequence of poles in the direction of rotation in a generator is *Ns* and *Sn*, where the capitals refer

to the main poles and the small letters refer to the commutating poles. Figure 286 shows an interpole separate from the machine. Figure 287 shows the frame and field coils for a commutating-pole motor. It will be noted that only two commutating poles are necessary in this 4-pole machine. Each pole has twice the strength that it would have if four such poles were used. Therefore, the proper commutating voltage is induced in but *one* side of the coil undergoing commutation.

In practice, commutating poles are so designed that they produce a flux of greater magnitude than is necessary. The entire commutating-pole circuit is then shunted by a low-resistance shunt, this shunt being adjusted until the best condition of commutation is obtained. The commutating-pole shunt is shown in Fig. 285. This shunt is sometimes made inductive so that the proper proportion of current will flow to the commutating poles on sudden changes of load, such as occur in railway generators.

Fig. 286.—Commutating pole and winding.

224. The Shunt Generator: Characteristics.—If a shunt generator, after building up to voltage, be loaded, the terminal voltage will drop. This drop in voltage will increase with increase of load. Such a drop in terminal voltage is undesirable, especially when it occurs in generators which supply power to incandescent lamps.

It is very important to know the voltage at the terminals of a generator for each value of current that it delivers, because the ability to maintain its voltage under load conditions determines in a large measure the suitability of a generator for a specified service.

To test a generator, in order to determine the relation of terminal volts to current, it is connected as shown in Fig. 259, page 312. The machine is self excited and a voltmeter is con-

nected across its terminals to indicate the terminal volts. An ammeter is connected in the line to measure the load current. In

FIG. 287.—Frame and field coil for Westinghouse 30-hp. direct-current, interpole motor.

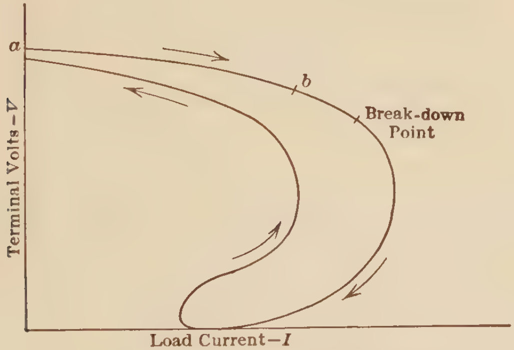


FIG. 288.—Shunt generator characteristic.

performing this test, it is often desirable to connect an ammeter in the field circuit so that the change in the field current as the load is applied may be determined.

In starting the test, rated load should first be applied and the field current adjusted until rated voltage is obtained. The load should then be thrown off and the no-load volts read on the voltmeter. The load should then be gradually applied, reading the volts and the current for each load. The speed of the generator should be maintained constant throughout. If the readings be plotted as shown in Fig. 288, the so-called shunt characteristic results. If, in a small generator, the load be carried far enough, the terminal voltage will begin to decrease rapidly as shown in Fig. 288. This is called the break-down point of the generator. Further application of load results in a very rapid decrease of voltage and beyond a certain point any attempt at increase of load results in a *decrease* of current rather than an increase. The load may even be carried to short-circuit conditions and yet the current will actually decrease as short-circuit is

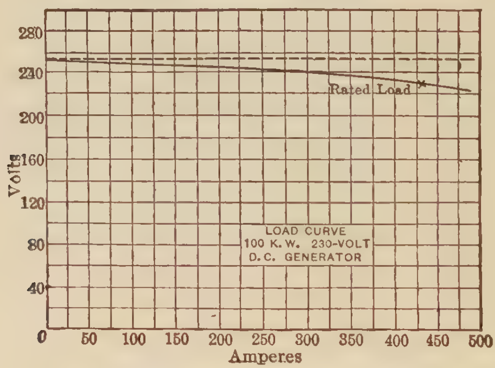


FIG. 289.—Typical shunt characteristic.

approached. This is due to the fact that the field is short-circuited and any current flowing at short-circuit is due only to the residual magnetism of the machine.

If the external resistance be now increased, the voltage will rise slowly and will ultimately reach a value not far below that at which it started. The fact that the voltage follows a different curve when the short-circuit is removed is primarily due to hysteresis. When the load is being applied, the voltage is dropping and the iron is on the part of the cycle represented by c (Fig. 255 (a), page 308). When the voltage starts to increase, it returns along the path a (Fig. 255 (a)). There is less flux for a

given field current and consequently less voltage is induced in the machine upon the return curve. This, together with a lesser field current resulting from the lower voltage, accounts for the return curve lying below the other.

In practice, machines are operated only on the portion *ab* (Fig. 288) of the characteristic. Figure 289 shows this portion of the curve for a 100-kw., 230-volt generator. The rated current is 100,000/230 = 435 amp. The generator field rheostat is set so that the generator terminal voltage is 230 when it is delivering this load of 435 amp.

There are three reasons for the drop in voltage under load of a shunt generator:

1. The terminal voltage is less than the induced voltage by the resistance drop in the armature. That is, the terminal voltage

$$V = E - I_aR_a \qquad (120)$$

where E is the induced volts, I_a the armature current, and R_a the armature resistance.

Example.—The voltage induced within the armature of a shunt generator is 600 volts. The armature resistance is 0.1 ohm. What is the terminal voltage when the armature delivers 200 amp.?

Applying equation (120),

$$V = 600 - (200 \times 0.1) = 600 - 20 = 580 \text{ volts. } \textit{Ans.}$$

2. Armature reaction weakens the field and so reduces the induced voltage.

3. The drop in terminal voltage due to (1) and (2) results in a decreased field current. This in turn results in a lesser induced voltage.

The effect of each of these three factors is shown in Fig. 290.

It might appear that the voltage of the generator would drop to zero, or practically so, of its own accord when the load first is applied, because the foregoing cycle is cumulative. That is, a lesser terminal voltage results in a weaker field, and a weaker field results in a lesser induced voltage and therefore a lower terminal voltage which still further weakens the field, etc. The above cycle *would* result in the terminal volts reaching zero, if the iron were not in some measure saturated. If a 10 per cent. drop in terminal voltage resulted in a 10 per cent. drop in flux, the generator would be unable to supply any appreciable load.

However, a 10 per cent. drop in terminal voltage, hence in field current, probably results only in a 1 or 2 per cent. drop in flux, due to saturation and also hysteresis, as is illustrated in Fig. 255 (*a*), page 308. Therefore, a generator when operating at

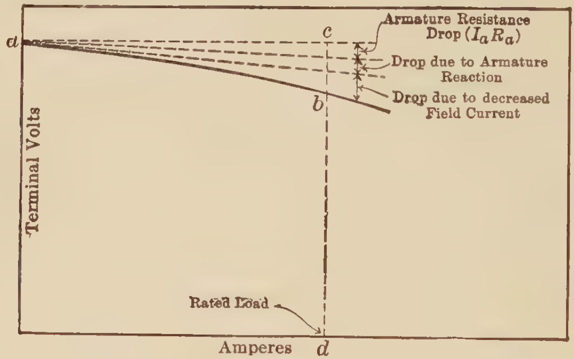


Fig. 290.—Voltage drops in shunt generator.

high saturation maintains its voltage better than when running at low saturation.

This is illustrated by Fig. 291, which shows two saturation curves for a 230-volt generator, one at 900 r.p.m. and the other

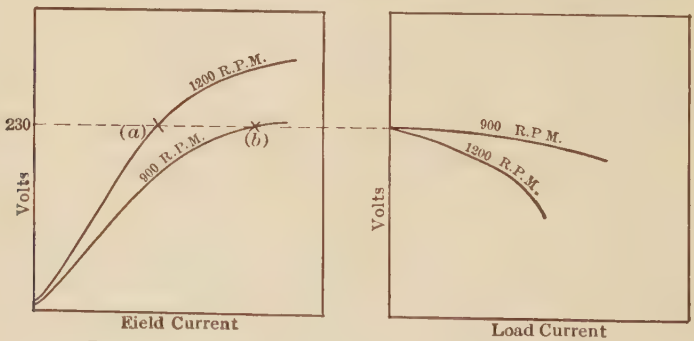


Fig. 291.—Relation of shunt characteristics to speed.

at 1,200 r.p.m. If the no-load voltage of the generator in each case is 230 volts, the generator will be operating at point (*a*) on the 1,200-r.p.m. curve and at point (*b*) on the 900-r.p.m. curve. As point (*b*) corresponds to a much higher saturation of the armature and field iron than (*a*), the generator will main-

tain its voltage better at 900 r.p.m. than at 1,200 r.p.m., as shown by the characteristics in Fig. 291.

225. Generator Regulation.—The ability of a generator to maintain its voltage under load is a measure of its suitability for constant-potential service. The *regulation* shows quantitatively the amount the voltage varies from rated load to no load.

Regulation is defined in the A. I. E. E. standards as follows:

> The regulation of a d.-c. generator is usually stated by giving the numerical values of the voltage at no load and rated load, and in some cases it is advisable to state regulation at intermediate loads. The regulation of d.-c. generators refers to changes in voltage corresponding to gradual changes in load, and does not relate to the comparatively large momentary fluctuations in voltage that frequently accompany instantaneous changes in load.

For example, the regulation of the generator whose characteristic is given in Fig. 289 is given by 250 volts at no load and 230 volts at rated load.

226. Total Characteristic.—Reference is often made to the total characteristic of a shunt generator. The *shunt* characteristic, to which reference has already been made, is the relation existing between *load* current and *terminal* volts. The *total* characteristic is the relation between *armature* current and *induced* volts.

The armature current differs from the load current by the amount of current flowing in the field.

The armature current

$$I_a = I + I_f, \tag{121}$$

where I is the load current and I_f the shunt-field current.

The induced volts

$$E = V + I_aR_a, \tag{122}$$

where V is the terminal voltage and R_a the armature resistance, including brush and brush-contact resistance. The total characteristic is the curve showing the relation of I_a and E. It may be found graphically from the shunt characteristic as follows:

Let qr (Fig. 292) be the shunt characteristic. Draw the field resistance line Oa, as was done in Figs. 260 and 261. The line will have the appearance of being nearly vertical, owing to the fact that the abscissas are plotted to armature current scale. The horizontal distances from the OY axis, Oq to Oa, give the value of field current for each value of voltage. By adding these distances horizontally to the shunt characteristic, the total cur-

rent is given by the resulting characteristic qe. For example, at point c on the shunt characteristic the distance $c'd'$ is added horizontally at cd, giving point d on the characteristic qe.

The armature resistance drop line Ob is then plotted, assuming that the brush contact resistance is constant. The voltage drop in the armature is then proportional to the current. It is only necessary to determine the drop $e'f'$ at some value of current Oe'. That is, the voltage drop

$$e'f' = (Oe')R_a.$$

Draw the line Of'. The vertical distances from the OX axis, Oe' to Of', give the armature drop for each value of current.

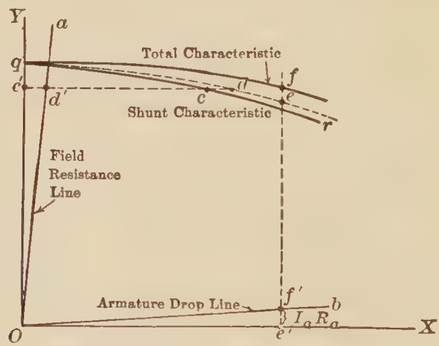


FIG. 292.—Total characteristic of shunt generator.

Adding these drops to the characteristic qe, as $ef = e'f'$ is added at the point e, the total characteristic qf is obtained.

It should be borne in mind that the total *induced* voltage multiplied by the total current gives the total power developed within the armature. All of this power is not available, however, for two reasons:

(1) Some of this power is lost in the armature itself, appearing as $I_a^2R_a$ loss in the armature copper.

(2) Some of the armature output is consumed in heating the shunt field.

Example.—A 20-kw., 220-volt, shunt generator has an armature resistance of 0.07 ohm and a shunt-field resistance of 100 ohms. What power is developed in the armature when it delivers its rated output?

Rated current

$$I = \frac{20,000}{220} = 90.9 \text{ amp.}$$

Field current

$$I_f = \frac{220}{100} = 2.2 \text{ amp.}$$

Armature current

$$I_a = 90.9 + 2.2 = 93.1 \text{ amp.}$$

Induced volts

$$E = 220 + (93.1 \times 0.07) = 226.5 \text{ volts.}$$

Power developed in armature

$$P = 226.5 \times 93.1 = 21.1 \text{ kw.} \quad \textit{Ans.}$$

The same result may be obtained by adding power losses as follows:
Field loss

$$P_f = \frac{(220)^2}{100} = 484 \text{ watts.}$$

Armature loss $\quad P_a = (93.1)^2 0.07 = 607$ watts.

Power developed in armature

$$P = 20{,}000 + 484 + 607 = 21{,}091 \text{ watts} = 21.1 \text{ kw.} \quad \textit{Ans.}$$

227. Relation of Shunt Characteristic to Saturation Curve.—It was pointed out in Par. 224 that the shunt characteristic depends on the degree of saturation of the iron, and hence on that

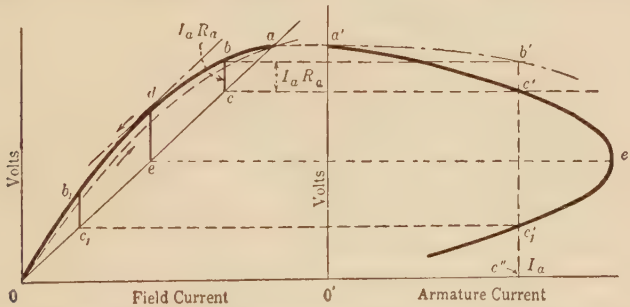


FIG. 293.—Relation of shunt characteristic to saturation curve—no armature reaction.

part of the saturation curve at which the machine is operating. Below, is shown a method of determining the shunt characteristic directly from the saturation curve and field-resistance line.

(a) *Armature Reaction Negligible.*—Assume first that the armature reaction is so small as to be negligible. This condition often occurs with interpole machines in which the brushes remain in the geometrical neutral. Figure 293 gives the saturation curve of a shunt generator taken at some fixed speed. The dotted line is taken with increasing values and the solid line with decreasing

values of field current. The field-resistance line is given by oa and the generator will build up along the dotted line to point a, neglecting the small drop in the armature due to the field current. Hence, a' on the right-hand plot will be the generator terminal voltage when the armature current is zero. As the load is applied to the armature, two effects will cause the terminal voltage to drop, the resistance drop (I_aR_a) in the armature and the decreased field current due to the lessened terminal voltage. The armature reaction has been assumed to be negligible. When the generator has reached a condition of stability, two conditions must be fulfilled. The induced e.m.f. must lie on the descending portion of the saturation curve abo, as at some point b; the corresponding terminal voltage must lie on the field-resistance line at point c, since the field-circuit terminals are connected directly to the armature terminals. Moreover, points b and c must have the same abscissa, for the same field current which gives the induced e.m.f. b is also caused to flow by the terminal voltage corresponding to c. Furthermore, the distance bc must be equal to I_aR_a, since the difference between the induced e.m.f. and the terminal voltage is equal to the armature resistance-drop. That is, the induced e.m.f. E at b is equal to the terminal voltage V at c plus the I_aR_a drop (see Eq. (122)). Hence, c gives one value of terminal voltage. The corresponding armature current I_a is found by dividing bc by R_a, that is $I_a = bc/R_a$. This value of armature current is laid off at $O'c''$ along the axis of abscissas in the right-hand graph. The ordinate c is projected horizontally to meet at c' the ordinate erected at c''. The point c' is a point on the characteristic. Since b gives the induced e.m.f. for this value of armature current, b' is a point on the total characteristic. If the external characteristic is desired, the load current may be found by subtracting the field current from I_a, either graphically by the method given in Fig. 292 or by subtracting numerically the value of field current corresponding to point c on the field-resistance line.

A study of Fig. 293 shows that ordinarily there is a second place, such as at b_1c_1, where bc may be fitted vertically between the saturation curve and the field-resistance line. Hence, there must be two values of terminal voltage corresponding to this same value of armature current. The load current will be slightly dif-

ferent in the two cases, since the values of field current are not the same. Point c_1', on the lower portion of the characteristic, gives the second value of terminal voltage corresponding to the this same value of armature current.

The maximum value of current may be found by drawing a tangent to the saturation curve at d parallel to the field-resistance line. The corresponding value of armature current is found by dividing the distance de by the armature resistance. This gives point e' on the characteristic.

With constant armature resistance, it is obvious that the distances bc, de, etc. are proportional to the armature current. Hence, the abscissas for the characteristic may also be found by drawing a number of vertical lines between the saturation curve and the field-resistance line, determining the armature current by scaling.

As is pointed out in Par. 222, page 330, the armature resistance is not constant but is a function of the armature current. A considerable error results in assuming the armature resistance constant. Also, the ordinates of the saturation curve depend so much on the previous magnetic history of the iron that the curve does not ordinarily repeat itself accurately. A small percentage change in either the saturation curve or the field-resistance line makes a much larger percentage change in their difference. Therefore, as would be expected, characteristics obtained by this method may only approximate the characteristics obtained by test. However, the method is extremely valuable in a qualitative sense. For example, if in Fig. 293, the speed be increased and the no-load voltage be maintained constant by increasing the field resistance, the distances bc, de, will be decreased. Hence, for any given terminal voltage, the armature current will be decreased, resulting in a more drooping characteristic (see Fig. 291, p. 340).

(*b*) *Armature Reaction Not Negligible.*—In Fig. 294, let *ofba* be the saturation curve with field *ampere-turns* as abscissas, and *oa* the field-resistance line in terms of field ampere-turns rather than field current. Consider some terminal voltage dd''. Point d must be on the field-resistance line since the field is connected across the armature terminals. If there were no demagnetizing action of the armature on the field, the induced

e.m.f., corresponding to the field ampere-turns od'', would lie on the saturation curve vertically over d, as b lies over c in Fig. 293. Under load, the armature reduces the total ampere-turns of the magnetic circuit by an amount $d''c''$, where $d''c''$ is equal to the armature demagnetizing ampere-turns (see Par. 217). Hence, the *net* ampere-turns acting on the field are given by oc''. The corresponding induced e.m.f. must be $c''b$.

The condition that the terminal voltage must be equal to the induced e.m.f. minus the I_aR_a drop must be fulfilled. Hence, bc must be equal to the I_aR_a drop in the armature, where cd is equal and parallel to $c''d''$. Therefore, triangle bcd must have such a

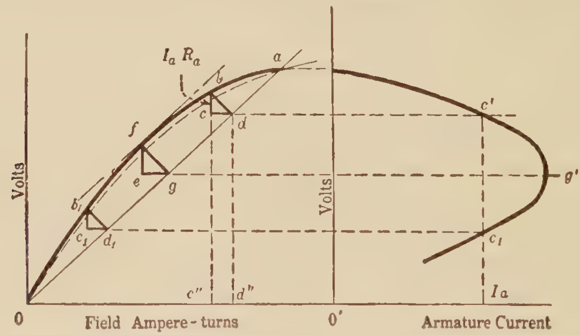


FIG. 294.—Relation of shunt characteristic to saturation curve—armature reaction.

position that point b lies on the saturation curve, point d lies on the field-resistance line and cd is parallel to the axis of abscissas. To determine the two sides of triangle bcd, side bc, or the I_aR_a drop, may be calculated as in (*a*), using the desired value of armature current. The armature demagnetizing ampere-turns per pole, cd, may also be calculated (see Par. 217). Knowing bc and cd, the right triangle bcd is determined. It is then merely necessary to fit this triangle between the saturation curve and the field-resistance line, so that cd is parallel to the axis of abscissas, point b lies on the saturation curve, and point d on the field-resistance line. The ordinate dd'' is obviously the value of terminal voltage corresponding to the chosen value of armature current. If the load current is desired, it may be readily found by the method outlined in (*a*). The point c' on the characteristic

is then found by projecting horizontally to meet the ordinate I_a as before.

If the saturation curve is plotted with field current as abscissas, rather than with field ampere-turns, it is merely necessary to divide the armature demagnetizing ampere-turns by N_f, where N_f is the number of shunt-field turns per pole.

Other values of armature current may be found as follows: If the armature resistance be assumed constant, both the sides *bc* and *cd* of triangle *bcd* are proportional to the armature current. Hence, the triangles for all values of armature current are similar, and when fitted properly between the saturation curve and the field-resistance line, their corresponding sides are parallel. Therefore, the hypotenuses are parallel to one another. That is, *fg* is parallel to *bd*, etc. To determine the armature current for the terminal voltage corresponding to *g*, draw *fg* parallel to *bd*, intersecting the saturation curve at *f*. The armature current will be equal to *fg* to the same scale as *bd*. Triangle *feg* is drawn to correspond to the maximum value of armature current, the tangent drawn at *f* to the saturation curve being parallel to the field-resistance line. Point g' on the characteristic is found by projecting horizontally to meet the ordinate corresponding to the value of armature current determined from *fg*. This method is open to the same errors as those of (*a*); that is, the armature resistance is not constant and the saturation curve varies because of variations in the magnetic history of the iron. In addition, it is not possible (on account of pole-tip saturation, etc.) to determine accurately the effect of the demagnetizing ampere-turns.

228. The Compound Generator.—The drop in voltage with load, which is characteristic of the shunt generator, makes this type of generator undesirable where constancy of voltage is essential. This applies particularly to lighting circuits, where a very slight change of voltage makes a material change in the candle-power of incandescent lamps. A generator may be made to produce a substantially constant voltage, or even a rise in voltage as the load increases, by placing on the field core a few turns which are connected *in series* either with the load or the armature. These turns are connected so as to *aid* the shunt turns when the generator delivers current (Fig. 295). As the load increases, the current through the series turns also increases and,

therefore, the flux through the armature increases. The effect of this increased flux is to increase the induced voltage. By proper adjustment of the series ampere-turns, this increase in armature voltage may be made to balance the combined drop in voltage due to armature reaction and to the resistance of the armature. If the terminal voltage is maintained substantially constant, the field current will not drop as the load increases. Therefore, the three causes of voltage drop, namely, armature reaction, I_aR_a drop, and drop in field current (Fig. 290), are neutralized more or less completely by the effect of the series ampere-turns.

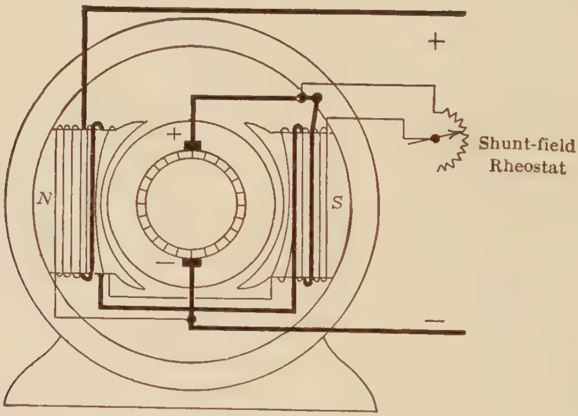


Fig. 295.—Connections of a compound generator (short shunt).

The shunt field may be connected directly across the armature terminals, Fig. 296 (*a*), in which case the machine is called *short* shunt. If the shunt field be connected across the machine terminals outside the series field (Fig. 296 (*b*)) the machine is *long* shunt. The operating characteristic is about the same in either case.

If the effect of the series turns is to produce the same voltage at rated load as at no load, the machine is said to be *flat* compounded (see Fig. 297). It is seldom possible to maintain a constant voltage for all values of current from no load to rated load. The tendency is for the voltage first to rise and then to drop again, reaching the same voltage at rated load as was obtained at no load. The particular shape of the characteristic

is due to the iron becoming saturated, so that the added series ampere-turns do not increase the flux at full load proportionately as much as they do at light load. When the rated-load voltage is greater than the no-load voltage, the machine is said to be *over* compounded. When the rated-load voltage is less than the

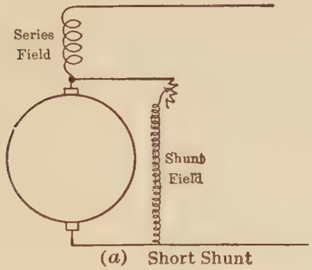


FIG. 296.—Compound-generator connections.

no-load voltage, the machine is said to be *under* compounded. Generators are seldom under compounded.

Flat-compounded generators are used principally in isolated plants, such as hotels and office buildings. The size of the conductors in the distribution system of such plants is determined

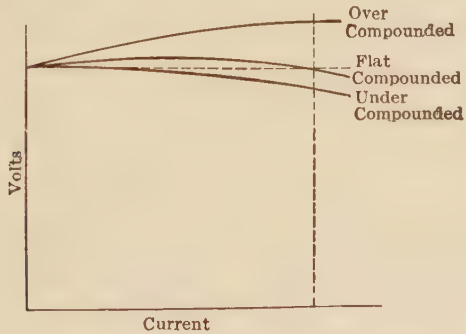


FIG. 297.—Compound-generator characteristics.

almost entirely by underwriters' requirements as to carrying capacity. Wires conforming to these requirements are usually of such size that only a very small voltage drop takes place between the generator and the various loads.

Over-compounded generators are used where the load is located at some distance from the generator. As the load

increases, the voltage at the load tends to decrease, due to the voltage drop in the feeder. If, however, the generator voltage rises just enough to offset this feeder drop, the voltage at the load remains constant.

Example.—Consider the conditions shown in Fig. 298 (*a*). A certain load is 4,000 ft. distant from the generator. The load is supplied over a 500,000-C.M. feeder. The no-load voltage of the generator is 500 volts. It is desired to maintain the load voltage at a substantially constant value of 500 volts from no load to the maximum demand of 300 amp. What must be the characteristic of the generator?

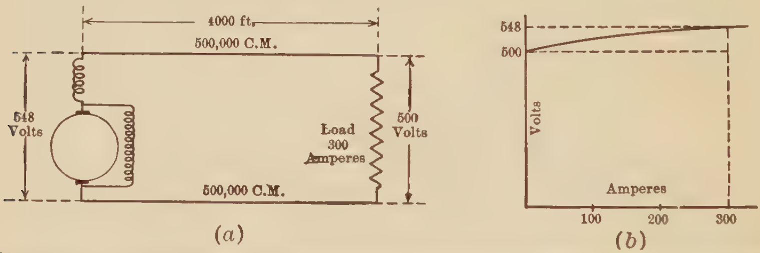


FIG. 298.—Over-compounded generator maintaining constant voltage at the end of a feeder.

If the cables were operated at the "normal" density the current would be 500 amp. or 0.001 amp. per cir. mil (Par. 71), and the drop would be 0.01 volt per foot, making a total drop of 80 volts.

The actual drop is

$$\frac{300}{500} \times 80 = 48 \text{ volts.}$$

The generator terminal voltage should rise from a no-load value of 500 volts to 548 volts when 300 amp. are being delivered to the load (Fig. 298 (*b*)).

Compound generators are usually wound so as to be somewhat over-compounded. The degree of compounding can then be regulated by shunting more or less current away from the series field. To do this a low-resistance shunt, called a *diverter*, is used (Fig. 299).

Compound generators which supply 3-wire distribution systems usually have two series field windings, one connected to each side of the armature. There are two separate series windings on each pole, one winding being connected to the positive terminal and the other to the negative terminal of the machine (see Fig. 369, p. 440).

In a compound generator the induced voltage in the armature is:

$$E = V + I_sR_s + I_aR_a, \tag{123}$$

where V is the terminal voltage, I_s the series field current, I_a the armature current, and R_s and R_a the series field and armature resistance respectively. (R_s is the equivalent parallel resistance of the series field and diverter, if a diverter is used. I_s then equals the combined current in the diverter and series field.) In a long shunt generator $I_s = I_a$.

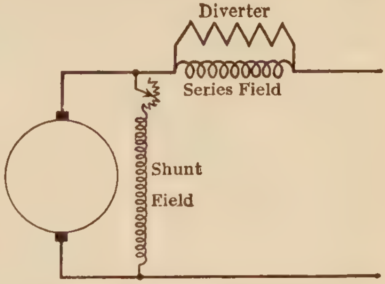


FIG. 299.—Series-field diverter.

Example.—A compound generator, connected short shunt, has a terminal voltage of 230 volts when it is delivering a current of 150 amp. The shunt-field current is 4 amp., the armature resistance 0.03 ohm, and the series-field resistance 0.01 ohm. Determine the induced voltage in the armature, the total power generated in the armature, and the distribution of this power.

The series-field current I_s = 150 amp., and the armature current I_a = 154 amp.

$$E = 230 + (150 \times 0.01) + (154 \times 0.03) = 236.1 \text{ volts}$$

Total power generated

$$P_a = 236.1 \times 154 = 36{,}400 \text{ watts} = 36.4 \text{ kw.}$$

Armature loss

$$P'_a = 154^2 \times 0.03 = \quad 711 \text{ watts.}$$

Series-field loss

$$P_s = 150^2 \times 0.01 = \quad 225 \text{ watts.}$$

Shunt-field loss

$$P_{sh} = (230 + 1.5)4 = \quad 926 \text{ watts.}$$

Power delivered

$$P = 230 \times 150 = 34{,}500 \text{ watts.}$$

Total 36,362 watts (check).

229. Effect of Speed.—Figure 300 shows the saturation curve of a 230-volt, compound generator, taken at 900 r.p.m. The shunt-field rheostat is so adjusted that the machine builds up to a no-load voltage of 230 volts. To produce this result a certain number of shunt-field ampere-turns are necessary, as indicated by the distance *oa*. When load is applied to the machine a

certain number of series ampere-turns are added. Let the number of series ampere-turns be represented by the distance *ab*. Neglecting armature reaction, the induced voltage will be increased by a value *cd* shown in heavy lines.

Let this same machine be speeded up to 1,200 r.p.m. (Fig. 300 (*b*)) and let the no-load terminal voltage still be 230 volts. The distance *oa* will now be less than it was in Fig. 300 (*a*), owing to the increased speed. But the distance *ab* will be the same in each case, as the increase of series-turns depends

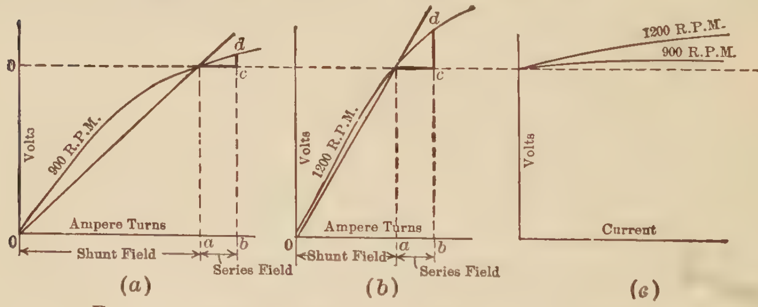


Fig. 300.—Effect of speed upon compound characteristic.

solely on the load. The *increase of voltage cd* is much greater in (*b*) than in (*a*), owing to the lesser saturation of the iron. Therefore, the higher-speed machine will have the more rising characteristic, as is shown in Fig. 300 (*c*). It will be noted that the effect of speed upon the compound characteristic is just opposite to the effect of speed upon the shunt characteristic (see Fig. 291). This is due to the fact that saturation opposes change of the flux in each case.

230. Relation of Compound Characteristic to Saturation Curve. The characteristic of the compound generator may be determined from the saturation curve and the field-resistance line just as with the shunt generator. In Fig. 301 is shown the saturation curve of a generator, plotted with shunt-field ampere-turns per pole as abscissas, and the field-resistance line *oa*. Assume that the machine is long shunt, so that the armature and series-field currents are the same. At no load, the voltage-drop in the armature and series field due to the shunt-field current may be neglected. Moreover, this small drop is offset by the series ampere-turns due to the shunt-field current flowing in the series

field at no load. Thus at no load the terminal voltage will be equal to $o'a'$. Consider some value of terminal voltage bb''. Since bb'' represents terminal voltage, point b must lie on the field-resistance line. The corresponding shunt-field ampere-turns are obviously equal to ob''. Let bc be equal to the series-field ampere-turns per pole, I_aN_s where I_a is the armature current and N_s the number of series turns per pole. The *total* ampere-turns per pole must be equal to the sum of the shunt ampere-turns ob'' and the series ampere-turns $b''c''$, giving a total of oc'' ampere-turns. Were it not for the demagnetizing ampere-turns, the induced e.m.f. would be found on the saturation curve at the point of intersection with ordinate cc''. The

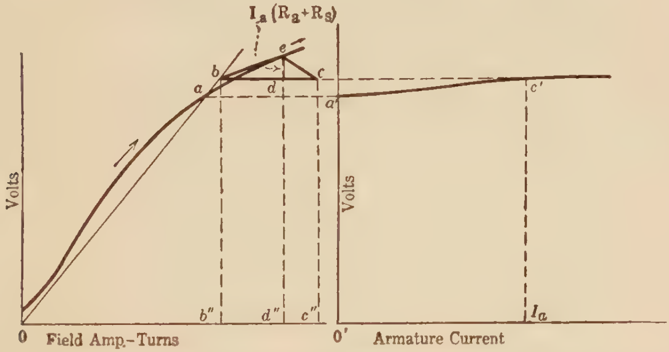


FIG. 301.—Relation of compound characteristic to saturation curve.

armature demagnetizing ampere-turns are given by cd or $c''d''$. These must be subtracted from the total ampere-turns oc'' in order to obtain the *net* field ampere-turns per pole od''. Hence, the induced e.m.f. is equal to $d''e$.

As before, the terminal voltage bb'' must be equal to the induced e.m.f. minus the $I_a(R_a + R_s)$ drop, where R_s is the resistance of the series field. Hence, ed is equal to $I_a\ (R_a + R_s)$.

To plot the characteristic, the desired value of I_a is assumed. The triangle bec is then determined; $bc = I_aN_s$; $cd = I_aN_D$; $ed = I_a(R_a + R_s)$ (N_D = armature demagnetizing turns per pole). This triangle is then so placed that bc is parallel to the axis of abscissas, point e lies on the saturation curve, and point b on the field-resistance line. The line bc is extended to the right to meet the ordinate erected at I_a, giving c' as a point on the

characteristic. If it is desired to plot the load current as abscissas, the field current may be subtracted, either graphically or numerically, its value being determined from the saturation curve ($= ob''/N$) (see page 347).

The method of determining some other point c_1' on the characteristic is indicated in Fig. 302, which shows the upper portion of the saturation curve of Fig. 301 to a larger scale. Assume that the point c_1' corresponds to an armature current $I_a' = I_a/2$. The triangle $b_1e_1c_1$, similar to bec, is constructed, each side being one-half the length of the corresponding side of triangle bec. Triangle $b_1c_1e_1$ is then so placed that side $b_1d_1c_1$ is parallel to the axis of abscissas, e_1 lies on the saturation curve, and b_1 on the field-resistance line. The point c_1' on the characteristic is found as before by extending the line $b_1d_1c_1$ to meet the ordinate at I_a'.

If the current is being increased, ae_1e must be the saturation curve taken with *increasing* values of field current; if the current

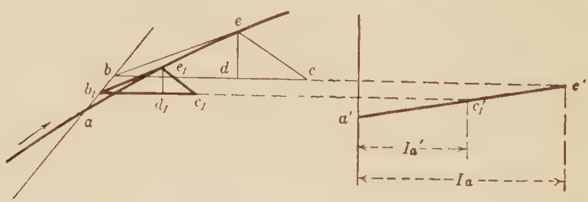


FIG. 302.—Compound characteristic and saturation curve.

is being decreased, ee_1a must be the saturation curve taken with decreasing values of field current.

It is obvious that if the saturation curve at ae_1e has curvature, the characteristic $a'c_1'c'$ will not be a straight line (see Figs. 297 and 298). If I_a is the rated armature current of the machine, and the machine is flat-compounded, points a and b coincide (Figs. 301 and 302).

231. Determination of Series Turns: Armature Characteristic.—It is often desired to determine experimentally the number of series turns which it is necessary to place upon the poles of a shunt generator in order to make it either flat-compounded or to give it any desired degree of compounding.

To make the determination, adjust the no-load voltage to its proper value. Let this value of shunt field current be I_1. Load

the generator to its rated load and by means of the field rheostat bring the terminal volts to the desired value. Let the corresponding value of field current be I_2. The necessary increase of field ampere-turns is

$$(I_2 - I_1)N_{sh}$$

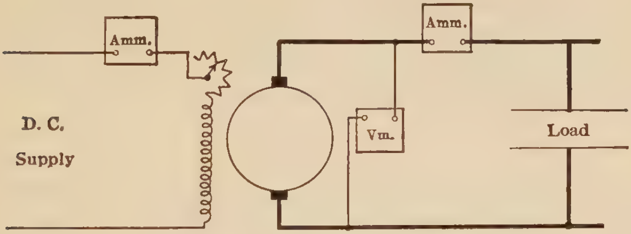


FIG. 303.—Connections for obtaining armature characteristic.

where N_{sh} = shunt-field turns (either turns per pole or total turns may be used).

Let I be the rated-load current of the machine, and N_s the necessary series turns.

Then
$$N_sI = (I_2 - I_1)N_{sh}$$
$$N_s = \frac{(I_2 - I_1)}{I}N_{sh}$$

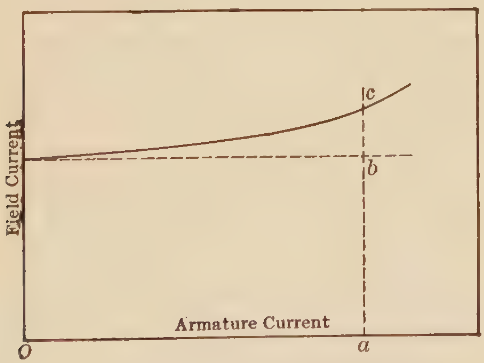


FIG. 304.—Armature characteristic.

The number of series turns for flat-compounding may also be obtained by means of the armature characteristic. The load is applied to the armature in the usual way. It is preferable to excite the field separately, as shown in Fig. 303. Load is applied and the terminal voltage is maintained constant by means of the

shunt-field rheostat. Corresponding values of field current and armature current are noted. When the two are plotted (as shown in Fig. 304) the resulting curve is the *armature characteristic.* The field current increases more rapidly than the armature current owing to saturation.

To determine the number of series turns necessary, multiply the increase of field current bc by the shunt turns and divide by the current Oa.

Series-field turns for flat compounding

$$N_s = N_{sh}\frac{bc}{Oa}$$

where N_{sh} is the number of turns of the shunt field.

232. The Series Generator.—In the series generator the field winding is connected in series with the armature and the external circuit. It must consist necessarily of a comparatively few turns of wire having a sufficiently large cross-section to carry the rated current of the generator.

The *series* generator in most instances is used for *constant current* work, in distinction to the *shunt* generator which maintains *constant potential.* Figure 305 shows the saturation curve of a series generator and also its characteristic. The saturation curve differs in no way from that of the shunt generator. The external characteristic is similar in shape to the saturation curve for low saturation. The voltage at each point is less than that shown by the saturation curve by the amount due to the drop through the armature and field, $I_a(R_a + R_s)$, and the drop due to armature reaction. The curve reaches a maximum beyond which armature reaction becomes so great as to cause the curve to droop sharply and the voltage drops rapidly to zero. These machines are designed to have a very high value of armature reaction.

The machine builds up as follows:

If the series field is connected in such a manner that the current due to the residual magnetism aids this residual magnetism, the generator will build up, provided the external resistance equals or is less than that indicated by the external resistance line Oa. The line Oa is therefore called the *critical* external resistance line. As the external resistance decreases, the external resistance line swings down to the right, as has already been discussed for the shunt generator, Par. 215. The line Ob is such a line. It would

be practically impossible to operate with an external resistance corresponding to the line Oa, or to any line cutting the curve to the left of d, as a small increase in external resistance would swing the resistance line away from the curve resulting in the generator's dropping its load. The machine is designed to operate along the portion bc of the curve, which corresponds to substantially constant current. The current is not affected by a considerable change in external resistance, corresponding to the line Ob swinging up or down. To obtain close regulation the series

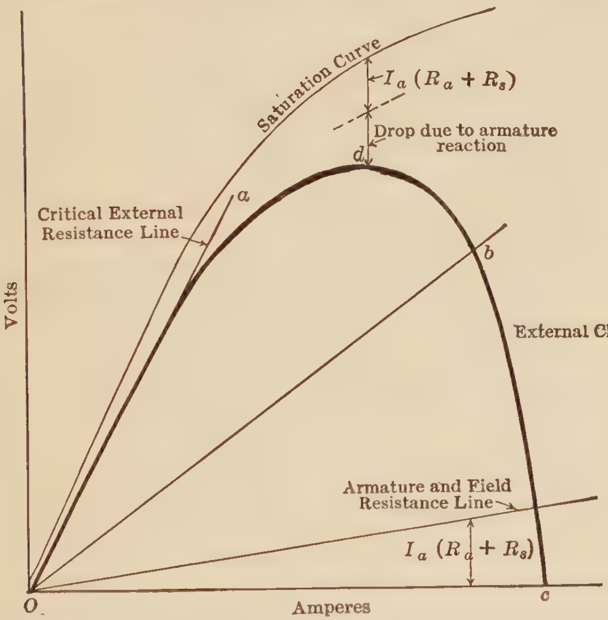


FIG. 305.—Series generator characteristic.

field is shunted by a rheostat. The resistance of this rheostat is controlled by a solenoid connected in series with the line. In this way the current delivered by the generator may be held substantially constant.

In the past, the series generator has been much used in series arc lighting. The Brush Arc machine and the Thomson-Houston generator are common examples of such machines. Both of these have open-coil armatures (see Par. 190). As the voltage on the commutator ranges from 2,000 volts to 3,000 volts, the com-

mutators have wide gaps between segments. In the Brush Arc generator there are as many as two or three separate commutators connected in series so as to reduce the voltage per commutator and also to smooth out the ripples in the voltage wave (see Fig. 214). There are but four segments per commutator.[1]

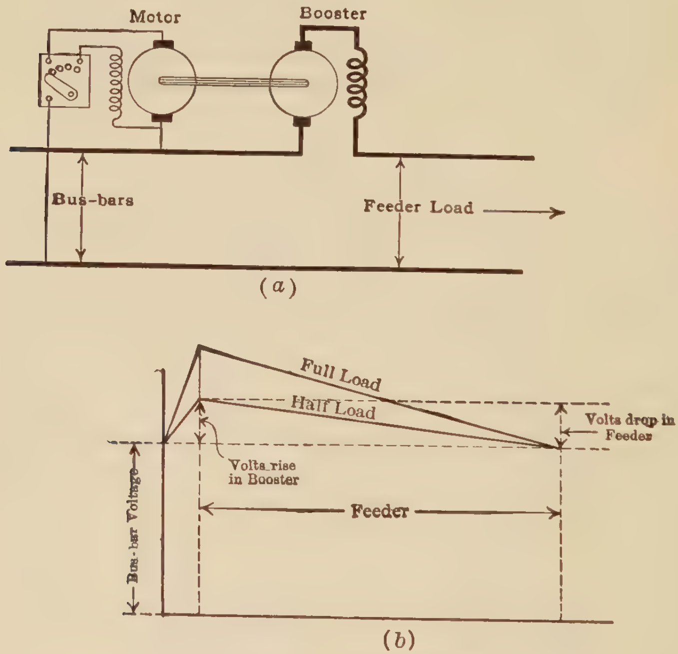


FIG. 306.—The series booster.

In Europe, power is transmitted by direct currents at potentials as high as 50,000 volts, in the Thury system.[2] This high voltage is obtained by connecting several series generators in series and transmitting at constant current. The voltage increases with the load. The generators have two commutators, one at each end of the armature. The potential may run as high as 5,000 volts per commutator. Regulation is obtained by

[1] For a more complete description see "Dynamo Electric Machinery," S. P. THOMPSON, Vol. I.

[2] See "Standard Handbook," 5th Ed., Chap. XI, McGraw-Hill Book Company, Inc.

shunting the fields. The power is utilized by series motors connected at the desired points in series with the line.

Series generators are often used as boosters on direct-current feeders. When a drop on a particular feeder becomes excessive, it may be cheaper to install a booster, and utilize it at the peak load, than to invest in more copper. The booster is a series generator operating on the straight portion of the magnetization curve, the terminal voltage being proportional to the current flowing through the machine. Likewise the voltage-drop in the feeder is proportional to the current in the feeder. If the generator be connected in series with the feeder (Fig. 306 (*a*)) and adjusted properly, its terminal volts may be made always equal to the drop in the feeder, as shown in Fig. 306 (*b*). Therefore, the voltage at the load may be maintained constant. The booster is direct-connected to a shunt motor taking its power from the bus-bars. If the driving power should in any way be removed, the series generator will reverse and operate as a motor. The speed of a series motor without load is practically unlimited, so that it will run away and tear itself to pieces. Therefore, such a booster should never be belt-driven and should have some protective device to prevent its running away.

233. Effect of Variable Speed upon Characteristics.—When a generator is being tested to determine its characteristic or its regulation, it is assumed that the generator speed is maintained at a constant value, the rated speed of the generator. Any drop in voltage resulting from a drop in speed of the prime mover or driving motor is not chargeable to the generator.

In practice, a drop in speed with load in the case of the prime mover is often unavoidable. Therefore, the regulation of the generator is made to include the voltage drop due to this decreased speed. When making out specifications, the regulation of the generator when driven by its prime mover should be specified. Speed correction applied to characteristics of generators is somewhat involved, because of the many factors which enter the computation.[1]

234. The Unipolar or Homopolar Generator.[2]—In the ordinary direct-current generator, the voltage as generated is alternating

[1] For a more complete discussion see "A Solution of an Acceptance Test Problem," by W. B. Kouwenhoven, *Elec. World*, Vol. 71, Jan. 19, 1918.

[2] For more complete discussion see the "Standard Handbook," 5th Ed., Sec. 8, Par. 226.

and the current must be rectified or commutated. In the unipolar generator, however, a direct current is generated, and no commutator is necessary.

The principle of the unipolar generator is that of Faraday's disc dynamo, Fig. 307 (*a*). If a disc be rotated between the poles of a magnet, an e.m.f. is generated between the center and the rim of the disc. A current can be taken from the disc by placing a brush at the center and another at the rim. The disc shown in Fig. 307 (*a*) would not be practicable because the electromotive force is generated only at one portion, so that

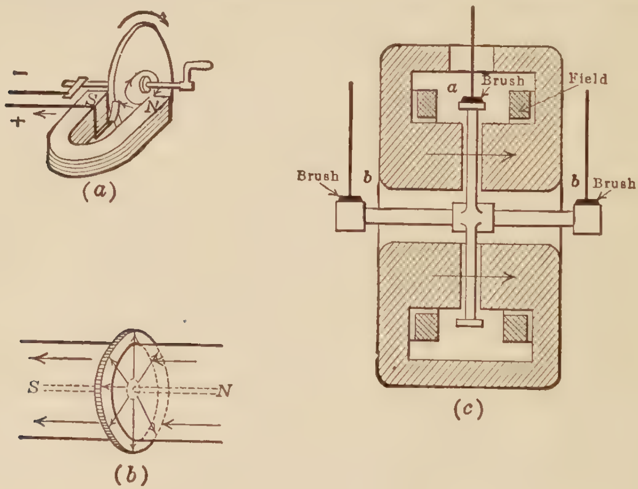


Fig. 307.—The unipolar generator.

current can flow back through the disc even when the external circuit is open. If an annular pole be used (Fig. 307 (*b*)) an equal electromotive force is generated along each radius, so that the current has no return path in the disc itself.

Figure 307 (*c*) shows a cross-section of a unipolar machine. The brushes *bb* are of one polarity and the brush *a* is of the opposite polarity. A hole in the casting allows access to brush *a*. Such generators are sometimes made with a rotating cylinder and are said to be of the axial type.

The chief disadvantage of the unipolar type of generator is the very low voltage generated, even at high speeds. It is necessary

to connect several discs in series in order to obtain working voltages. The generator in Fig. 307 (*c*), having an armature diameter of about 20 in., and running at 3,000 r.p.m., would give only about 40 volts. Another disadvantage is the difficulty of conducting the current from the disc at the high speeds at which these machines are necessarily run.

Such generators are manufactured by both the General Electric Company and the Westinghouse Company. Their field of application is that of a high-speed, turbo-driven generator, designed for high currents at low voltages.

AUTOMATIC VOLTAGE REGULATORS

It has been pointed out that the voltage of a generator varies with the load, speed, etc. By means of an automatic regulator, the voltage of a generator can be maintained constant even under rapid fluctuations of load. In addition, compensation may be made for line drop. These regulators usually operate through the field of the generator or of an exciter to vary the field current with changes in line voltage.

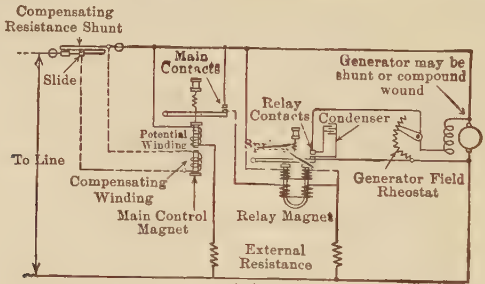
235. The Tirrill Regulator.—In the Tirrill regulator, the voltage is controlled by small relay contacts, which short-circuit the shunt-field rheostat, the duration of the short-circuit depending upon the amount of regulation required. The field rheostat is usually set so that the generator voltage is 35 per cent. below normal when the regulator is disconnected.

The diagram of the apparatus is shown in Fig. 308. The relay magnet is U-shaped and has two solenoids, differentially wound, upon its core. One winding is directly across the line. The other is connected across the line through the main contacts. The relay contacts intermittently short-circuit the generator field rheostat.

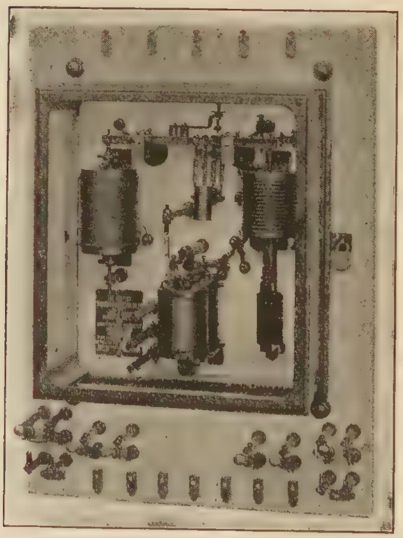
The main control magnet can open the main contacts or allow them to close. These contacts are normally held closed by a spring. Assume that the voltage rises. The potential winding of the main control magnet strengthens this magnet and opens the main contacts. This opens one of the windings on the relay magnet and so nullifies the differential action. The relay contacts are then pulled open and the short-circuit removed from the generator field rheostat. This immediately reduces the

generator voltage. The reverse action takes place when the voltage drops.

As a matter of fact both relays are constantly vibrating so that the changes in the generator voltage are very small.



(*a*)



(*b*)

FIG. 308.—The Tirrill regulator.

The relay contacts are shunted by a condenser to reduce sparking. Owing to the fact that these contacts can carry only a very small current, it is usually necessary to have the regulator act on an exciter field, and so maintain the bus-bar voltage constant through the exciter.

A compensating winding on the main control magnet may be connected across a series shunt to give the system a rising voltage characteristic and so compensate for line drop.

236. Counter Electromotive Force Regulator.—A counter e.m.f. regulator has recently been developed by the General Electric Company. A simplified diagram of connections is given in Fig. 309. The armature of a small counter e.m.f. motor is in series with the generator field. The field of this motor, in series with

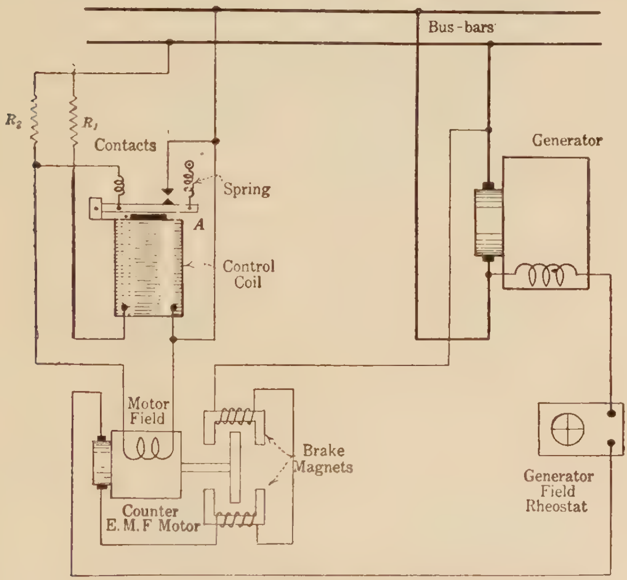


FIG. 309.—Counter e.m.f. regulator.

the resistance R_2, is connected across the bus-bars. This motor field is shunted when two contacts, one on the armature A of the control coil, close. A solenoid or control coil in series with R_1 is also connected across the bus-bars. This solenoid tends to pull down the armature A against the action of the spring and thus open the two contacts. A small eddy-current brake is attached to the shaft of the motor. The disc of this brake rotates between the poles of electromagnets which are excited by the field current of the generator.

Assume that the bus-bar voltage rises. The control coil overcomes the action of the spring and opens the contacts. This opens

the shunt around the field of the motor and increases its counter e.m.f., provided too great a drop in speed does not result. This decreases the field current of the generator and at the same time decreases the excitation of the brake magnets. Accordingly, the bus-bar voltage drops and the weakening of the magnets tends to prevent the speed of the motor decreasing.

A drop in voltage is followed by the closing of the contact, the shunting of the motor field, a decrease in counter e.m.f. of the motor and an increase in the field current of the generator. This strengthens the brake magnets, tending to prevent the speed of the motor rising to offset the decrease in counter e.m.f.

Actually, the contacts vibrate at from 500 to 800 times a minute, depending on the voltage.

CHAPTER XII

THE MOTOR

It was stated in Chap. XI that a generator is a machine for converting mechanical energy into electrical energy.

In a similar way the motor is a machine for converting *electrical* energy into *mechanical* energy. The same machine however, may be used either as a motor or as a generator.

237. Principle of the Motor.—Figure 310 (*a*) shows a magnetic field of uniform strength or intensity in which is placed a conductor that carries no current. In (*b*) the conductor is shown as

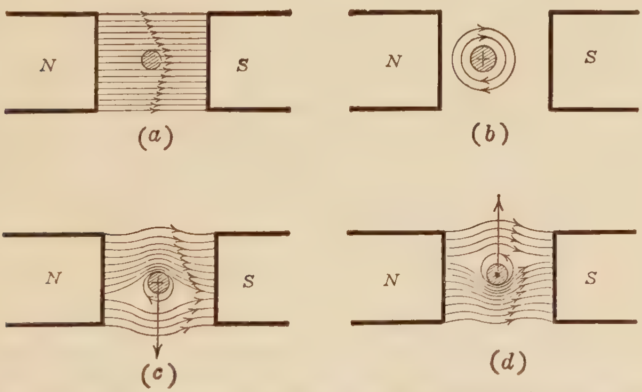


Fig. 310.—Force acting on a conductor carrying current in a magnetic field.

carrying a current into the paper, but the field due to the N and S poles has been removed. A cylindrical magnetic field now exists about the conductor due to the current in it. The direction of this field, which may be determined by the corkscrew rule, is clockwise.

Figure 310 (*c*) shows the resultant field obtained by combining the main field and that due to the current. The field due to the current in the conductor acts in conjunction with the main field

above the conductor, whereas it opposes the main field below the conductor. The result is to crowd the flux in the region directly *above* the conductor and to reduce the flux density in the region directly *below* the conductor.

It will be found that a force acts on the conductor, trying to push the conductor *down*, as shown by the arrow.

It is convenient to think of this phenomenon as being due to the crowding of the lines on one side of the conductor. Magnetic lines of force may be considered as acting like elastic bands under tension. These lines always are endeavoring to contract so as to be of minimum length. The tension in these lines on the upper side of the conductor is tending to pull it down as shown in the figure.

If the current in the conductor be reversed, the crowding of the lines will occur *below* the conductor, which will tend to move it *upward*, as shown in Fig. 310 (*d*).

The operation of the electric motor depends upon the principle illustrated by Fig. 310. A conductor carrying current in a magnetic field tends to move at right angles to the field.

238. Force Developed with Conductor Carrying Current.—The force acting on a conductor carrying a current in a magnetic field is directly proportional to three quantities: the strength of the field, the magnitude of the current, and the length of the conductor lying in the field. The force in *dynes* is given by

$$F = Bl\frac{I}{10} \text{ dynes.} \tag{124}$$

where B is the flux density in lines per square centimeter or gausses, l the active length of the conductor in centimeters, and I the current in amperes. The direction of the field, the conductor, and the direction of the force are all mutually perpendicular to one another.

Example.—A coil consisting of 20 turns lies with its plane parallel to a magnetic field (see Fig. 315), the flux density in the field being 3,000 lines per square centimeter. The axial length of the coil is 8 in. The current per conductor is 30 amp. Determine the force in pounds which acts on each side of the coil (see arrows in Fig. 315 (*a*)).

$$B = 3{,}000,$$
$$l = 8 \times 2.54 = 20.32 \text{ cm.}$$
$$I = 30,$$
$$F_1 = 3{,}000 \times 20.32 \times {}^{30}\!/_{10} = 182{,}900 \text{ dynes.}$$

As there are 20 turns,

$$F = 20 \times 182{,}900 = 3{,}658{,}000 \text{ dynes.}$$

$$\frac{3{,}658{,}000}{981} = 3{,}730 \text{ gm.}$$

$$= 3.73 \text{ kg.}$$

$$3.73 \times 2.204 = 8.22 \text{ lb.} \quad \textit{Ans.}$$

239. Fleming's Left-hand Rule.—The relation among the direction of a magnetic field, the direction of motion of a conductor in that field, and the direction of the *induced* electromotive force is given by Fleming's Right-hand rule (see Par. 189).

In a similar manner, the relation among the direction of a magnetic field, the direction of a current in that field and the direction of the resulting *motion* of the conductor can be determined by using Fleming's Left-hand rule.

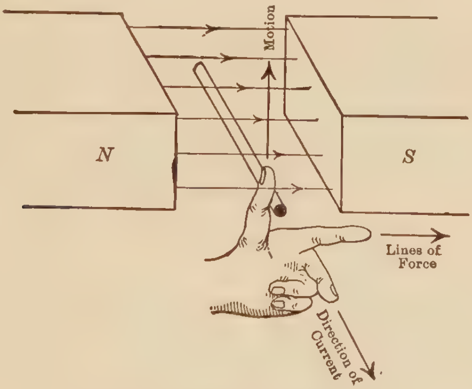


FIG. 311.—Fleming's left-hand rule.

Fleming's left-hand rule:

Point the forefinger in the direction of the field or flux, the middle finger in the direction of the current in the conductor, and the thumb will point in the direction in which the conductor tends to move. This is illustrated by Fig. 311.

Another convenient method for determining the above relation is to make use of the fact that the crowding of the magnetic lines behind the conductor tends to push it along. It is necessary merely to sketch the main field and the lines about the conductor, as shown in Fig. 312 (*a*). It is evident that the lines will be crowded at the right of the conductor so that the direction of motion is to the left.

In Fig. 312 (*b*) is shown a similar condition for a generator. In this case the conductor, as a generator, moves to the right. Hence in a generator the conductor must move *against* a force tending to oppose its motion, and so the conductor requires a driving force to keep it in motion. This driving force is supplied by the prime mover to which the generator is connected.

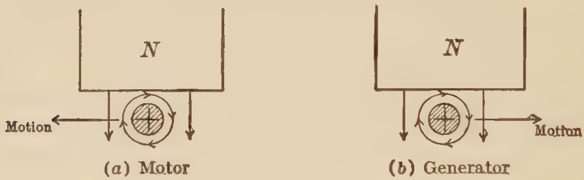


FIG. 312.—Motor and generator action.

240. Torque.—When an armature, a fly wheel, or any other device is revolving about its center, a tangential force is necessary to produce and maintain rotation. This force may be developed within the machine itself as in a motor or steam engine, or it may be applied to a driven device such as a pulley, a shaft, a

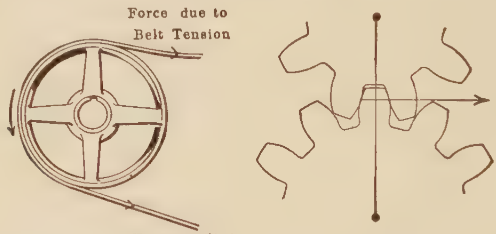


FIG. 313.—Torque developed by a belt and by gears.

generator, the driving gears on the wheels of a street car, etc. (Fig. 313). The total effect of the force is determined not only by its *magnitude* but also by its *arm*, or radial distance from the center of the pulley or gear to the line of action of the force.

The product of this force and its perpendicular distance from the axis is called *torque*.

Torque may also be considered as a mechanical couple tending to produce rotation. It is expressed in units of force and distance.

In the English system, torque is usually expressed in pounds-feet. (This distinguishes it from foot-pounds which represent *work*.)

In the c.g.s system the unit of torque is the dyne-centimeter (a very small unit), and in the metric system the unit is the kilogram-meter.

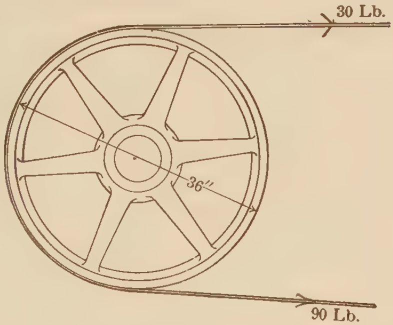


FIG. 314.—Example of torque produced upon a pulley by a belt.

Example.—A belt is driving a 36 in. pulley as shown in Fig. 314. The tension in the tight side of the belt is 90 lb. and that in the loose side is 30 lb. Determine the torque applied to the pulley.

The two sides of the belt are acting in opposition so that the net pull on the rim of the pulley is

$$90 - 30 = 60 \text{ lb.}$$

This force is acting 18 in. or 1.5 ft. from the center of the pulley. Therefore the torque

$$T = 60 \times 1.5 = 90 \text{ lb.-ft.} \quad \textit{Ans.}$$

241. Torque Developed by a Motor.—Figure 315 (*a*) shows a coil of a single turn, whose plane lies parallel to a magnetic field.

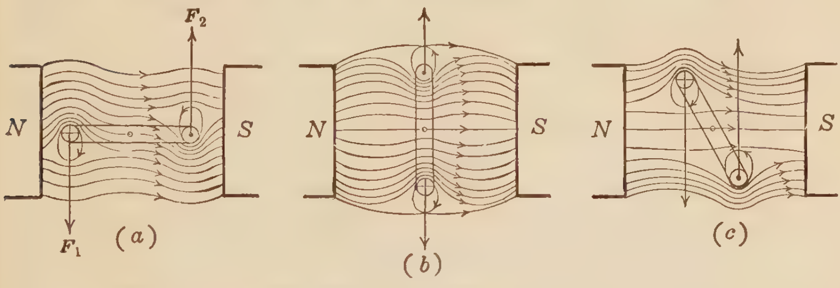


FIG. 315.—Torque developed at different positions of a coil.

Current flows into the paper in the left-hand side of the coil and out of the paper in the right-hand side of the coil. Therefore, the left-hand conductor tends to move downward with a force F_1 and the right-hand conductor tends to move upward with an equal force F_2. These two forces tend to rotate the coil about its axis. Both act to turn it in a counterclockwise direction and so develop a torque. As the current in each of these conductors is

the same and they lie in magnetic fields of the same strength, force $F_1 = F_2$. In (*a*) the coil is in the position of maximum torque because the perpendicular distance from the coil axis to the forces acting is a maximum.

When the coil reaches the position (*b*) neither conductor can move any farther without the coil itself spreading. This is a position of zero torque because the perpendicular distance from the coil axis to the line of action of the forces is zero.

If, however, the current in the coil be reversed when the coil reaches position (*b*) and the coil be carried slightly beyond the dead center, as shown in (*c*), a torque is developed which still tends to turn the coil in the counterclockwise direction.

To develop a continuous torque in a motor, the current in each coil on the armature must be reversed just as it is passing through the neutral plane or plane of zero torque and a commutator is therefore necessary. This is analogous to using a commutator in connection with a generator in order that the current delivered to the external circuit may be unidirectional.

A single-coil motor, like that shown in Fig. 315, would be impracticable as it has dead centers and the torque which it develops is pulsating. A 2-coil armature would eliminate the dead centers, but the torque developed would still be more or less pulsating in character.

The best results are obtained when a large number of coils is used, just as in the armature of a generator. In fact there is no difference in the construction of a motor armature and a generator armature. In Fig. 316 (*a*) an armature and a field are shown for a 2-pole machine and the torque developed by each individual conductor is indicated. Figure 316 (*b*) shows an armature and a field for a 4-pole machine. The direction of the torque developed by each belt of conductors is indicated by the arrow at that belt.

In armatures of this type a very small proportion of the total number of coils is undergoing commutation at any one instant. Therefore, the variation in the number of active conductors is so slight that the torque developed is substantially constant, for constant values of armature current and main flux.

From Eq. 124, the torque developed by any armature can be shown to be

$$T = K_t'ZI\Phi, \qquad (125)$$

where K'_t = a constant of proportionality, involving the diameter of the armature, the parallel paths through the armature, the choice of units, etc.

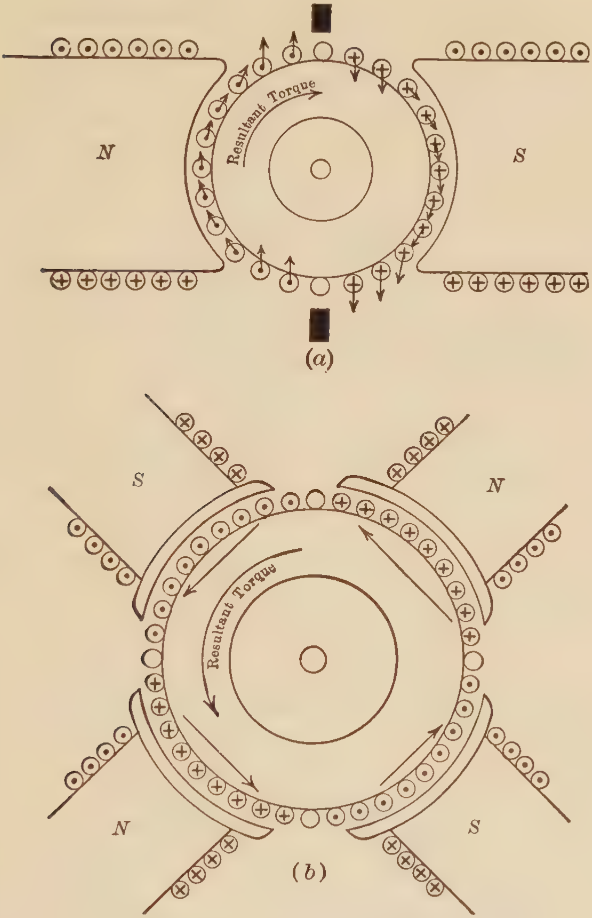


FIG. 316.—Torque developed by belt conductors in motor armatures.

Z = number of conductors on the surface of the armature.

I = current supplied to the armature, in amperes.

Φ = flux from one north pole entering the armature.

For any particular machine Z is a fixed quantity, so that the torque

$$T = K_t I \Phi, \tag{126}$$

where K_t is a new constant of proportionality.

That is, in a given motor, *the torque is proportional to the armature current and to the strength of the magnetic field.*

This is a very important relation to keep in mind, for by its use the variation of torque with load in the various types of motors can be readily determined.

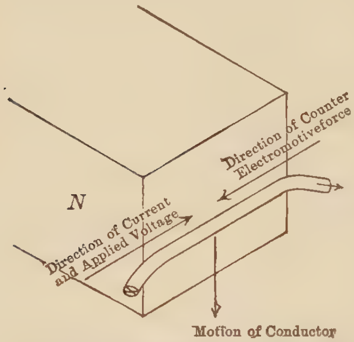


FIG. 317.—Relation of the direction of currents and voltages in a motor conductor.

Example.—When a certain motor is taking 50 amp. from the line it develops 60 lb.-ft. torque. If the field strength is reduced to 75 per cent. of its original value and the current increases to 80 amp., what is the new value of the torque developed?

If the current remained constant the new value of torque, due to the weakening of the field, would be

$$0.75 \times 60 = 45 \text{ lb.-ft.}$$

Due to the increase in the value of the current, however, the final value of torque will be

$$\tfrac{80}{50}\, 45 = 72 \text{ lb.-ft.} \quad \textit{Ans.}$$

It must be remembered that the torque expressed by the above equations is the entire torque or the internal torque developed by the armature. The torque available at the pulley will be slightly less than this, due to the torque lost in overcoming friction and in supplying the iron losses of the armature.

242. Counter Electromotive Force.—The resistance of the armature of the ordinary 10-hp., 110-volt motor is about 0.05 ohm. If this armature were connected directly across 110-volt mains, the current, by Ohm's law, would be

$$I = \frac{110}{0.05} = 2{,}200 \text{ amp.}$$

This value of current is not only excessive but unreasonable, especially when one considers that the rated current of such a motor is in the neighborhood of 90 amp. When a motor is

in operation, the *current through the armature is evidently not determined by its ohmic resistance alone.*

The armature of a motor is in every way similar to that of a generator. The conductors on its surface, in addition to carrying current and so developing torque, are cutting flux. Therefore, they *must* be generating an electromotive force.

Consider Fig. 317 which shows a conductor, on the armature of a motor, moving downwards in front of a north pole. In order to move downwards, the current in this conductor must be inwards or from left to right.

If the right-hand rule be applied to determine the direction of the e.m.f. *induced* in this conductor, due to its downward motion (see Fig. 317), it will be found to be acting from right to left or in opposition to the current.

If the direction of the induced e.m.f. in any conductor on a motor armature be similarly determined, it will be found to act always in *opposition* to the current. That is, it *opposes* the current entering the armature. This induced e.m.f. is called the *counter electromotive force* or *back electromotive force.* As the counter electromotive force opposes the current it must also oppose the line voltage. Therefore, the net electromotive force acting in the armature circuit is the difference of the line voltage and the back electromotive force. Let V equal the line voltage and E the back electromotive force. The net voltage acting in the armature circuit is

$$V - E.$$

The armature current follows Ohm's law and is

$$I_a = \frac{V - E}{R_a} \tag{127}$$

where R_a is the armature resistance.

This equation may be transposed and written

$$E = V - I_a R_a. \tag{128}$$

This should be compared with Eq. (122), page 341, which is the similar equation for a generator.

In a generator the induced e.m.f. is equal to the terminal voltage *plus* the armature resistance drop. In a motor the induced e.m.f. is equal to the terminal voltage *minus* the armature resistance drop. The counter electromotive force must always be less than the terminal or impressed voltage if current is to flow *into* the armature at the positive terminal.

Example.—Determine the back electromotive force of a 10-hp. motor when the terminal voltage is 110 volts and its armature is taking 90 amp. The armature resistance is 0.05 ohm.

$$E = 110 - (90 \times 0.05) = 110 - 4.5 = 105.5 \text{ volts.} \quad \textit{Ans.}$$

An interesting experiment for demonstrating the existence of counter electromotive force is shown in Fig. 318. A lamp bank is connected in series with the armature of a shunt motor. First close switch S_2 which closes the field circuit. Then close S_1. At the instant of closing S_1 the lamps will burn brightly, being practically up to candle power. As the armature speeds up, these lamps will become dimmer and dimmer, showing that the armature is generating a *counter* electromotive force which opposes the line voltage and so results in less voltage for the

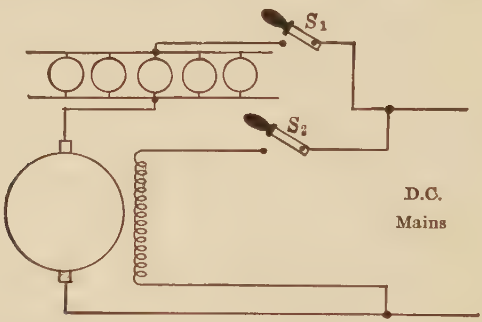


FIG. 318.—Demonstration of counter electromotive force.

lamps. When the armature is up to speed, the lamps will be very dim. If, however, the field switch S_2 now be opened, the flux and, therefore, the counter electromotive force will be immediately reduced to zero practically, which will be shown by the lamps again coming up to full candle-power. (In practice when a motor is in operation, *the field circuit should not be opened under any conditions whatsoever.*)

Equation (118), page 306, for the induced electromotive force in a generator will obviously apply to a motor. That is, the counter electromotive force

$$E = \frac{\phi s P Z}{p 10^8} \text{ volts,}$$

where ϕ is the total flux entering the armature from one north pole, s the speed of the armature in revolutions per second,

P the number of poles, Z the number of conductors on the surface of the armature, and p the parallel paths through the armature.

As Z, P, p, and 10^{-8} are all constant for any given motor, the counter electromotive force becomes

$$E = K_1\phi S,$$

which is identical with Eq. (119), page 307, S being given in r.p.m.

Solving for speed

$$S = K\frac{E}{\phi}, \tag{129}$$

where

$$K = \frac{1}{K_1}.$$

The speed of a motor is directly proportional to the counter electromotive force and inversely proportional to the field.

Substituting for E in Eq. (129) its value given in Eq. (128), the speed becomes

$$S = K\frac{V - I_aR_a}{\phi}. \tag{130}$$

This is a very important equation for it shows the law of speed variation of a motor with changes of load.

Example.—A certain motor has an armature resistance of 0.1 ohm. When connected across 110-volt mains the armature takes 20 amp. and its speed is 1,200 r.p.m. What is its speed when the armature takes 50 amp. from these same mains, with the field increased 10 per cent.?

Applying Eq. (130)

$$\frac{S_2}{S_1} = \frac{K\dfrac{110 - 50 \times 0.1}{\phi_2}}{K\dfrac{110 - 20 \times 0.1}{\phi_1}} = \frac{\dfrac{105}{\phi_2}}{\dfrac{108}{\phi_1}} = \frac{105}{\phi_2}\cdot\frac{\phi_1}{108}.$$

$$S_1 = 1{,}200.$$

Therefore:

$$S_2 = 1{,}200\,\frac{105}{108}\cdot\frac{\phi_1}{\phi_2}.$$

But

$$\phi_2 = 1.10\phi_1.$$

Therefore

$$S_2 = 1{,}200\,\frac{105}{108}\cdot\frac{\phi_1}{1.10\phi_1} = 1{,}060 \text{ r.p.m.} \quad \textit{Ans.}$$

243. Counter Electromotive Force and Mechanical Power.—Let the input to a motor armature be VI_a watts. A portion of this power is lost in heating the armature, and is given by

$$P_a = I_a^2R_a,$$

where R_a is the armature resistance. The remainder of the power VI_a must appear as mechanical power, P_m. This follows from the law of the conservation of energy. That is

$$P_m = VI_a - I_a^2R_a$$
$$= (V - I_aR_a)I_a.$$

But $V - I_aR_a$ is the counter e.m.f. of the motor (Eq. 128).

Hence, *the mechanical power developed within the armature of a motor is equal to the product of the counter e.m.f. and the armature current.* The power available at the pulley is slightly less, since some of this internal power is accounted for by friction, windage, and iron losses or is lost as stray power (see p. 416).

Example.—Determine the mechanical power developed in the armature of the motor given in the example on page 374. The counter e.m.f. is 105.5 volts and the armature current is 90 amp. Hence the internal power,

$$P_m = 105.5 \times 90 = 9{,}495 \text{ watts. } \textit{Ans.}$$
$$= 12.73 \text{ hp. } \textit{Ans.}$$

244. Armature Reaction and Brush Position in a Motor.—Figure 319 (a) shows a motor armature carrying current the directions of the current corresponding to clockwise rotation and to a north pole at the left. Due to the armature ampere-turns, a magnetomotive force F_A is produced in the armature, and the direction of flux produced by this m.m.f. is upwards and at right angles to the polar axis. Figure 319 (b) shows the vectors representing the magnitudes and directions of the armature m.m.f. F_A and the field m.m.f. F. By adding these two m.m.fs. vectorially, the resultant m.m.f. F_o is obtained. The total flux produced by F_o is distorted as shown in Fig. 319 (c). It will be noted that (1) the flux has been crowded into the *leading* pole tips, and (2) the neutral plane perpendicular to the resultant field has moved *backward*. Therefore in a motor it is necessary to move the brushes *backward* with increase of load, whereas in a generator they are moved *forward*. Were it not for the electromotive force of self-induction (see Par. 221), the brush axis would coincide with the neutral plane. Due, however, to the necessity of counteracting this last electromotive force, the

brushes are set behind this load neutral plane, as is shown in Fig. 319 (*c*). That is, in both the motor and the generator it is necessary to set the brushes beyond the load neutral plane in order to counteract this electromotive force of self-induction.

This backward movement of the brushes is accompanied by a demagnetizing action of the armature upon the field, as indicated

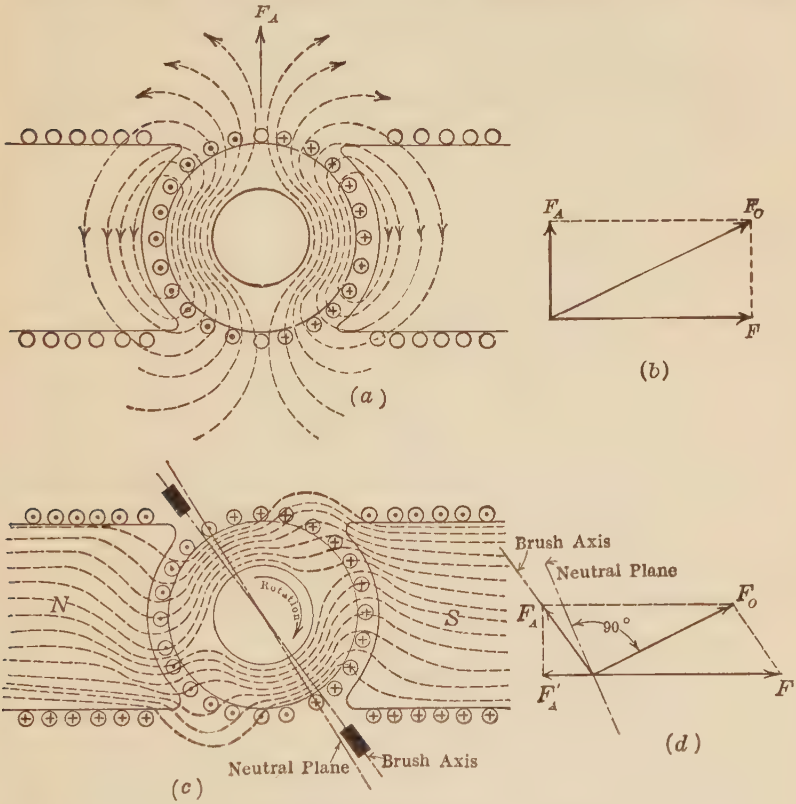


FIG. 319.—Armature reaction in a motor.

in Fig. 319 (*d*), where F'_A is the demagnetizing component of F_A. Therefore, as the load is increased on a motor the armature reaction tends to increase the motor speed. In fact instances have been known where motors with short air-gaps (producing high armature reaction) have run away when the load was applied.

Figure 320 shows the armature conductors of a motor carrying current and passing under successive north and south poles. The directions of the currents correspond to a direction of rotation from left to right. These directions are such that the armature reaction F_A in the first interpolar space is *upward* (see Fig. 267, page 321). Therefore, if a commutating pole is to be used it must be a *north* pole, in order to oppose this magnetomotive force of the armature by tending to send a flux down into the armature. F'_A must likewise be opposed by a south pole. Therefore in a motor, the relation of main poles and commutating poles, in the direction of rotation, is *Nn Ss*, or opposite to the corresponding relation for a generator (see Fig. 285, page 335).

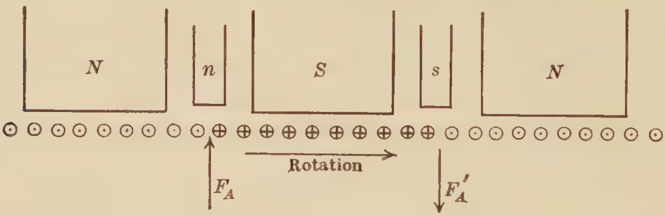


FIG. 320.—Relation of commutating poles to main poles in a motor.

The polarity of the interpoles should be carefully investigated with a compass, if a motor happens to be sparking badly from some unknown cause, as the sparking may be due to their being incorrectly connected.

245. The Shunt Motor.—The shunt motor is connected in the same manner as a shunt generator, that is, its field is connected directly across the line in parallel with the armature.

A field rheostat is usually connected in series with the field. If the *applied* torque to any rotating power-transforming device is increased, the resulting reactions must be such as to cause an increase in the *developed* torque. Otherwise the device will cease to operate.

If load is applied to any motor it immediately tends to slow down. With the shunt motor, the flux remains substantially constant. The decrease of speed, therefore, lowers the back electromotive force.

If the back electromotive force is decreased, more current flows into the armature (see Eq. (127), page 373). This continues

until the increased armature current produces sufficient torque to meet the demands of the increased load.

The suitability of a motor for any particular duty is determined almost entirely by two factors, the variation of its *torque* with load and the variation of its *speed* with load.

In the shunt motor the flux is substantially constant. Therefore, from Eq. (126), the torque will vary almost directly with the armature current. For example, in Fig. 321, when the armature current is 30 amp. the motor develops 40 lb.-ft. torque, and when the current is 60 amp. the motor develops 80 lb.-ft. torque. That is, when the current doubles the torque doubles.

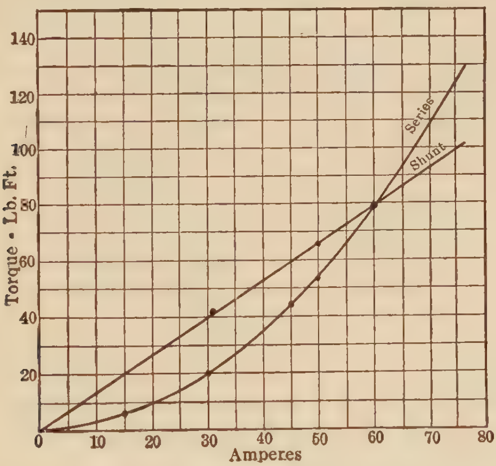


FIG. 321.—Shunt and series motors; torque-current curves.

The speed of a motor varies according to Eq. (130), where

$$S = K\frac{V - I_aR_a}{\phi}.$$

$T \propto I$

In the case of the shunt motor, K, V, R_a, and ϕ are all substantially constant. Therefore, the only variable is I_a. As the load on the motor increases, I_a increases and the numerator of this equation decreases. As a rule the denominator changes only a small amount. The speed of the motor will then drop with increase of load, as shown in Fig. 322. As I_aR_a is ordinarily from 2 to 6 per cent. of V, the percentage drop in speed of the motor is of this order of magnitude. For this reason the shunt

motor is considered a constant speed motor, even though its speed does drop slightly with increase of load.

Owing to armature reaction, ϕ ordinarily decreases slightly with increase of load and this tends to maintain the speed constant. Occasionally the armature reaction is sufficiently great to give a rising speed characteristic with increase of load.

Speed regulation is defined by the A. I. E. E. Standards Rules as follows:

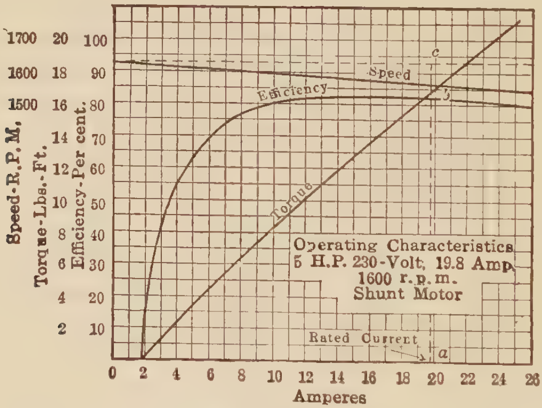


Fig. 322.—Typical shunt motor characteristics.

In constant-speed direct-current motors the regulation is the ratio of the difference between rated-load and no-load speeds to the rated-load speed at the final temperature attained at operation under rated load for the time specified in the rating.

That is, in Fig. 322, the speed regulation is

$$\frac{ca - ba}{ba} = \frac{cb}{ba}$$

Example.—The speed of a shunt motor falls from 1,100 r.p.m. at no load to 1,050 r.p.m. at rated load. What is its speed regulation?

$$\text{Regulation} = \frac{1{,}100 - 1{,}050}{1{,}050} = 0.0476 = 4.76 \text{ per cent.} \quad \textit{Ans.}$$

The speed regulation is a measure of a motor's ability to maintain its speed when load is applied.

Figure 322 shows the three essential characteristics of a shunt motor, the torque, the speed, and the efficiency, each plotted against current. The effect of the machine losses upon the effi-

ciency will be discussed in the next chapter. It will be noted that the shunt motor has a definite no-load speed. Therefore it does not run away when the load is removed, provided the field circuit remains intact.

Shunt motors are used where a substantially constant speed is required, as in machine shop drives, spinning frames, blowers, etc.

There is an erroneous impression that shunt motors have a low starting torque and therefore should not be started under load. Starting boxes are usually designed to allow 125 per cent. of full-load current to flow through the armature on the first notch. Therefore, the motor develops 125 per cent. of full-load torque at starting. By decreasing the starting resistance, the motor could be made to develop 150 per cent. of full-load torque without trouble. Ordinary starting boxes however will overheat under these conditions, if the starting period is too long. Therefore controllers, which are able to carry currents of this magnitude without over-heating are used under these conditions (see Par. 248).

246. The Series Motor.—In the series motor the field is connected in series with the armature, as shown in Fig. 323. The field has comparatively few turns of wire and this wire must be of sufficient cross-section to carry the rated armature current of the motor.

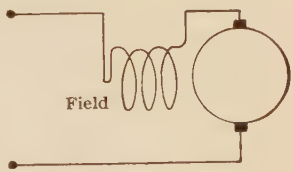


FIG. 323.—Connections of a series motor.

In the series motor the flux, ϕ, depends entirely on the armature current. If the iron of the motor is operated at moderate saturation, the flux will be almost directly proportional to the armature current. Therefore, in the expression for torque,

$$T = K_t I \phi \qquad T \propto I_a^2$$

if ϕ is assumed to be proportional to I, the expression becomes

$$T = K_t' I^2, \tag{131}$$

where K_t' is a constant.

The torque is proportional to the *square* of the armature current, as shown in Fig. 321. When the current is 30 amp. the torque is 20 lb.-ft.; at 60 amp. the torque is 80 lb.-ft. That is, the doubling of the armature current results in the quadrupling

of the torque. It will be noted that as the current increases above 60 amp., the torque rises very rapidly. This characteristic of the series motor makes its use desirable where large increases of torque are desired with moderate increases in current. In practice, saturation and armature reaction both tend to prevent the torque increasing as rapidly as the square of the current.

When Eq. (130) is applied to the series motor, the speed

$$S = K\frac{V - I_a(R_a + R_s)}{\phi} \tag{132}$$

where K is a constant, V the terminal voltage, I_a the motor current, R_a the armature resistance including brushes, R_s the series field resistance, and ϕ the flux entering the armature from a north pole. R_s, the resistance of the series field, is now added to the armature resistance in order to obtain the total motor resistance. Both I_a and ϕ vary with the load.

As the load increases, the voltage drop in the field resistance and the armature resistance increases because this voltage drop is proportional to the current. Therefore, the back e.m.f. becomes less, which tends to decrease the speed, although this effect is only of the magnitude of a few per cent. The flux ϕ, however, increases almost directly with the load. Therefore the speed must drop, in order that the back e.m.f. be of the proper value, which is usually a few per cent. less than the terminal voltage. Both effects tend to slow down the motor. The resistance drop is ordinarily from 2 to 6 per cent. of the terminal voltage V so its effect on the speed is only of this magnitude. The speed is, however, inversely proportional to the flux ϕ and a given percentage change in ϕ produces the same percentage change in the speed.

When the load is decreased, the flux ϕ correspondingly decreases and the armature must speed up in order to develop the required back e.m.f. If the load be removed altogether, ϕ becomes extremely small, resulting in a very high speed. It is dangerous to remove the load from series motors, as their armatures are almost certain to reach speeds where centrifugal action will wreck them.

Figure 324 shows the characteristic curves of a series motor plotted with current as abscissas. The torque curve concaves

upward for the reasons which have just been stated. The speed is practically inversely as the current, that is, at large values of current the speed is low and at small values of current the speed is high. The characteristics cannot be determined for small values of current because the speed becomes dangerously high.

The efficiency increases rapidly at first, reaches a maximum at about half load and then decreases. This is due to the fact that at light loads the friction and iron losses are large as compared with the load. The effect of these becomes less as the load

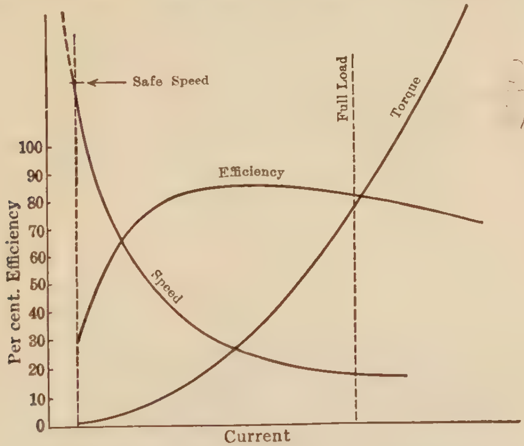


FIG. 324.—Typical series motor characteristics.

increases. The field and armature loss varies as the *square* of the current (I^2R), so that these losses increase rapidly with the load. The maximum efficiency occurs when the friction and iron losses are practically equal to the copper losses. These curves should be carefully compared with the corresponding characteristic curves of the shunt motor (Fig. 322).

Series motors are used for work which demands large starting torque, such as street cars, locomotives, cranes, etc. In addition to the large starting torque, there is another characteristic of series motors which makes them especially desirable for traction purposes. Assume that a shunt motor is used to drive a street car. When the car ascends a grade, the shunt motor maintains the speed of the car at approximately the

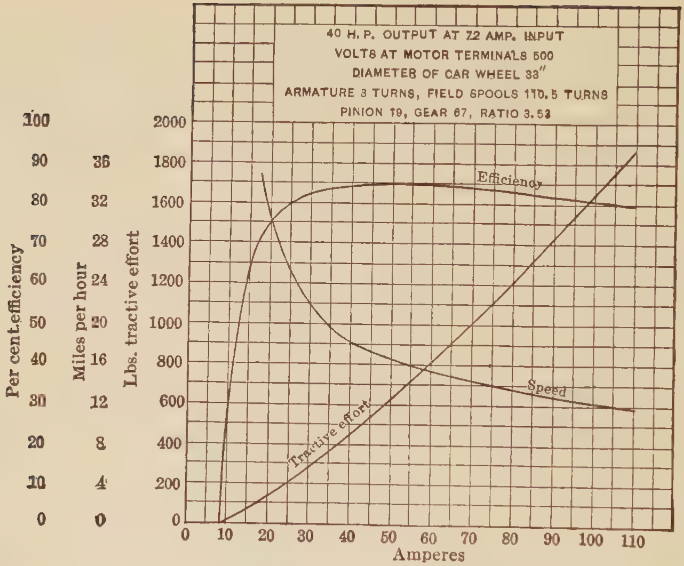


FIG. 325.—Typical railway motor characteristics.

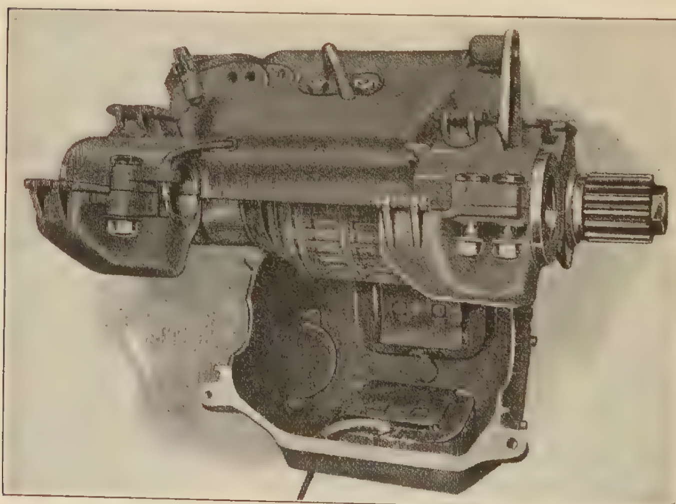


FIG. 326.—Railway motor with frame lowered for inspection.

same value that it has when the car is running on level ground. The motor therefore tends to take an excessive current. A series motor, on the other hand, automatically slows down upon reaching such a grade, because of the increased current. It therefore develops more torque at reduced speed. The drop in speed allows the motor to develop a large torque with but a moderate increase of power. Hence, a series motor could be smaller than a shunt motor operating under the same conditions.

When the characteristics of railway motors are plotted, the curves refer to the output at the track and not at the motor shaft. Figure 325 gives such characteristics for a 500-volt, 40-hp., General Electric railway motor. It will be noted that tractive effort is plotted rather than torque. The speed of the car in miles per hour is given rather than the r.p.m. of the motor armature. These curves differ from the curves of torque and r.p.m. respectively by a constant of proportionality, determined by the gear ratio and by the diameter of the driving wheels. The efficiency curve is also the efficiency at the rails. These curves resemble closely the characteristic curves of Fig. 324. Figure 326 shows a typical railway motor with half of the casing lowered.

247. The Compound Motor.—A shunt motor may have an additional series winding in the same manner as a shunt generator. This winding may be connected so that it aids the shunt winding, in which case the motor is said to be *cumulative compound;* or the series winding may oppose the shunt winding, in which case the motor is said to be *differential compound.*

The characteristics of the cumulative compound motor are a combination of the shunt and series characteristics. As the load is applied, the series turns increase the flux, causing the torque for any given current to be greater than it would be for the simple shunt motor. On the other hand, this increase of flux causes the speed to decrease more rapidly than it does in the shunt motor. These characteristics are shown in Fig. 327. The cumulative compound motor develops a high torque with sudden increase of load. It also has a definite no-load speed, so does not run away when the load is removed.

Its field of application lies principally in driving machines which are subject to sudden applications of heavy load, such as occur in rolling mills, shears, punches, etc. This type of motor

is used also where a large starting torque is desirable but where a straight series motor cannot be conveniently used. Cranes and elevators are representative of such loads. In elevators the series turns are usually short-circuited when the motor reaches speed.

In the *differential* compound motor, the series field opposes the shunt field so that the flux is decreased as the load is applied. This results in the speed remaining substantially constant or even increasing with increase of load. This speed characteristic is obtained with a corresponding decrease in the rate at which the torque increases with load. Such motors are used where a very constant speed is desired. Because of the substantially constant speed of the shunt motor there is little occasion to use the differential motor. In starting a differential compound motor, the series field should be short-circuited, as the large starting current passing through the series field may be sufficient to overbalance the shunt field ampere-turns and to cause the motor to start in the wrong direction. Typical torque and speed curves of the differential compound motor are shown in Fig. 327.

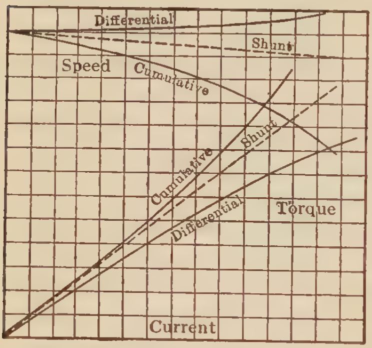


FIG. 327.—Torque and speed characteristics of shunt and compound motors.

To reverse the direction of rotation in any motor, either the armature alone or the field alone must be reversed. If both are reversed the direction of rotation remains unchanged. Therefore, in so far as the direction of rotation of the motor is concerned, it is immaterial which line is positive.

248. Motor Starters.—It was shown in Par. 242 that if a 10-hp., 110-volt motor were connected directly across 110-volt mains, the resulting current would be $\frac{110}{0.05}$ or 2,200 amp. Such a current would not be permissible under commercial conditions. Hence, resistance must be connected in series with the motor *armature* when starting. This resistance may be gradually cut out as the armature comes up to speed and develops a back electromotive force.

Figure 328 shows the use of a simple resistance R for starting a shunt motor. It will be noted that this resistance is in the *armature* circuit and that the field is connected directly across the line and outside the resistance. If the field were connected across the armature terminals, putting the resistance R in series with the whole motor, there would be little or no voltage across the field at starting. There would be little torque developed and difficulty in starting would be experienced.

Figure 329 shows a 3-point starter. This does not differ fundamentally from the connections shown in Fig. 328. One line connects directly to an armature and a field terminal tied together. It makes no connection whatever with the starting box. The other line goes to the line terminal of the starting box which is connected directly to the starting arm. The starting arm moves over contacts set in the slate front of the starting box. These contacts connect with taps distributed along the starting resistance. The armature terminal of the starting box, which is the right-hand end of the starting resistance, is connected to the other armature terminal of the motor. The field connection in the starting box is connected from the first starting contact, through the hold-up magnet, to the field terminal of the box. This field terminal is connected directly to the other terminal of the shunt field.

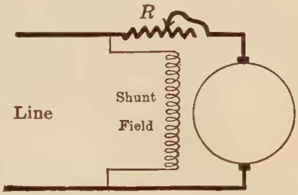
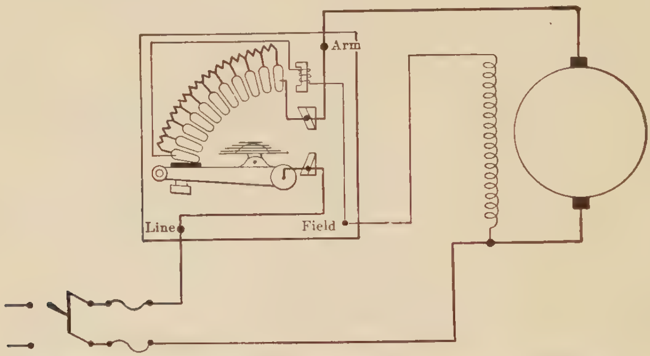


FIG. 328.—Resistance used for starting purposes.

When the starting arm makes connection with the first contact, the field is connected directly across the line and at the same time all the starting resistance is in series with the armature. As this arm is moved, the starting resistance is gradually cut out. When the arm reaches the running position, the starting resistance is all cut out and, to insure good contact, the line and armature conductors frequently are connected directly by a laminated copper brush, shown in Fig. 329. The field current now feeds back through the starting resistance. This resistance is so low, compared with the resistance of the field itself, that it has no material effect upon the value of the field current. A spring tends to pull the starting arm back to the starting position.

When the arm reaches the running position, it is held against the action of this spring by a soft-iron magnet (hold-up magnet), connected in series with the shunt field. (A soft-iron armature is often attached to the starting arm as shown in the figure.) If for any reason the line is without voltage, the starting arm will



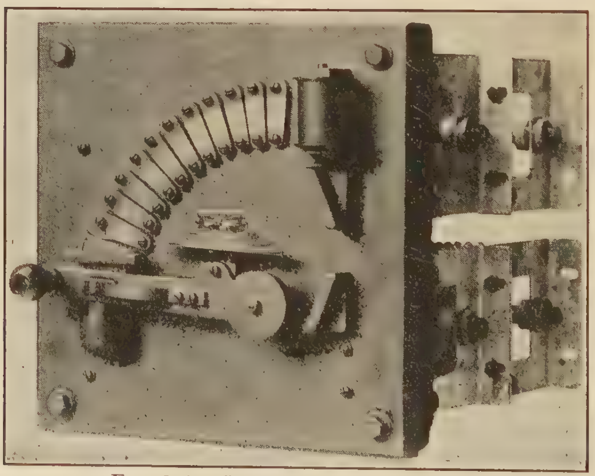


FIG. 329.—Three-point starting box.

spring back to the starting position. Otherwise, if the voltage returned to the line after a temporary shut-down, the stationary motor armature would be thrown directly across the line and a short-circuit would result.

The advantage of connecting the hold-up coil in series with the field is that, should the field circuit become opened, the arm

springs back to the starting position and so prevents the motor running away.

The 3-point starting box cannot be used to advantage on variable speed motors having field control. Such motors frequently have a speed variation of five to one. This results in the field current having approximately this same range. The

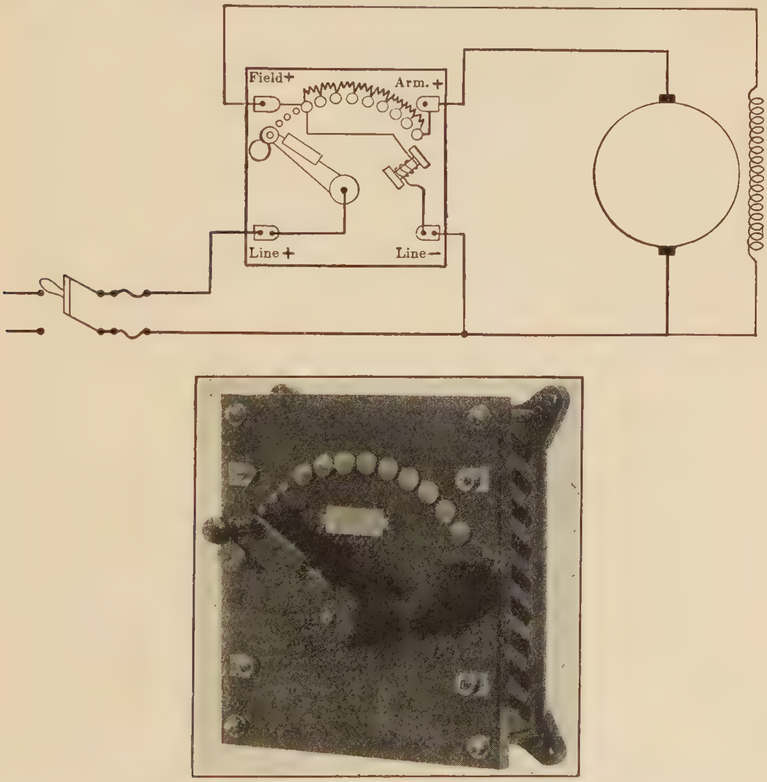
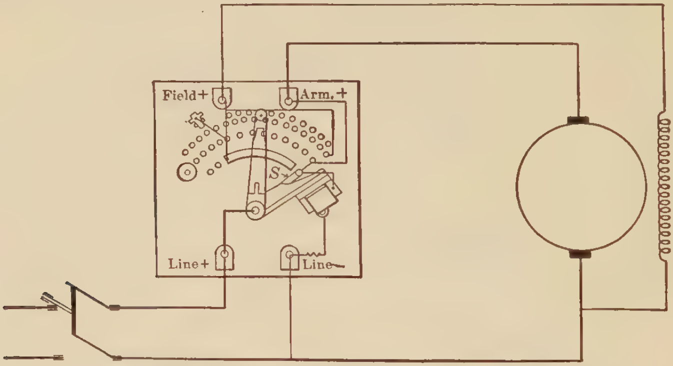


FIG. 330.—Connections for a 4-point starting box.

hold-up magnet may be too strong, therefore, at the higher values of field current and too weak at the lower values. To obviate this difficulty a 4-point box is used (Fig. 330). It is similar to the box shown in Fig. 329, except that the hold-up coil is of high resistance and is connected *directly across the line.* The only difference in the connection is that the "line − terminal" must be connected to the side of the line which runs directly to the com-

mon armature and field terminals. When the voltage leaves the line, the hold-up coil becomes dead and allows the arm to spring back to the starting position.

Sometimes the field resistance is contained within the starting box. The box then has two arms, as shown in Fig. 331. The shorter arm is pushed up by the longer arm and cuts out the



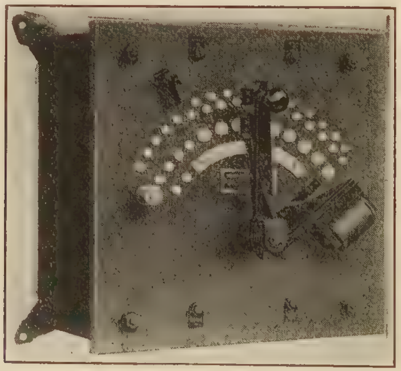
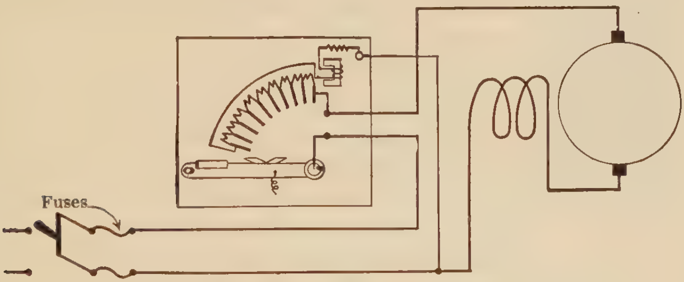


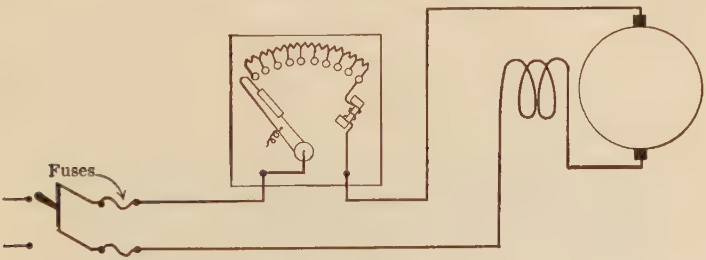
FIG. 331.—Westinghouse starting and speed-adjusting rheostat.

armature resistance in the ordinary manner. During the starting period the field rheostat is short-circuited by the finger S (Fig. 331). When the starting resistance is all cut out, the shorter arm is held by the magnet and the short-circuit of the field resistance is removed by this arm pushing S to the right. The longer arm, which has no spring, inserts resistance into the field circuit when moved backward. When the voltage goes off, the shorter arm springs back carrying the longer one with it.

In stopping a motor, the line switch should always be opened rather than throwing back the starting arm. With shunt motors, the line switch can be opened with no appreciable arc, since the motor develops a back electromotive force nearly equal to the line voltage and the net voltage across the switch contacts is small. The electromagnetic energy stored in the field does not appear at the switch but is discharged gradually through the armature. On the other hand, if the starting arm is thrown back, the field circuit is broken at the last contact button. Owing to



(*a*) Series starter, no-voltage release



(*b*) Series starter, no-load release

FIG. 332.—Series motor starters.

the inductive nature of the field, this results in a hot arc which burns the contact. To prevent the contact from being burned, a small finger breaks the arc (Fig. 331).

The series motor starter needs no shunt-field connection. There are two principal types, one having a no-voltage release, shown in Fig. 332 (*a*), and one having a no-load release, shown in Fig. 332 (*b*). In the former type, the hold-up coil is connected directly across the line and releases the arm when the voltage goes off the line. In the latter type, the hold-up coil consists of a few turns in series with the motor. When the motor current

falls below the desired value, the starting arm is released. This last type is particularly adapted to series motors where there is a possibility of the load dropping to such a low value that the motor speed may become dangerous.

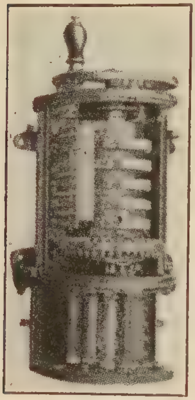


FIG. 333.—Controller, showing drum, armature reversing contacts, field regulating points and field resistors.

Controllers of the type shown in Fig. 333 are used where the operation of the motor is continually under the direct control of an operator, as in street car, crane, and elevator motors. The controller must be more rugged than the starting box, since the controller is used for constant starting, stopping, and reversing the motor while operating. Such controllers usually have an external resistance which is cut in and out by fingers in the controller. A shunt motor field rheostat may also be incorporated in the controller. Controllers are usually fitted with a "reverse," so that the motor may be run in either direction. The controller shown in Fig. 333 contains both of these features.

Automatic starters are often used in practice. They have many advantages over the hand-operated starter. They cut out the starting resistance at a definite rate,

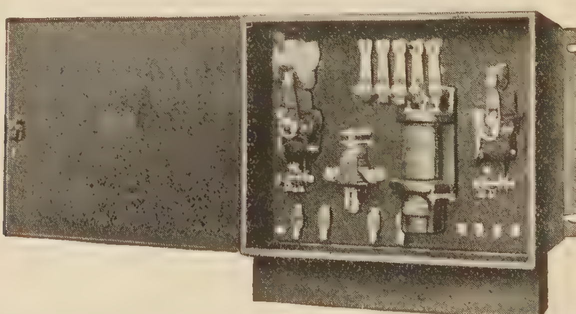


FIG. 334.—Cutler-Hammer automatic starter with door open, showing overload relay mounting.

so that the blowing of fuses and the opening of circuit breakers, due to too rapid acceleration, are avoided. In many installations where a motor is used intermittently, it may be started and stop-

ped by merely turning a snap switch. Employees will be more likely to shut the motor down when the power is not being used, because of the ease with which starting and stopping are effected. In the larger sizes of motors, especially when extremely rapid operation is necessary as in rolling mills, automatic starters alone can give satisfactory results.

Figure 334 shows a Cutler-Hammer starter, having contactor fingers which close in sequence, cutting out the starting resistance. The accelerating movement is controlled by an oil-filled dash-pot. A motor may be brought up to speed in 30 sec. without the current exceeding 150 per cent. of its rated value. The starter is put into operation by pushing a starting button. The starter is provided with low-voltage release, which shuts down the motor when the voltage falls below a safe value or when it fails altogether. The starter is also provided with a circuit breaker for overload protection, and time-limit protection can also be provided. When large motors are to be started, these contactors may be connected as relays to operate large solenoid-operated contactors, which control the starting resistance of the larger motors.

By using 3- and 4-way switches, this type of controller may be operated from widely separated points. Instead of a simple snap switch, the motor may be controlled by a float switch, a pressure switch, or any other automatically-operated switch.

Figure 335 shows a simple and ingenious type of starter of the contactor type. The contactors themselves operate as follows: Figure 335 (*a*) and (*b*) represent a rectangular iron frame *FF* and plunger *P*. The plunger *P* is narrower at the bottom than at the top and the narrow part of it fits loosely in an opening in the bottom of the frame *FF*. There are two air-gaps *DD* between the plunger *P* and the bottom of the frame *FF*, and one air-gap *U* between the plunger *P* and the top of the frame *FF*. A coil is placed around the plunger *P*, as shown in the figure, where the black circles represent the cross-sections of the wires of the coil *CC*. If a heavy current flows through the coil, magnetic lines will stream through the plunger *P* across the air-gap *U*, back through the frame *FF*, and through the narrow part of the plunger *P*, and also across the air-gaps *DD* (Fig. 335 (*a*)). The reason that some of the lines go through the air-gaps *DD* is

that the narrow part of the plunger P is saturated, or, in other words, it cannot easily carry any more magnetic lines. These lines, therefore, are forced to pass through the air-gaps DD when a large current flows through the coil. The magnetic lines in the air-gap U cause an upward pull on the plunger, but the weight of the plunger and the downward pull of the magnetic lines in the air-gaps DD hold the plunger down. In Fig. 335 (b) conditions are the same as in Fig. 335 (a) except that less current flows through the coil CC, with the result that there are not so many lines existing through the plunger P, the air-gap U, and the frame

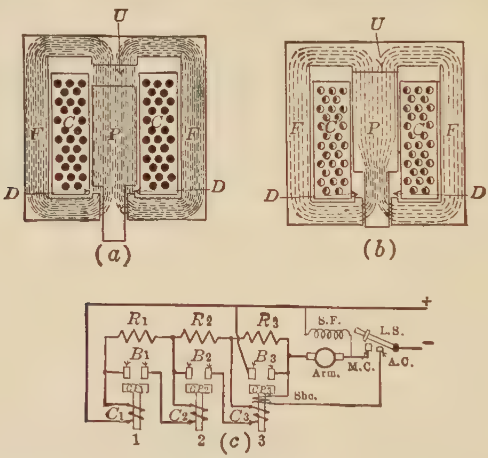


FIG. 335.—Electric Controller and Mfg. Co. automatic starter.

FF. Most of these lines now pass through the narrow part of the plunger, but there are still a few in the air-gaps DD. The downward pull, due to the lines passing through the gap DD is now small and the pull in the gap U is sufficient to raise the plunger.

The operation of the starter is shown in Fig. 335 (c). When the line switch is closed, the current flows from the positive main through the coil C_1 of contactor 1, the resistances R_1, R_2, and R_3 in series and the motor armature to the negative main. A shunt coil, Shc, on contact CC_3 is also put across the line but it is not sufficiently strong to raise the plunger of 3.

When the current falls to a sufficiently low value, the plunger $CP1$ rises, as already described, closing the contact points $B1$,

which short-circuits R_1. This causes an increase of current which now passes through the coil C_2. When the current drops again, due to the motor coming up to speed, contactor $CP2$ operates, short-circuiting R_2 and causing the current to feed through C_3. When the current drops again, C_3 operates and short-circuits all the resistances and coils so that the plungers of 1 and 2 fall back. 3 is held up by the shunt coil *Shc*.

249. Magnetic Blow-outs.—Controllers and circuit breakers are often equipped with magnetic blow-outs. Their function is to extinguish the arc, resulting from opening a circuit, so that

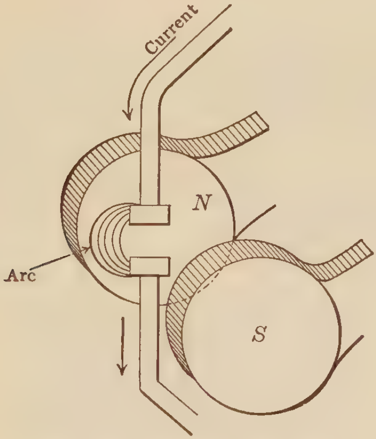


Fig. 336.—Magnetic blow-out.

the arc does not persist and so burn the contacts. The principle of blow-outs is as follows: The contacts between which the arc is to be broken are placed between the poles of a magnet, as shown in Fig. 336. When the contacts open, the current tends to persist in the form of an arc. This arc finds itself in a magnetic field so that motor action immediately follows. The arc starts to move across the field according to Fleming's left-hand rule. In doing so it draws itself out to such an extent that it is broken.

250. Resistance Units.—Starting boxes are usually designed for starting duty only. They can carry the starting current of the motor safely for the short period of starting, but they cannot carry such a current continuously. The box resistance units are usually of the type shown in Fig. 337. In the smaller types

the wire is wound in the form of a helix. It may be self supporting or it may be wound on asbestos or porcelain forms, as shown in Fig. 337. In the larger types, cast-iron grids are used. These grids are bolted together. Current lugs are clamped on at suitable points so that the desired ranges of resistance are readily obtainable.

Some types of starter are built in the form of controllers. The resistance, usually of the grid type, is designed to carry the rated current of the motor continuously so that it may be used to secure speed control.

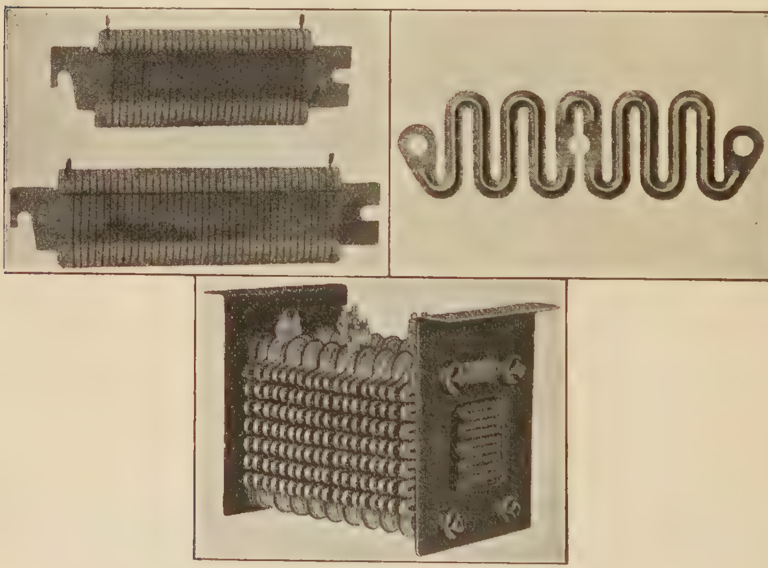


FIG. 337.—Starting box resistance units.

251. Speed Control.—In the equation for motor speed, $S = KE/\phi$, there are but two factors that can be changed to secure speed control without making changes in the motor construction. These factors are the back electromotive force E and the flux ϕ.

Armature Resistance Control.—In this method, the speed control is obtained by connecting a resistance directly in series with the motor *armature*, keeping the field across the full line potential, as shown in Fig. 338 (a). Because of the voltage-drop in this resistance, the back e.m.f. E is changed. A wide range of speed

can be obtained by this method and at the same time the motor will develop any desired torque over its working range, for the *torque* depends only upon the *flux* and armature *current.*

The principal objections to this method of speed control are that an excessive amount of power is lost in the armature series resistance and the speed *regulation* is very poor. In Fig. 338 (*b*) there is shown for comparison the speed-load curves of a shunt motor with and without resistance in series with the

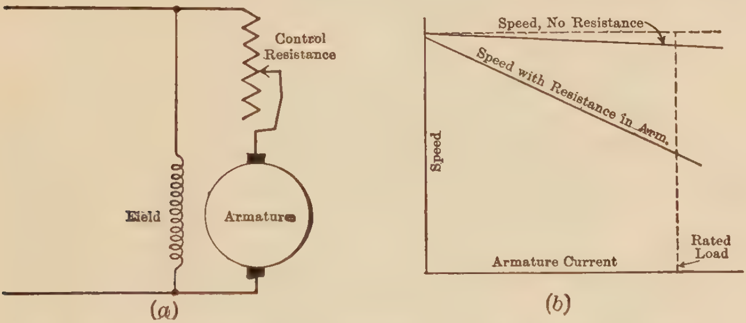


FIG. 338.—Speed control and regulation—armature resistance method.

armature. The speed-load curve with series armature resistance shows that half speed is obtained at rated load. It will be observed that the speed at no load rises to a value which is practically equal to the speed of the motor when there is no series armature resistance. In this case the speed regulation with resistance is about 100 per cent. and about 50 per cent. of the power supplied to the armature is lost in the series resistance. Without series resistance the speed regulation is the usual 3 or 4 per cent.

Example.—A 220-volt, 7-hp. motor has an armature resistance of 0.25 ohm. When running without load at 1,200 r.p.m. the armature takes 6 amp. (*a*) What resistance should be connected in series with the armature to reduce the speed of the motor to 600 r.p.m. at its rated load of 30 amp.? (*b*) How much power is lost in the resistance? (*c*) What percentage of the power delivered to the armature circuit is delivered at the armature terminals? (*d*) What is the speed regulation of the armature? Neglect armature reaction.

(*a*) $$E_1 \text{ (at no load)} = 220 - (6 \times 0.25) = 218.5 \text{ volts.}$$

$$E_2 \text{ (at 600 r.p.m.)} = \frac{600}{1{,}200}\,218.5 = 109.3 \text{ volts.}$$

$$\text{Total } (R + R_a) = \frac{220 - 109.3}{30} = \frac{110.7}{30} = 3.69 \text{ ohms.}$$

Subtracting the armature resistance,

$$R = 3.69 - 0.25 = 3.44 \text{ ohms.} \quad \textit{Ans.}$$

(b) Power lost in the series resistance

$$P_1 = (30)^2 \times 3.44 = 3{,}096 \text{ watts.} \quad \textit{Ans.}$$

(c) Power delivered to armature circuit

$$P_2 = 220 \times 30 = 6{,}600 \text{ watts.}$$

Power delivered to armature

$$P_3 = 6{,}600 - 3{,}096 = 3{,}504 \text{ watts.}$$

Percentage power delivered to armature

$$= \frac{3{,}504}{6{,}600} = 53.1 \text{ per cent.} \quad \textit{Ans.}$$

(d) Speed regulation

$$\frac{1{,}200 - 600}{600} = 100 \text{ per cent.} \quad \textit{Ans.}$$

Multivoltage System.—In this system several different voltages are available at the armature terminals of the motor.

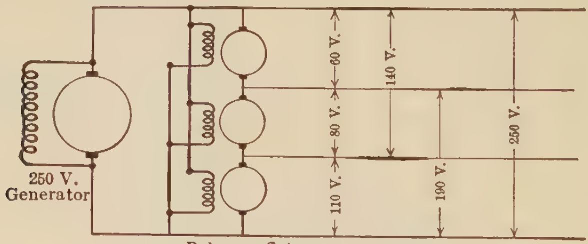


Fig. 339.—Multivoltage speed control.

These voltages are often supplied by a balancer set, Fig. 339. The shunt field of the motor is connected permanently across a fixed voltage and, with the 4-wire system shown, six voltages are available for the armature. Intermediate speed adjustments can be made with a limited field control. Owing to the necessity of having a balancer set, or its equivalent, and due to the large number of wires necessary, this system is little used in this country for ordinary power drive. It is used extensively, however, to give the starting voltages for direct-current elevators, where a large group of elevators is involved.

Ward Leonard System.—In this system, shown in Fig. 340, variable motor voltage is obtained by means of a separate generator, G, driven by a motor, M_1. By varying the field of the generator, the desired voltage across the motor terminals, M_2, is obtained. The motor field is connected across the supply

mains in parallel with the fields of the other two machines. In Fig. 340, M_1 is a motor driving generator G. G in turn supplies variable voltage to the armature of motor M_2 whose speed is to be varied. This system is very flexible and gives close adjustment of speed. The chief disadvantages are the necessity of having the two extra machines and the low over-all efficiency of the system, especially at light loads. This system has been

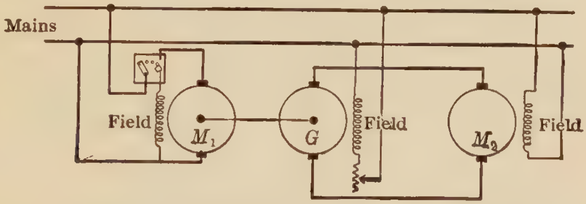


Fig. 340.—Ward Leonard system of speed control.

used extensively for turning the turrets of battleships, but is now superseded for this purpose.

Field Control.—In the foregoing methods of speed control, the armature volts have been varied. A change of speed may also be obtained by varying the flux, ϕ, by means of a field rheostat. This method is very efficient so far as power is concerned and for any particular speed adjustment the speed regulation

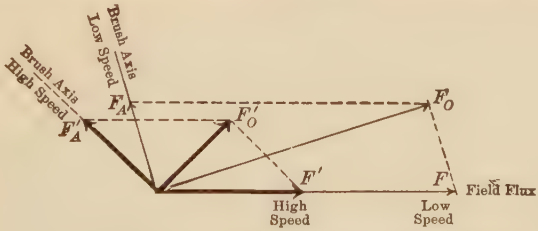


Fig. 341.—Effect of a weak field upon brush position.

from no load to full load is excellent. The range of speed obtainable by this method with the ordinary motor is limited by commutation difficulties. Referring to Fig. 341, F is the field flux at low speed and F_A is the corresponding armature flux. The resultant flux, neglecting saturation, is F_o. If it be attempted to double the speed of the motor by weakening its field, the new field flux will be F'. It will now be necessary to move the brushes farther backward so that the armature flux will be at the position shown at F'_A. The resultant field is F'_o.

It is evident that the neutral plane has been moved backward to a considerable extent and that the armature flux is about equal to the field flux. In addition to severe sparking at the commutator, the strong armature field may so weaken the main field that the motor tends to run away. In order to eliminate the demagnetizing action due to the moving of the brushes, commutating-pole motors only should be used where the speed range is large. A range of 5 to 1 in speed variation is obtainable with properly designed machines having commutating poles.

The Stow Motor.—In this type of motor, shown in Fig. 342, the field cores slide in and out of the yoke and are actuated by a hand wheel through a rod and bevel gear mechanism. By varying the length of the air-gap, the flux, and therefore the speed of the motor, may be varied. As the reluctance to the armature flux is increased at the higher speeds, with the increased air-gaps, there is little difficulty with commutation. In other words, the ratio of field ampere-turns to armature ampere-turns does not change. This type of motor is little used.

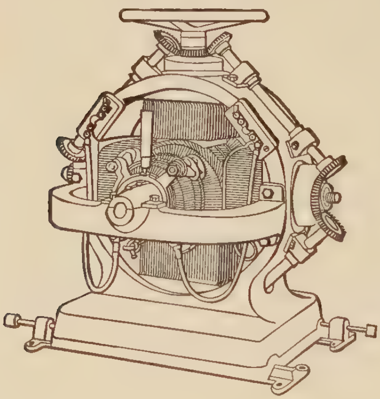
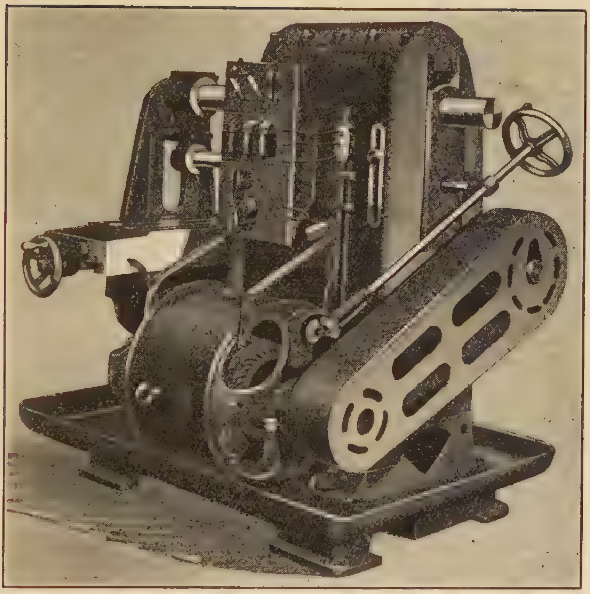


FIG. 342.—The Stow motor.

The Lincoln Motor.—In the Lincoln motor, made by the Reliance Electric and Engineering Company, the flux entering the armature is varied by moving the rotating armature in and out of the field structure, as shown in Fig. 343. As the armature is moved out of the field the length of armature conductor cutting flux is reduced. Therefore the armature must rotate faster in order to develop the requisite electromotive force. This gives a finely graduated speed control over wide ranges, ratios as high as 10 to 1 being obtained. These motors are provided with commutating poles.

252. Railway Motor Control.—In a 2-motor trolley car, two different speeds can be efficiently obtained. The motors are first connected in series through a starting resistance R as shown in Fig. 344 (a). This resistance is gradually cut out by the con-



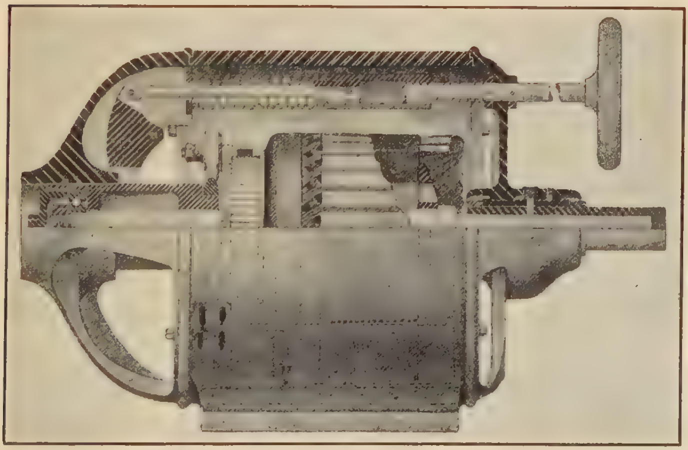


FIG. 343.—Lincoln adjustable speed motor.

troller as the car comes up to speed and then each motor receives one-half the line voltage. This is the first running position. For any given value of armature current each motor will run at half its rated speed. As there is no external resistance in the circuit, the motors are operating at an efficiency very nearly equal to that obtainable with full-line voltage across the terminals of each.

When it is desired to increase the speed of the car, the two motors are connected in parallel with each other and in series with a portion of the resistance R. This resistance is gradually cut out and when the running position is reached, each motor receives full-line voltage, as shown in Fig. 344 (*b*).

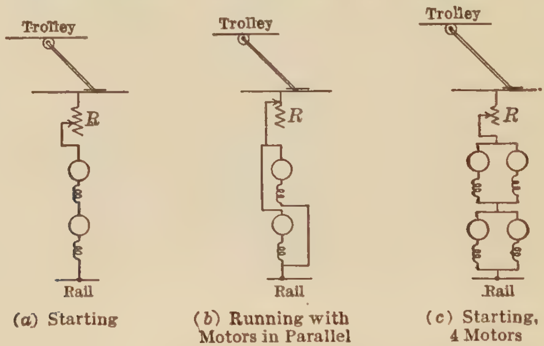


FIG. 344.—Series-parallel control of series motors.

In a 4-motor car, the motors are usually divided into two groups, each group consisting of two motors which are always in parallel with each other. In starting, these two groups are connected in series, each group taking the place of the single motor of a 2-motor car. This starting condition is shown in Fig. 344 (*c*). When the full-speed running position is reached, both groups are connected in parallel across the line. Each motor then receives full-line voltage.

Multiple-unit Control.—In the heavier electric cars and locomotives, the currents become so large that direct platform control is out of the question from the standpoint of the size of controller, safety, and expense. Moreover, when cars are operated in trains, it is necessary that the motors on all the cars shall be under a single control and that they shall operate simultaneously.

In the multiple-unit system, all the heavy-current switching is done by solenoid-operated contactors located beneath the car.

These contactors in turn are operated by an auxiliary circuit called the train line, which runs the entire length of the train (Fig. 345). The train line is made continuous through plug and socket connectors located in the car couplers. The wires of this train line receive their power through the master controller operated by the motorman. As this train line current is only of the magnitude of 2.5 amp., a small platform controller can be used. Another distinct advantage of this system is that the rate of

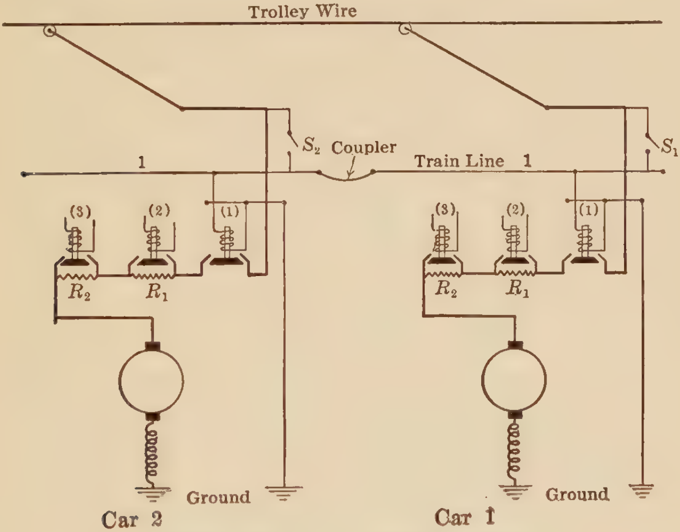


FIG. 345.—Principle of multiple-unit control.

cutting out the starting resistance during the acceleration periods is outside the control of the motorman, being accomplished by automatically-operated contactors which close in sequence at the proper times. This insures uniform acceleration and eliminates the opening of the car circuit breakers and the shocks to the equipment caused by too rapid acceleration when manual operation is used.

Figure 345 shows the underlying principle of the system, no attempt being made to give the many details which must necessarily accompany such a system. Each car has its own trolley or third-rail shoe for collecting the current. A train line of small wires runs the entire length of the train, the connections being

made by the use of couplers between cars. This line usually consists of six wires. Solenoids, operating contactors, are connected across the train lines. Some of the contactors are operated directly by the controller in the hands of the motorman and others operate automatically after the controller has been turned to the desired position. For example, in Fig. 345 are shown two motors, one in each car. One line of the train line is shown running between cars and connected by the coupler. It is assumed that the train is to be operated from car 1. If the switch S_1 in the controller of car 1 be closed, train line 1-1 becomes alive. This energizes relay (1) (1) in each car and both relays simultaneously close the motor circuits, the starting resistances R_1, R_2 being in series with each motor respectively. As the motors "pick up," the current drops and relays (2), (2), become automatically energized and some of the starting resistance R_1, R_1 is cut out in each car. The next set of relays become energized in a similar manner, until all the starting resistance is cut out and the motors are across the line.

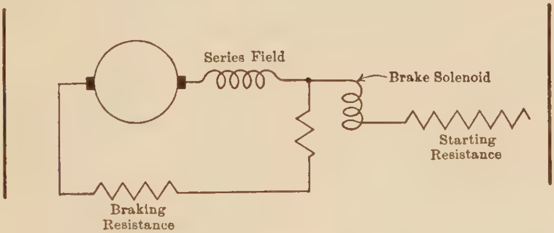
The above is merely an abbreviated description of the system. In the complete system there are six train lines, some of which reverse, change from series to parallel, etc. The great advantage of this system is that every motor on the train can be operated from either controller on any one car, that all the motors act simultaneously, the acceleration cannot exceed a certain value irrespective of the motorman, and as there are driving wheels on every car, high accelerations can be obtained. This system is also used extensively on single cars.

253. Dynamic Braking.—It is often desirable to brake a motor when it is being driven by its load, as in the case of descending elevators, cranes, etc. This is often done by using a controller which leaves the field connected across the line and at the same time puts a resistance load across the armature terminals. This produces generator action and therefore retards the armature. If series motors are used, their fields must be connected across the line in series with a resistance. Such braking is not effective for completely stopping the motor armature, as the braking action ceases when the armature is stationary.

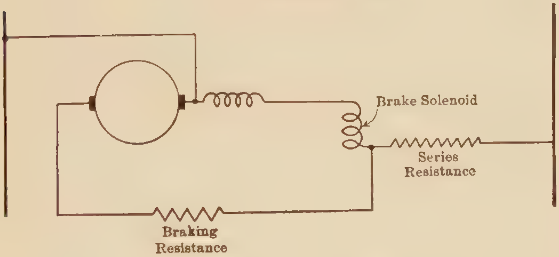
Dynamic braking for a series motor is shown in Fig. 346. In (*a*), which shows the holding or "off" position, the motor is

totally disconnected from the line. The solenoid of the mechanical brake becomes de-energized, resulting in the brake being set (see Fig. 34, page 25). In (*b*), the brake solenoid and the series field are connected across the line in series with a resistance. The armature has a resistance connected across its terminals through the brake solenoid and series field on one side. The brake is released, the armature acts as a generator sending current through the braking resistance and so is retarded.

Regenerative braking is based on this same principle, except that the power is returned to the line rather than wasted in resist-



(*a*) "Off" or Holding Position



(*b*) Braking or Lowering

FIG. 346.—Dynamic braking.

ance. Such a system is used on the electric locomotives of the Chicago, Milwaukee and St. Paul Railroad.

254. Motor Testing—Prony Brake.—It is often necessary to determine the efficiency of a motor at certain definite loads and frequently over its entire range of operation. A knowledge of the efficiency may be necessary, as in the case of an acceptance test; further, the motor may be used as a power-measuring device for determining the power taken by some machine, such as a generator, pump, blower, etc. Knowing the motor input, which can be measured with an ammeter and a voltmeter, and also

knowing the motor efficiency, the output for any given input can be computed. This output will be the power delivered to the generator, the pump, etc.

The most common method of making direct measurements of efficiency in motors up to about 50 hp. is to use a prony brake. Such brakes are made in various forms. One typical form is shown in Fig. 347. It consists of a wooden arm of the proper length, a canvas brake band and a hand wheel for applying ten-

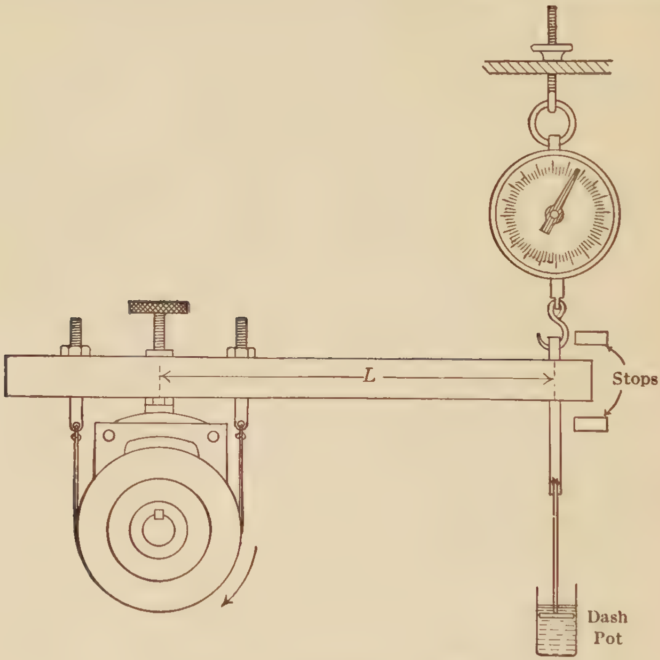


Fig. 347.—Typical prony brake.

sion to the brake band. By means of this hand wheel the motor load can be controlled. An oil dash-pot is advisable, to prevent vibrations of the brake arm.

The balance measures the pull on the arm due to the rotation of the drum, plus the dead weight of the arm. By multiplying the net balance reading by the distance L, the torque of the motor can be determined.

There are two simple methods for determining the dead weight or tare of the brake arm. The brake band is loosened and some

sort of knife edge, such as a pencil, is placed between the top of the drum and the brake carriage. This acts as a substantially frictionless fulcrum, so that the balance registers the dead weight of the arm alone. Another and easier way is to turn the drum toward the balance by hand, stop and read the balance. In this case the friction of the brake causes the balance to read too high. If this operation be repeated by rotating the drum in the opposite direction, the balance reading will be too low, due to the same friction. The average of these two balance readings will give very nearly the correct value for the dead weight of the arm.

Brakes of this type are cooled ordinarily by pouring water into the hollow brake drum. This water prevents the drum from becoming excessively hot. As the maximum temperature which water can reach in the open air is 100° C., the drum temperature cannot much exceed this. The heat developed in the drum is utilized in converting the water into steam. As a considerable number of heat units are required to convert a small amount of water into steam, a moderate amount of water will keep the drum comparatively cool.

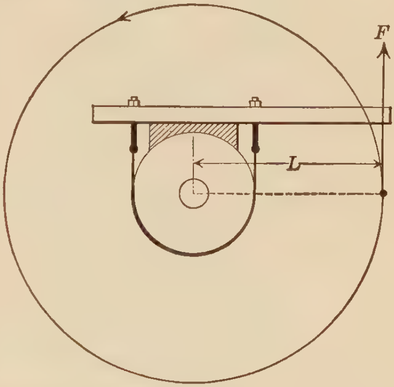


FIG. 348.—Work developed by a prony brake.

To determine the equation for the horsepower absorbed by such a brake, consider Fig. 348. Let F be the net force in pounds acting at a perpendicular distance L feet from the center of the drum. First assume that the drum is stationary and that the arm is pulled around the drum by means of the force F. The distance per revolution through which the force F acts is $2\pi L$. The work done in one revolution of this arm around the drum is the force times the distance $= F(2\pi L)$.

The work done in S revolutions $= F(2\pi L)S$.

If S is the revolutions per minute, the horsepower

$$\text{Hp.} = \frac{2\pi(FL)S}{33{,}000},$$

but FL is the torque, T, therefore

$$\text{Hp.} = \frac{2\pi TS}{33{,}000}.$$

$$\frac{2\pi}{33{,}000} = 0.00019.$$

Therefore $$\text{Hp.} = 0.00019\ TS. \tag{133}$$

Obviously, the same amount of work is done on the brake surface whether the drum is stationary and the arm rotates or the arm is stationary and the drum rotates. Therefore, equation (133) applies to brakes of the type shown in Figs. 347 and 348. It will be noted that in this particular type of brake the horsepower is independent of the diameter of the drum.

Example.—In a brake test of a shunt motor, the ammeter and voltmeter measuring the input read 34 amp., 220 volts. The speed of the motor is found to be 910 r.p.m. and the balance on a 2-ft. brake arm reads 26.2 lb. The dead weight of the arm is found to be +2.4 lb. (*a*) What is the output of the motor? (*b*) What is its efficiency at this particular load?

(*a*) Net reading of balance = 26.2 − 2.4 = 23.8 lb.
The torque T = 23.8 × 2 = 47.6 lb.-ft.
Hp. output = 0.00019 × 47.6 × 910 = 8.23 hp. *Ans.*

(*b*) Output = 8.23 × 746 = 6,140 watts.
Input = 220 × 34 = 7,480 watts.

$$\text{Efficiency, } \eta = \frac{6{,}140}{7{,}480}\,100 = 82.1 \text{ per cent.} \quad \textit{Ans.}$$

In brakes of this type, the brake arm should be kept approximately level.

Another simple type of brake is the rope brake shown in Fig. 349. A rope is given a turn and a half around a drum and the two free ends are each held by a spring balance. The larger balance is on the end of the rope which is being pulled downward by the rotation of the drum. Let F_1 be the reading of the larger balance and F_2 that of the smaller balance. As F_1 and F_2 pull in opposite directions with respect to the rotation of the drum, the net pull at the drum periphery is $F_1 - F_2$.

The torque in pound-feet is

$$T = (F_1 - F_2)R$$

where R is the radius of the pulley in *feet*.

Example.—In a rope brake of the type shown in Fig. 349, F_1 = 32.4 lb. and F_2 = 8.2 lb. The drum is 10 in. in diameter. If the motor speed is 1,400 r.p.m., what horsepower does the motor develop?

The torque

$$T = (32.4 - 8.2)\ {}^5\!/\!_{12} = 24.2 \times {}^5\!/\!_{12} = 10.08 \text{ lb.}$$

The horsepower

$$\text{Hp.} = 0.00019 \times 10.08 \times 1{,}400 = 2.68. \quad \textit{Ans.}$$

255. Measurement of Speed.—The measurement of the speed of machines is as a rule much simpler than the measurement of torque. The most common method is to use a simple revolution counter having a conical rubber tip which fits into the counter-

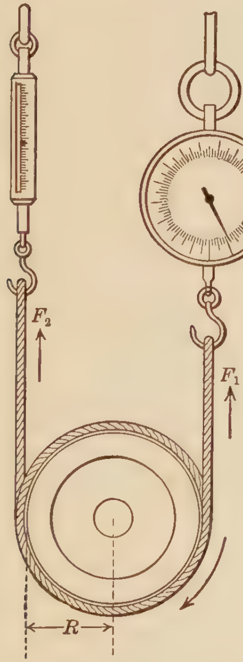


FIG. 349.—Rope brake.

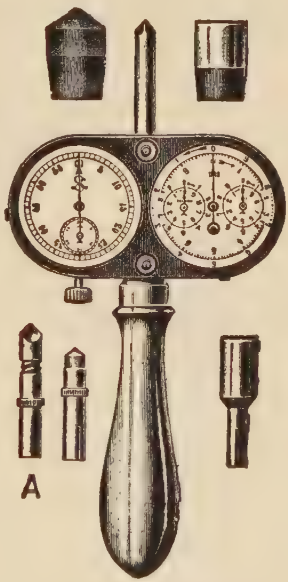


FIG. 350.—Jagabi tachoscope.

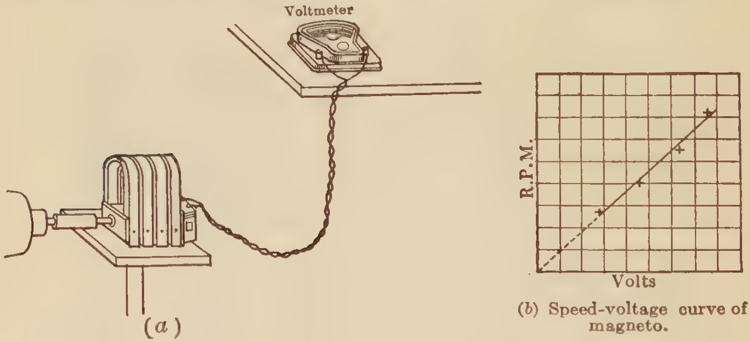
sink of the shaft. The Veeder type is a convenient form of revolution counter. The revolutions are recorded directly on the counter. As this counter cannot be set to zero, the actual speed must be found by subtracting the counter reading before from that after the measurement.

The Jagabi tachoscope, Fig. 350, is a combination of speed counter and stop watch. The spindle may be inserted in the counter-sink of the shaft without recording. A little pressure,

however, causes the counter and stop watch to start simultaneously. They also stop simultaneously when the pressure on the tachoscope is removed. Measurements made with this type of instrument are free from personal error.

Tachometers indicate the instantaneous value of speed. There are mechanical tachometers, where the indicator is actuated by centrifugal action. This type should be carefully checked at each occasion of use, as it is especially subject to error after having been in service for some time.

A simple and convenient type of tachometer is the combination of a direct-current magneto and a voltmeter, as shown in Fig. 351 (*a*). In the magneto the flux is produced by permanent magnets and so is constant. Therefore, the voltage induced in



(*b*) Speed-voltage curve of magneto.

Fig. 351.—Speed measurement with magneto and voltmeter.

the magneto armature is directly proportional to the speed. If this voltage be measured with a voltmeter, the voltmeter reading multiplied by a constant gives the speed directly. The relation of speed to volts may be plotted as shown in Fig. 351 (*b*) and the speed read directly from the plot. This plot is ordinarily a straight line through the origin, which makes one point accurately determined. It is convenient to attach the magneto to the shaft of the machine whose speed is being measured, by a piece of rubber tubing. It is usually necessary to thread a small stud into the end of the shaft whose speed is to be measured, as shown in Fig. 351 (*a*).

256. The Dynamotor.—The A. I. E. E. standards give the following definition of a dynamotor:

A dynamotor is a transforming device combining both motor and generator action in one magnetic field, either with two armatures, or with one armature having two separate windings and independent commutators.

The two windings of the dynamotor armature, when the machine has but a single armature, may or may not lie in separate slots. The machine is ordinarily used to convert direct-current power from one voltage to another. One winding develops motor action and causes rotation; the other develops generator action and delivers electrical energy. Since the motor and generator currents are in opposite directions, the net armature ampere-turns are small, being just sufficient to overcome the torque due to rotational losses. Hence, there is but slight armature reaction.

As both windings cut the same magnetic field at the same speed, their induced voltages must be directly proportional to their turns. The voltage ratio cannot be changed by changing the field excitation. If, for example, the field is strengthened, the motor conductors as well as the generator conductors are cutting more magnetic lines. This, however, will slow down the armature (motor principle) and the induced voltage will remain unchanged. If the field is weakened, the speed will increase and again the induced volts do not change. The ratio between terminal volts is affected to some extent by the armature resistance drops.

The dynamotor is also used as both starter and generator in one type of automobile electric system.

CHAPTER XIII

LOSSES; EFFICIENCY; OPERATION

257. Dynamo Losses.—A certain portion of the energy delivered to any motor or generator is lost within the machine itself, being converted into heat, and therefore wasted. This represents not only energy lost, but has the further objection that it heats the machine and so limits its output. If the energy loss in the machine becomes excessive, the resulting temperature rise may injure the insulation by carbonizing it.

As a motor and a generator are similar, they have the same types of losses throughout. Therefore, the following applies to either a motor or a generator.

COPPER LOSSES

Armature.—The armature windings have a certain resistance and when current flows through them a certain amount of

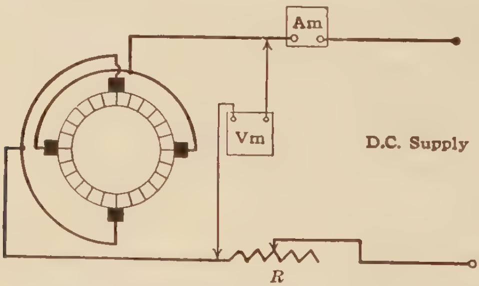


Fig. 352.—Measurement of armature resistance.

power must be lost. In addition to the loss in the armature copper, there is an electrical loss in the brushes and in the commutator. Let this total power loss be P_a. Then,

$$P_a = I_a^2 R_a \qquad (134)$$

where I_a is the armature current and R_a is the armature resistance measured between the terminals of the machine and includ-

ing, therefore, the brushes and their contact resistance. This contact resistance is not exactly constant, but little error is made in assuming it to be so (see Par. 222). The resistance measurement is often made by the connections shown in Fig. 352. The resistance R is inserted to limit the current flowing through the stationary armature (see Par. 127). The measurement should be made with the armature in three or four different positions in order to obtain an average value of resistance. As the low-reading scale of the voltmeter is ordinarily used in making this measurement, the instrument may be injured on opening the circuit by the rise of voltage due to the self-inductance of the armature. Therefore, the voltmeter should be disconnected when the circuit is being opened or closed and when the armature is being turned.

Shunt Field.—The field takes a current I_f at the terminal voltage V of the generator or motor. Therefore, the power lost in the field is

$$P_f = VI_f. \qquad (135)$$

This includes the power lost in the field rheostat as this is chargeable to the field circuit.

Series Field.—The series-field loss is

$$P_s = I_s^2R_s \qquad (136)$$

where I_s is the series-field current, which may or may not be equal to the armature current, depending on whether the machine is long or short shunt.

R_s is the series-field resistance. If a series-field shunt or diverter is used, R_s is the equivalent parallel resistance of this diverter and the series field and I_s is the current of the series field plus that of the diverter.

The losses in the commutating-pole circuit are determined in the same way as are those of the series field.

The foregoing losses are all copper losses and can be either measured directly or calculated with a high degree of precision from instrument readings.

IRON LOSSES

Eddy Currents.—As the armature iron rotates in the same magnetic field as the copper conductors, voltages are also induced in this iron. As the iron is a good conductor of electricity and

the current paths are short and of large cross-section, large currents would be set up in the armature iron were it a solid mass as shown in Fig. 353 (*a*). These currents represent an excessive power loss which could not be tolerated in a commercial machine. By laminating the armature iron in the manner indicated in Fig. 353 (*b*), the paths of these currents are broken up and their magnitude is reduced to a very low value. Laminating does not

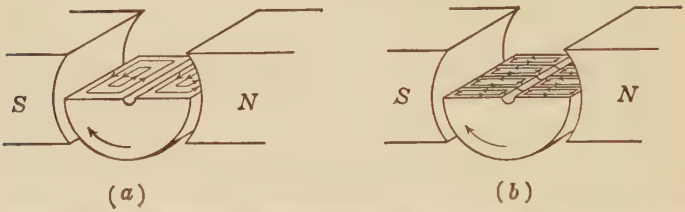


FIG. 353.—Eddy currents in armature iron without and with laminations.

entirely eliminate these eddy-current losses, but it does reduce them to a small value. It will be noted that although the laminations break up the eddy-current paths, they do not interpose reluctance in the magnetic circuit, since they are parallel to the direction of the magnetic flux.

These eddy currents are proportional to both the speed and the flux. As the loss varies as the square of the current (I^2R), the eddy current *loss varies* as the *square* of both the speed and the flux.

Example.—The eddy-current loss in a certain machine is 600 watts when the total flux is 2,000,000 lines per pole and the speed is 800 r.p.m. What is the loss when the flux is increased to 2,500,000 lines and the speed increased to 1,200 r.p.m.?

$$P_e = 600 \times \left(\frac{2{,}500{,}000}{2{,}000{,}000}\right)^2 \times \left(\frac{1{,}200}{800}\right)^2 = 2{,}100 \text{ watts.} \quad \textit{Ans.}$$

Hysteresis.—It was shown in Chap. VIII that when iron is carried through a cycle of magnetization (Par. 158) there results an energy loss proportional to the area of the hysteresis loop. The iron in an armature undergoes a similar cyclic change of magnetization when the armature rotates. Consider the small section of the armature iron at (*a*) (Fig. 354) when it happens to be under a north pole. This small section has a north and a south pole at its ends. When the section reaches position (*b*) its poles have

become reversed, as shown. Obviously, nearly all the armature iron is continually going through similar cycles of magnetic reversals. Therefore, there results a hysteresis loss in the armature iron as the armature rotates. This loss is directly proportional to the speed and is proportional to the 1.6 power of the maximum flux density by the Steinmetz formula (Eq. 72, p. 207). Laminating the iron does *not* affect the hysteresis loss.

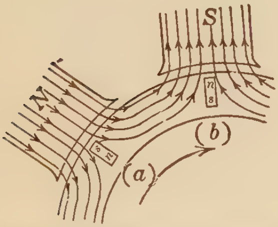


Fig. 354.—Reversal of magnetic flux in armature iron.

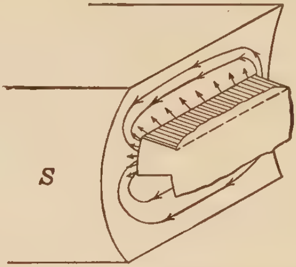


Fig. 355.—Pole-face loss due to tufts of flux from teeth.

Pole-face Loss.—The flux enters and leaves the armature in tufts through the teeth as has already been shown (p. 30, Fig. 40, Chap. II). As these tufts of flux pass across the pole face, they produce flux pulsations in the pole face. These pulsations set up eddy currents in the pole face, as shown in Fig. 355. This results in a power loss. A hysteresis loss also accompanies these flux pulsations. These combined losses are some function of the flux and of the speed. They are reduced, being in part due to eddy currents, by laminating the pole faces (see Fig. 244).

FRICTION LOSSES

These losses consist of bearing friction, brush friction, and windage, and all are functions of the speed.

SUMMARY

The foregoing losses may be summarized as follows:

Copper losses:

Armature $I_a{}^2R_a$.
Shunt field VI_f.
Series field $I_s{}^2R_s$.

$$\text{Stray power}\begin{cases}\text{Iron losses (armature and pole face):}\\ \qquad\text{Eddy current—function of flux and speed.}\\ \qquad\text{Hysteresis—function of flux and speed.}\\ \text{Friction losses (bearings, brushes, windage)—}\\ \qquad\text{function of speed.}\end{cases}$$

The copper losses can be accurately measured or can be calculated. The iron and friction losses can neither be so accurately calculated nor so readily measured as separate losses. Moreover, since they are all some function of the flux, or speed, or both, these losses are combined and are called *stray losses;* the power that they represent being called *stray power*.

As stray power is a function of the speed and the flux only, it will be constant in a given machine provided the flux and the speed be kept constant. Therefore, no matter what the load is, the stray power does not change unless either the flux or the speed changes.

In distinction to the copper losses the stray power is all supplied *mechanically*. For example, in a motor, a mechanical torque is required to supply these losses, making the torque available at the pulley less than that developed by the armature. In a generator these losses are supplied by the prime mover and not by the generator itself. On the other hand, the electrical losses are supplied by the generator itself.

258. Efficiency.—The efficiency of a machine is the ratio of output to input. Thus:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

This may also be written in either of the following ways:

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} \qquad (137)$$

$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}} \qquad (138)$$

Therefore, if the losses in a machine be known, the efficiency may be found for any given input or output.

Example.—A shunt motor takes 40 amp. at 220 volts. The total motor losses are 1,800 watts. What is the motor efficiency?

Using Eq. (138)

$$\text{Efficiency} = \frac{(220 \times 40) - 1{,}800}{220 \times 40} = 79.6 \text{ per cent.} \quad \textit{Ans.}$$

As electrical units, rather than mechanical quantities, are ordinarily used in efficiency determinations, Eq. (137) is used for generators (output is electrical) and Eq. (138) for motors (input is electrical).

259. Efficiencies of Motors and Generators.—The efficiency of electrical apparatus is high as a rule.

The following table gives the approximate weights and efficiencies of a few typical motors. The efficiencies of generators, having corresponding ratings, are practically the same.

Horsepower	Speed, revolutions per minute	Weight of motor alone, pounds	Efficiencies, per cent.		
			Load		
			½	¾	Full
½	1,140	54	64	71	74
¾	1,725	54	66	71	73
¾	1,140	82	70	73	76
1	1,725	80	68	74	76
2	1,725	170	71	77	79
3	1,725	180	73	78	80
5	1,750	260	74	80	83
10	1,150	500	82	85	86
25	1,150	890	84	86	87
50	850	1,640	85	88	89
100	850	2,890	87	89	90
200	850	4,410	89	91	92

The efficiency of a motor may be determined from simultaneous measurements of its input and its output as was shown in Par. 254, where a prony brake was used.

Theoretically, the efficiency of a generator may be determined in a similar manner by measurements of its input and output. The output is readily measured with an ammeter and a voltmeter. The input, however, is very difficult to measure. The difficulty lies in the measurement of the torque transmitted to the generator. Torsion dynamometers have been devised but they are unsatisfactory as a rule. The generator may be suspended in a "cradle," as shown in Fig. 356. The ends of the generator shaft are supported in bearings, so that the frame is free to turn. The torque is determined by measuring the torque necessary to prevent the frame's turning. Such a cradle is

expensive, is not readily adaptable to all generators, and necessitates the generator shaft's protruding beyond both generator bearings.

In any direct measurement of efficiency any percentage error in the measurement of either output or input introduces the same percentage error into the efficiency.

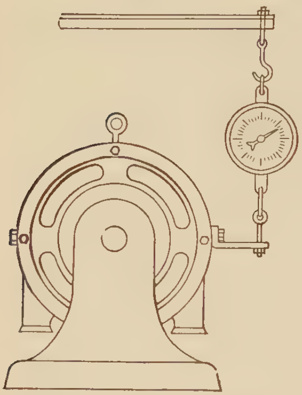


FIG. 356.—Cradle dynamometer.

In the direct measurement of efficiency the power necessary for the test must be equal to the rating of the machine. In addition to *supplying* this power there must be means for *absorbing* it. This is not a serious matter with small machines, but when large machines are tested, supplying and absorbing the necessary power may be difficult, if not quite impossible.

Because of the foregoing reasons, it is often desirable and even necessary to obtain the efficiency by determining the losses.

Example.—A 250-kw. 230-volt compound generator is delivering 800 amp. at 230 volts. The field current is 20 amp. The armature resistance is 0.005 ohm and the series-field resistance is 0.002 ohm. The stray power at this load is 2,500 watts. The generator is connected long shunt. What is the generator efficiency at this load?

Output = 230 × 800 = 184,000 watts.

Shunt-field loss	$= 230 \times 20$	=	4,600 watts.
Armature loss	$= 820^2 \times 0.005$	=	3,360 watts.
Series-field loss	$= 820^2 \times 0.002$	=	1,340 watts.
Stray power		=	2,500 watts.
	Total loss	=	11,800 watts.

$$\text{Efficiency} = \frac{184{,}000}{184{,}000 + 11{,}800} = \frac{184{,}000}{195{,}800} = 94 \text{ per cent.} \quad \textit{Ans.}$$

260. Measurement of Stray Power.—It is necessary merely to duplicate the flux and the speed in a motor or a generator in order to duplicate the stray-power loss. As the speed from Eq. (129) (p. 375) is $S = KE/\phi$, it is only necessary to duplicate the speed S and the electromotive force E in order to obtain the proper value of ϕ.

To measure stray power, the machine, whether it be a motor or a generator, is run light (without load) as a motor, as shown in Fig. 357. The field is connected across the line in series with a rheostat.

The total power input to the machine is:

$$VI = V(I_a + I_f) = VI_a + VI_f.$$

This power is distributed as follows: Some goes to supply the field loss, some supplies the armature $I_a^2R_a$ loss and the remainder is the stray power, S.P., the output being zero. Therefore

$$VI_a + VI_f = VI_f + I_a^2R_a + \text{S.P.}$$

$$\text{S.P.} = VI_a - I_a^2R_a. \quad (139)$$

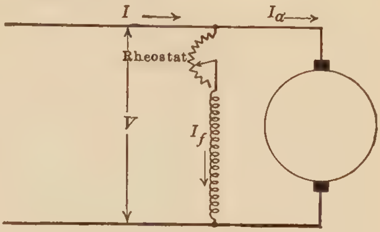


FIG. 357.—Determination of stray power in a dynamo.

The *stray power* is equal to the total input to the *armature* minus the armature-resistance loss.

Example.—A shunt generator when running light as a motor takes 12 amp. from 115-volt mains. The field current is 7 amp. and the armature resistance is 0.03 ohm. What is the stray-power loss of the machine at this particular value of flux and speed?

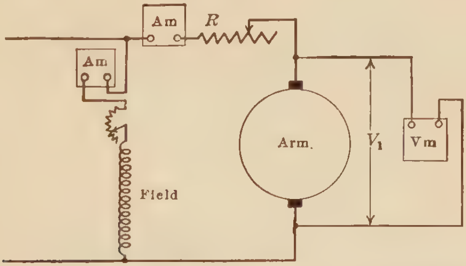


FIG. 358.—Connections for stray power measurement.

The armature current $I_a = 12 - 7 = 5$ amp.

The stray power, S.P. $= 115 \times 5 - (5)^2 0.03 =$

$575 - 0.75 = 574$ watts. *Ans.*

It will be observed that the armature $I_a^2R_a$ is negligible in this instance.

Assume that the above generator is delivering 100 amp. at 110 volts at 1,000 r.p.m. The field current is 7 amp. It is

desired to determine the value of its stray power under these conditions.

If the full-load electromotive force E and speed S be duplicated when the generator is running light, the stray power will be the same in both cases. When the machine is running light as a motor the stray power is readily measured as follows:

When carrying the above load, the induced e.m.f.

$$E = 110 + (107 \times 0.03) = 113.2 \text{ volts}$$

$$S = 1{,}000 \text{ r.p.m.}$$

To make these adjustments of E and S, the generator is run as a motor, connected as shown in Fig. 358. A rheostat R and an ammeter are connected directly in the armature circuit and a voltmeter is connected directly across the armature terminals. The rheostat R is first adjusted so that $V_1 = 113.2$ volts, the small armature drop at this load being negligible. The field rheostat is then adjusted to give a speed of 1,000 r.p.m. The machine is now operating at the same value of speed and flux as it did under load. Therefore, the stray power is the same in the two cases and is equal to $V_1I_a - I_a^2R_a$.

As an example, assume that the current I_a is 4.8 amp. and $V_1 = 113.2$ volts. (This neglects the small drop in the armature, 4.8×0.03.) The stray power

$$\text{S.P.} = 113.2 \times 4.8 - (4.8)^2\,0.03 = 543 \text{ watts.} \quad \textit{Ans.}$$

The efficiency of the generator can now be determined.

$$\begin{aligned} \text{Output under load} &= 110 \times 100 = 11{,}000 \text{ watts.} \\ I_a^2R_a &= (100 + 7)^2\,0.03 = \quad 344 \text{ watts.} \\ VI_f &= 110 \times 7 = \quad 770 \text{ watts.} \\ \text{S.P.} &= \quad 543 \text{ watts.} \\ \hline \text{Total loss} &= 1{,}657 \text{ watts.} \end{aligned}$$

$$\text{Efficiency} = \frac{11{,}000}{11{,}000 + 1{,}657} = \frac{11{,}000}{12{,}660} = 86.9 \text{ per cent.} \quad \textit{Ans.}$$

261. Stray-power Curves at Different Field Currents.—It is sometimes desired to determine the stray power of a machine over a considerable range, in order to have sufficient data for obtaining the stray power under various operating conditions. Stray power is a function of two variables, flux and speed, and a single curve cannot express the relationship under all conditions. To plot the relation, one quantity, either flux or speed, is held

constant and the other is varied. Because it is more convenient, the flux is usually held constant and the speed is varied, the connections being shown in Fig. 358. The flux is held constant by means of the field rheostat and the speed is varied by means of the rheostat R in the armature circuit.

The flux is a function of the field current. Hence, stray power may be considered as a function of field current and speed. This introduces small errors. The flux at no load, in the stray-power test, may be greater than its value when the machine is under

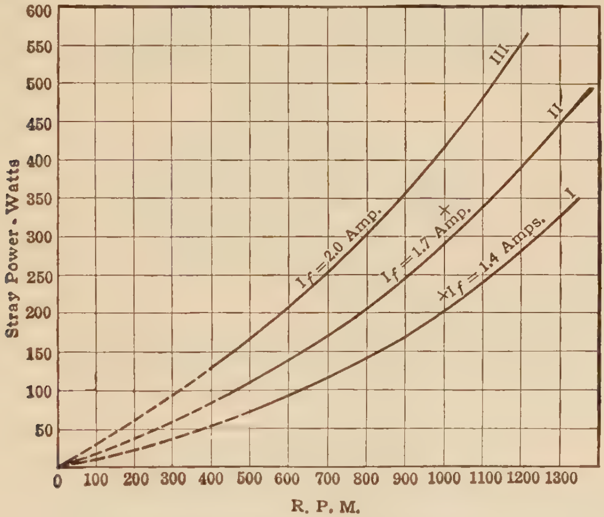


FIG. 359.—Typical stray-power curves.

load and with the same excitation, because armature reaction tends to reduce the flux.

Therefore, if the field current instead of the flux is used to determine stray power, the stray power may be too large with the machine running light owing to armature reaction. This is in part compensated by the fact that the flux "peaks" under load so that the loss for any value of total flux is increased, even though the average flux be the same (see p. 322, Fig. 268). Also, the flux for a given value of field current may vary due to hysteresis. This may cause errors.

In a stray-power run, the field current may be held at a definite value and the speed varied over the probable working

range of the machine. The field current may then be adjusted to another value and the run repeated. At least three values of field current should be used, the maximum and the minimum value under which the machine is likely to operate and an intermediate value. Curves similar to those shown in Fig. 359 are obtained in this manner.

Example.—The curves of Fig. 359 were obtained from a 10-kw., 230-volt generator by the method just described, the generator being run as a motor when these curves were obtained. The rated current of this machine is 43.5 amp. and its armature resistance is 0.14 ohm.

Determine its efficiency as a generator at half load and at rated load, the voltage being the same in each case, the respective values of field current being 1.5 and 1.8 amp. The speed is constant at 1,000 r.p.m.

At half load,

$$I = 43.5/2 = 21.8 \text{ amp.}$$
$$I_a = 21.8 + 1.5 = 23.3 \text{ amp.}$$
$$I_a^2R_a = (23.3)^2\, 0.14 = 76 \text{ watts.}$$
$$VI_f = 230 \times 1.5 = 345 \text{ watts.}$$

From Fig. 359, on the 1,000 r.p.m. ordinate, one-third the distance from curve I to curve II (= 1.5 amp.), the stray power is found to be 230 watts.

The efficiency at this load is:

$$\text{Efficiency} = \frac{230 \times 21.8}{230 \times 21.8 + 76 + 345 + 230} = \frac{5{,}000}{5{,}650} = 88.5 \text{ per cent.} \quad \textit{Ans.}$$

At rated load,

$$I = 43.5.$$
$$I_a = 43.5 + 1.8 = 45.3 \text{ amp.}$$
$$I_a^2R_a = (45.3)^2\, 0.14 = 287 \text{ watts.}$$
$$VI_f = 230 \times 1.8 = 414 \text{ watts.}$$

In Fig. 359, on the 1,000-r.p.m. ordinate, one-third the distance from curve II to curve III, corresponding to 1.8 amp., the stray power is found to be 330 watts.

$$\text{Efficiency} = \frac{230 \times 43.5}{230 \times 43.5 + 287 + 414 + 330} = \frac{10{,}000}{11{,}030} = 90.7 \text{ per cent.} \quad \textit{Ans.}$$

Assume that it is desired to determine the efficiency of this machine when running as a motor at 900 r.p.m. and taking 45 amp. at 230 volts from the line. Under these conditions the field current is found to be 1.6 amp.

$$I_a = 45 - 1.6 = 43.4 \text{ amp.}$$
$$I_a^2R_a = (43.4)^2\, 0.14 = 264 \text{ watts.}$$
$$VI_f = 230 \times 1.6 = 368 \text{ watts.}$$

On the 900-r.p.m. ordinate (Fig. 359) two-thirds the distance from curve I to curve II (= 1.6 amp.), the stray power is found to be 225 watts.

$$\text{Efficiency} = \frac{230 \times 45 - 264 - 368 - 225}{230 \times 45} = \frac{9{,}490}{10{,}350} = 91.6 \text{ per cent.} \quad \textit{Ans.}$$

262. Stray Power as a Function of Flux and Speed.—As was pointed out in Par. 261, strictly speaking stray power is not a function of the field current. The net flux depends not only on the field current but on the armature reaction as well. It is possible, however, to determine stray power as a function of flux only and not field current.

It is shown that stray power is a function of flux and speed. That is,

$$\text{S.P.} = f(\phi, S).$$

It is not a simple matter to measure the flux with ordinary instruments. But from Eq. (119), page 307, the induced e.m.f. is given by

$$E = K\phi S,$$

where K is a constant depending on the winding, number of poles, etc., ϕ is the total flux entering the armature from one north pole, and S is the speed. Solving for the flux,

$$\phi = \frac{E}{KS}. \tag{140}$$

For any given machine, K is constant. Hence, the stray power is a function of E/S and S; that is,

$$\text{S.P.} = f'\left(\frac{E}{S}, S\right). \tag{141}$$

If the ratio E/S and S are the same for two operating conditions, the stray power must be the same, neglecting the changes in the shape of the flux distribution curve. The following example illustrates the application of this equation.

Example.—A 20-kw., 220-volt shunt generator, having an armature resistance of 0.096 ohm, is delivering 85 amp. at 222 volts. Its speed is 920 r.p.m. and the field current is 2.24 amp. (*a*) Under what no-load conditions must it be operated in order that its no-load stray power may be equal to that which exists under this condition of operation? (*b*) If the armature input at no load is 4.02 amp. at 230.4 volts, what is the value of the stray power? (*c*) Determine the efficiency of the machine under the foregoing operating conditions.

(*a*) With the load as given,

$$E = 222 + (85 + 2.24)\,0.096 = 230.4 \text{ volts.}$$

$$\frac{E}{S} = \frac{230.4}{920} = 0.251.$$

Although the machine is a generator, it is operated as a motor to determine its stray power. The connections are as shown in Fig. 358. The *field* rheostat is adjusted until the terminal voltage divided by the speed is equal to 0.251. The armature resistance drop is negligible at no load. The speed is then adjusted by means of the *armature* rheostat until it is equal to 920 r.p.m. Changing the armature rheostat does not change the flux and hence does not change the ratio E/S,

(*b*) From Eq. (139), the stray power is

$$\text{S.P.} = 230.4 \times 4.02 - (4.02)^2\, 0.096 = 926 \text{ watts.} \quad \textit{Ans.}$$

(Also see point *a*, Fig. 360.)

(*c*) Output $= 222 \times 85 = 18{,}870$ watts.

$$\begin{aligned} I_a^2R_a = (85 + 2.24)^2\, 0.096 &= \phantom{2{,}}730 \text{ watts.} \\ VI_f = 222 \times 2.24 &= \phantom{2{,}}497 \text{ watts.} \\ \text{S.P.} &= \phantom{2{,}}926 \text{ watts.} \\ \hline \text{Total losses} &= 2{,}153 \text{ watts.} \end{aligned}$$

$$\eta = \frac{18{,}870}{18{,}870 + 2153} = 89.8 \text{ per cent.} \quad \textit{Ans.}$$

Owing to the peaking of the flux by armature reaction (see p. 322, Fig. 268) the stray-power loss will be slightly greater than it is with a flat-top flux curve, even though the total areas of the two curves are the same. This is due to the fact that the stray-power losses are not directly proportional to the flux density but to a power higher than the first.

263. Stray-power Curves with Constant Flux.—Let it be required to plot a series of stray-power curves, similar to those of Fig. 359, covering the maximum probable range of operation of a machine both as motor and generator. The stray power is to be plotted as a function of speed for constant values of flux rather than field current. Three curves are usually sufficient.

Consider the generator of Par. 262. Let it be required to determine three stray-power curves that shall cover the probable operating range of the machine both as motor and generator. This range of operation will depend on each particular machine and the conditions under which it is to be used. The values given below would apply to an ordinary constant-speed machine.

The maximum value of flux occurs when E is a maximum and S is a minimum. E is a maximum for generator operation at overload. Assume 25 per cent. overload. The rated current is 91 amp. Assume 3 amp. for the field current.

$$E_1 = 220 + (114 + 3)(0.096) = 231.2 \text{ volts.}$$

The machine will probably not operate at less than 900 r.p.m. Hence, the maximum value of flux is determined by

$$\frac{E}{S} = \frac{231.2}{900} = 0.26 \text{ (nearly).}$$

The minimum value of flux occurs when E is a minimum and S is a maximum. E is a minimum under motor operation at overload. Assume 25 per cent. overload.

$$E = 220 - 114(0.096) = 209.0 \text{ volts.}$$

Assume that the motor speed will probably not exceed 1,050 r.p.m.

Hence, the minimum value of flux is determined by

$$\frac{E}{S} = \frac{209.0}{1{,}050} = 0.2 \text{ (nearly).}$$

An intermediate value of flux would be determined by 0.23.

To obtain these three curves, the connections shown in Fig. 358 are used, the machine being operated as a motor in all

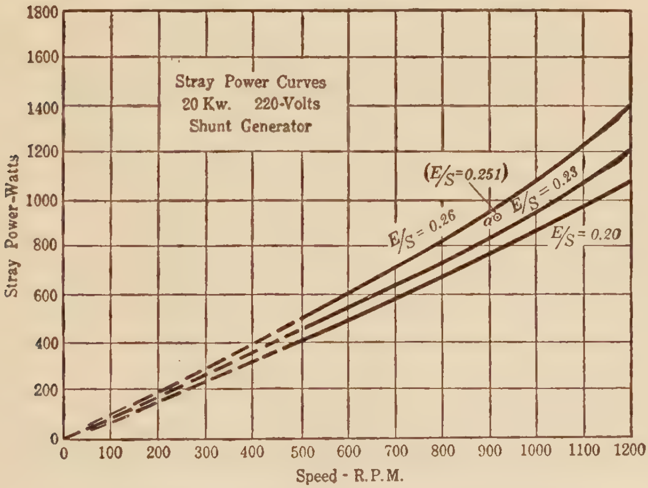


Fig. 360.—Stray power as a function of flux and speed.

the stray-power tests. It may be necessary to excite the field from a source greater than 220 volts to obtain the maximum value of flux.

The field rheostat is adjusted until $E/S = 0.26$ and a run is made with varying speed. The speed is varied with the arma-

ture rheostat. Changing this rheostat does not change the flux and hence does not change E/S. The other two curves are obtained in a similar manner, the field rheostat being adjusted to give values of $E/S = 0.23$ and $E/S = 0.20$. A set of curves obtained in this manner is shown in Fig. 360.

In order to determine the efficiency, input, etc., these curves are used in precisely the same manner as those in Fig. 359, as is illustrated in Par. 261.

It is also possible to determine the stray power of a machine by driving it without load by means of a smaller machine whose efficiency is known. In using this method it is possible to separate the friction and windage losses from the core loss by measuring the power delivered to the machine when the field circuit is closed and again when it is opened.

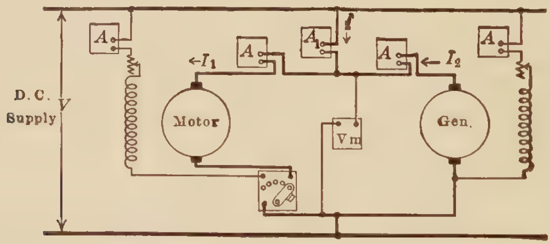


Fig. 361.—Kapp opposition method for determining losses.

Since stray power is a small proportion of the total input of a machine, large errors in its determination have but small effect on the efficiency.

264. Opposition Test—Kapp Method.—The objection to the foregoing stray-power methods of measuring losses is that the machine is not under load when the losses are being measured, so their values may be in error. If two similar machines are available, their losses may be determined when both machines are loaded, and yet the line supplies only the *losses* of the two machines. The connections for making such a test are shown in Fig. 361.

The two similar machines are coupled together mechanically and are then connected to the line, as shown. The motor should have a starting box. Five ammeters are used, one in each field,

one in each armature circuit, and one in the line supplying the two armatures. The fields are connected directly to the line so that their currents are not indicated by the ammeter A_1.

The operation of the set is as follows: The motor supplies mechanical power to the generator. This in turn supplies electrical power to the motor. The power delivered by the generator is less than that required by the motor, owing to the losses in the two machines. Therefore, this deficit must be made up by the line which supplies the current I.

The total input to the two armatures is VI.

This power is distributed as follows:

Motor armature loss $= I_1^2R_1$.
Generator armature loss $= I_2^2R_2$.
Motor stray power.
Generator stray power.

where R_1 and R_2 are the motor and generator armature resistances.

As the generator field is necessarily stronger than that of the motor, because it requires the higher internal voltage, its stray power will be greater than that of the motor, as stray power increases with increase of flux. As a close approximation, the total stray-power loss may be divided between the two machines in proportion to their induced voltages.

Let E_1 equal the motor induced volts and E_2 the generator induced volts.

$$E_1 = V - I_1R_1.$$
$$E_2 = V + I_2R_2.$$

Let P_1 and P_2 be the values of stray power in the two machines. Then:

$$\frac{P_1}{P_2} = \frac{E_1}{E_2}. \qquad (142)$$

The total input to the two machines goes to supply their armature and stray-power losses, because the output of the system is zero and the field power is supplied separately. By sub-

tracting the armature losses from the input, the total stray power $(P_1 + P_2)$ remains.

That is:

$$P_1 + P_2 = VI - I_1^2R_1 - I_2^2R_2.$$

The field losses are measured directly by the ammeter in each field circuit.

The advantages of this method are that each machine is operating under load conditions; the regulation of each machine may be determined; the line need supply only the losses.

The principal disadvantage is that it requires two similar machines. The assumptions made in regard to the stray-power distribution may be slightly in error.

The machines are brought into operation by first starting the motor with the starting box. The generator voltage is then made equal to the motor terminal voltage and the generator terminals are then connected directly across the motor terminals, just as generators are connected in parallel. Care should be taken that the correct polarity is observed. The generator field is then strengthened and the motor field weakened until the desired conditions of load and speed are obtained.

Example.—Two similar 120-volt, 7.5-hp. motors are connected in the manner shown in Fig. 361. The armature resistance of each is 0.12 ohm. The fields are so adjusted that the motor current I_1 is 57 amp., and the generator current I_2 is 45 amp. Under these conditions the line is supplying a current I of 12 amp. at 120 volts. Find the stray power of each machine under these conditions of load.

The power supplied by the line

$$P = 120 \times 12 = 1{,}440 \text{ watts.}$$

$$I_1^2R_1 = 57^2 \times 0.12 = 390 \text{ watts.}$$

$$I_2^2R_2 = 45^2 \times 0.12 = 243 \text{ watts.}$$

$$\text{Total} = 633 \text{ watts.}$$

Total stray power $= 1{,}440 - 633 = 807$ watts.

$$E_1 = 120 - (57 \times 0.12) = 113.2 \text{ volts.}$$

$$E_2 = 120 + (45 \times 0.12) = 125.4 \text{ volts.}$$

The motor stray power

$$P_1 = \frac{113.2}{113.2 + 125.4} 807 = 383 \text{ watts.}$$

The generator stray power

$$P_2 = \frac{125.4}{113.2 + 125.4} 807 = 424 \text{ watts.}$$

Knowing the stray power and the armature and field losses, the efficiency is readily calculated.

265. Ratings and Heating.—Practically all power apparatus, whether it be steam engines, gas engines, or dynamos, has definite power ratings. These ratings are determined by the manufacturer and are supposed to give the power which the apparatus can safely or efficiently deliver. It is interesting to consider what, in general, determines the rating of various power devices.

Both a steam engine and a steam turbine are usually rated at the load for which their *efficiency* is a maximum. These two types of prime mover can carry a high overload without difficulty. Ordinarily, they can carry at least 100 per cent. overload easily, but at reduced efficiency.

Owing to their excessive weights and costs, large gas engines are usually rated as high as possible, which is near the point at which they cease to operate. Their thermal efficiency is ordinarily so much greater than that of the steam engine or turbine that the question of weight is more important than the question of efficiency.

Electrical apparatus is usually rated at the load which it can safely carry without *overheating*. (Commutation may at times limit the output of direct-current machines.)

If the temperature of electrical apparatus becomes too high, the cotton insulation on the armature and the field conductors, and the insulating varnishes, become carbonized and brittle. This may result ultimately in grounds and short-circuits within the machine. The A. I. E. E. Standardization Rules[1] specify insulations and their safe temperature limits when embodied in machines as follows:

Class O Insulation.—Cotton, silk, paper, and similar organic materials when neither impregnated nor immersed in oil.

Class A Insulation.—Cotton, silk, paper, and similar materials when impregnated or immersed in oil; also enamel as applied to conductors.

Class B Insulation.—Inorganic materials such as mica and asbestos in built-up form combined with binding substances.

The limiting "hottest spot" temperatures for these materials are as follows:

[1] Direct-current Rotating Machines, Generators, and Motors. *Am. Inst. Elec. Eng.*, Standards, **5**, July, 1925.

Class *O* material	90° C.
Class *A* material	105° C.
Class *B* material	125° C.

The difficulty in determining the "hottest spot" temperatures lies in the fact that these temperatures are within coils and are not accessible for easy temperature measurement.

The following three methods are used for determining temperatures:

Thermometer Method.—A mercury or alcohol thermometer or thermocouple is applied to the hottest part of the machine accessible to the mercury or alcohol thermometers. The bulbs of the thermometers are covered by felt pads cemented to the machine by oil putty or cotton waste.

Resistance Method.—This method consists in the determination of temperature by comparison of the resistance of a winding at the temperature to be determined with the resistance at a known temperature.

Embedded-detector Method.—This method consists in the determination of the temperature by thermocouples or resistance temperature detectors, built into the machine. Since none of the foregoing methods gives the "hottest spot" temperatures, the following "conventional allowances" are added to each:

Thermometer method	15° C.
Resistance method	10° C.
Embedded-detector method	5° C.

From the foregoing data the A. I. E. E. has drawn up the following specifications as to the upper limits of operating temperatures of the important parts of direct-current machines.

Limiting Temperature Rise for Machines Having Continuous and Short-time Ratings.—The temperature rise of each of the various parts, (of d.-c. machines) above the temperature of the cooling medium when tested in accordance with the rating, shall not exceed the values given in the following table. All temperatures shall be determined by the Thermometer method.

TABLE I

Item		Type of enclosure	Limiting temperature rise, degrees Centigrade		
			Class O insulation	Class A insulation	Class B insulation
1	Armature windings, wire field windings and all windings other than 2	All types except totally enclosed	35	50	70
		Totally enclosed	40	55	75
2	Single-layer field windings with exposed uninsulated surfaces and bare copper windings	All types except totally enclosed	45	60	80
		Totally enclosed	45	60	80
3	Cores and mechanical parts in contact with or adjacent to insulation	All types except totally enclosed	35	50	70
		Totally enclosed	40	55	75
4	*Commutators and collector rings	All types except totally enclosed	50	65	85
		Totally enclosed	50	65	85
5	Miscellaneous parts (such as brush-holders, brushes, pole-tips, etc.) other than those whose temperatures affect the temperature of the insulating material may attain such temperatures as will not be injurious.				

*These limits for the temperature rise of commutator and collector rings apply where Class *O*, *A*, or *B* insulation, respectively, as the case may be, is employed in the commutator, or is adjacent thereto and its life would be affected by the heat from the commutator. It is recognized that the heating of the commutator could correspond to Class *B* even if the slot insulation were of Class *O*, on condition that the insulation of connections was not affected by the heating of the commutator.

In the foregoing table, 40° C. = 104° F. has been assumed as the surrounding or ambient temperature. Also, the "conventional allowance" for thermometer measurement has been made. For example, 35° C. is given as the limiting temperature rise for Class *O* insulation under 1. Adding this to the 40° C. room

temperature and then adding the 15° conventional allowance gives 90° C.

266. Temperature Measurement by Resistance Method.—It is frequently more accurate and more convenient to determine the temperature by the resistance method.

It has already been shown that the resistance of copper conductors changes with the temperature. By utilizing this principle, an idea of the average temperature within a winding may be obtained. For copper the increase of resistance per degree rise of temperature may be obtained from the formula $1/(234.5 + t)$,[1] where t is the surrounding or ambient temperature. For example, at an ambient or room temperature of 30° C., the increase of resistance per degree rise is $1/264.5 = 0.00378$.

Example.—With an ambient temperature of 30° C. the resistance of the field of a shunt generator increases from 104 to 112 ohms. What is its temperature rise?

$$\text{The fractional change in resistance is } \frac{112 - 104}{104} = 0.077.$$

$$\text{Temperature } rise = 0.077/0.00378 = 20.^\circ 4 \text{ C.} \quad \textit{Ans.}$$

Owing to the time required to reach a constant temperature, motors and generators should be run from 6 to 18 hr. in order that an accurate test of their temperature may be made. As such a long time is usually prohibitive, the heating is often accelerated by running overload for an hour or so and then dropping back to rated load. By this procedure a very good idea of the ultimate temperature may often be obtained in a run of 2 or 3 hr.

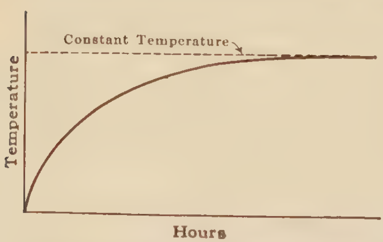


Fig. 362.—Curve of temperature rise with time, for a dynamo.

In order to obtain an idea as to how close a machine is to its ultimate temperature, it is often desirable to plot a curve of temperature rise during the test. A typical curve of this type for a shunt field is shown in Fig. 362.[2] At the beginning of the test,

[1] See Par. 46, p. 45.

[2] This curve is identical in character with the curve giving the rise of current with time in an inductive circuit (see Fig. 174, p. 213).

there is but a slight difference of temperature between the field coils and the room. Therefore, but a small amount of heat is given out by the coils and as a result the temperature rises rapidly. As the difference between the coil temperature and the room temperature increases, more and more heat is given out by the coils, and the temperature rises less rapidly. Therefore, the rate of temperature increase becomes less as the time increases. This is illustrated by the curve of Fig. 362. When the curve becomes practically horizontal, the total heat developed in the coil is equal to the heat dissipated by the coils and the coils have reached a constant temperature. Similar curves would hold for other parts of the machine.

Care must be taken in measuring the armature resistance when determining temperature rise. The object of this measurement is not to determine the resistance with the idea of calculating the loss, but to determine the *change of resistance in the armature copper, due to change of temperature.* Therefore, it is essential that the resistance of the *copper alone* be measured and that the current path through the copper be the same in every measurement. To exclude all resistance except that of the copper, the brush and contact resistances must not be included in the measurement. Therefore, the voltmeter leads must be held on the commutator segments inside the brushes, as shown in Fig. 363 (*a*). Moreover, these segments should be marked and in every subsequent measurement they should be directly under the same brushes. This insures the same conducting path for each measurement.

When a multipolar armature is so measured, the *division* of current in the various paths is determined in part by the brush contact resistance. Thus in Fig. 363 (*b*), the current from brush a to brush b is I_1 and that from brush a to brush c is I_2. The total current entering the brush a is their sum, I amp. The division of the current I between brushes b and c is in part determined by the contact resistance at these two brushes. As contact resistance is a variable quantity, the current *division* in the armature may change considerably with different measurements. To keep the current in definite paths, two brushes may be insulated as shown in Fig. 363 (*c*). In this case the current paths are not symmetrical, but the division of current is deter-

mined not by brush contact resistance but by the copper resistance itself.

In measuring the shunt-field resistance, the voltmeter should be connected directly across the winding so as to exclude the drop in the rheostat.

This resistance method gives an average value of the temperature of the windings. To find the hottest-spot temperature, 10° C. should be added according to the A. I. E. E. rules.

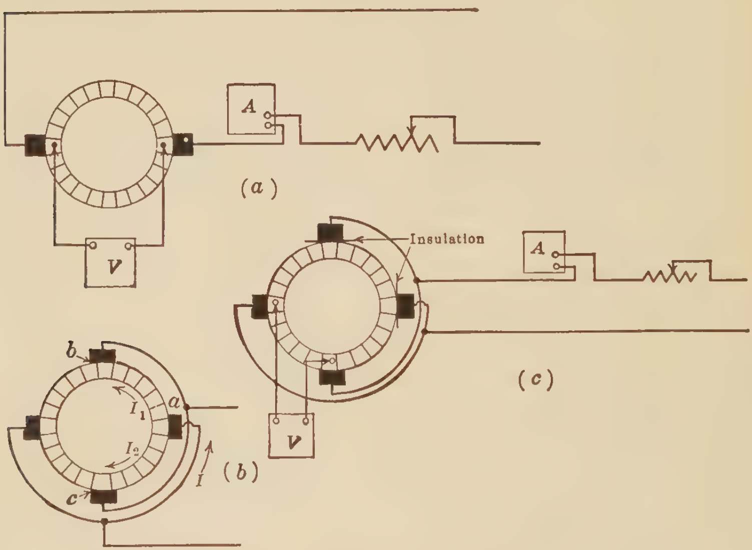


FIG. 363.—Measurement of armature resistance for temperature test.

In addition to measuring the temperature of the windings, the rise of temperature of bearings and of commutator should be measured with a thermometer.

In modern machines of large size, thermocouples are inserted in the windings and are connected to millivoltmeters on the switchboard, so that the operator can determine the "hot spot" temperatures at any time. A "conventional allowance" of 5° C. is added to give the hottest-spot temperature.

267. Parallel Running of Shunt Generators.—In most power plants it is necessary and desirable that the power be supplied by several small units rather than by a single large unit.

(*a*) Several small units are more reliable than a single large unit, for if a unit is disabled the entire power supply is not cut off. (*b*) The units may be connected in service and taken out of service to correspond with the load on the station. This keeps the units loaded up to their rated capacity which increases the efficiency of operation. (*c*) Units may be repaired more readily if there are several in the station. (*d*) Additional units may be installed to correspond with the growth of station load. (*e*) The station load may exceed the capacity of any single available unit.

Shunt generators, because of their drooping characteristic, are particularly well suited for parallel operation. In Fig. 364 are shown the characteristics of two shunt generators which will be designated as No. 1 and No. 2 respectively. It will be noted that generator No. 1 has the more drooping characteristic. If the two generators are connected in parallel (Fig. 365) their terminal voltages must be the same, neglecting any very small voltage drop in the connecting leads. Therefore, for a common terminal voltage, V_1 (Fig. 364) generator No. 1 delivers I_1 amperes and generator No. 2 delivers I_2 amperes. That is, the machine with the more drooping characteristic carries the smaller load.

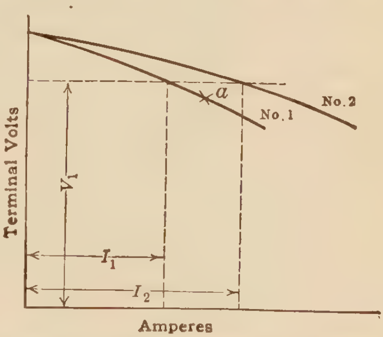


FIG. 364.—Characteristics of shunt generators in parallel.

Assume that some condition arises which temporarily causes generator No. 1 to take more than its share of the load. This condition might arise from a temporary increase in the speed of its prime mover, or it might be occasioned by change of load on the system. Generator No. 1 would immediately tend to operate at some point a on its characteristic. This results in a drop in its terminal voltage, which tends to make it take *less* load. Therefore, any tendency of one machine to take more than its share of the load results in a change of voltage which opposes this tendency. Hence, shunt generators in parallel may be said to be in *stable* equilibrium. The reactions of the system are such

as to hold the generators in parallel. Moreover, if any *change* of load on the system occurs, each machine must carry some of the increase or decrease of load.

The connections for operating shunt generators in parallel are shown in Fig. 365. Each generator should have its own ammeter. A common voltmeter is sufficient for all the machines. The individual machines can be connected to the voltmeter or potential bus through suitable plug connectors or selective switches. Assume that No. 2 is out of service and that No. 1 is supplying all the load. It is desired to put No. 2 in service. The prime

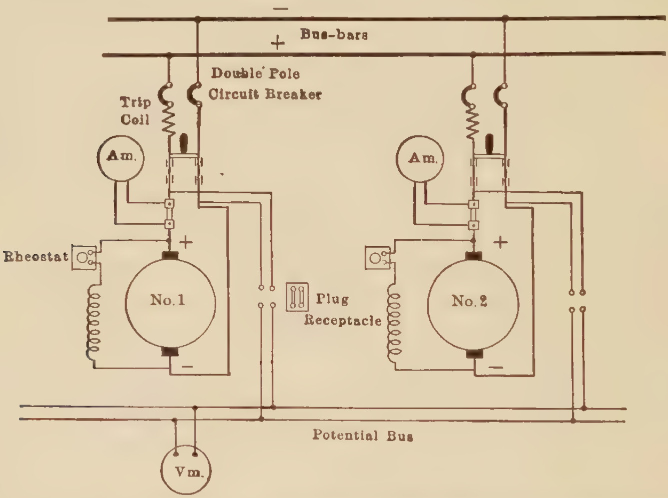


FIG. 365.—Connections for the parallel operation of shunt generators.

mover of No. 2 is started and No. 2 is brought up to speed. Its field is then adjusted so that its voltage is just equal to that of the bus-bars, which condition may be determined by the voltmeter. The breaker and switch are now closed and No. 2 is connected to the system. Under these conditions, however, it is not taking any load, as its *induced* voltage is just equal to the bus-bar voltage and no current will flow between points at the same potential. Its induced voltage must be greater than that of the bus-bars in order that it may deliver current. Therefore, the field of No. 2 is strengthened until the generator takes its share of the load. It may be necessary to

weaken the field of No. 1 simultaneously in order to maintain the bus-bar voltage constant.

To take a machine out of service, its field is weakened and that of the other machine is strengthened until the load of the first machine is zero. The breaker and then the switch are opened,

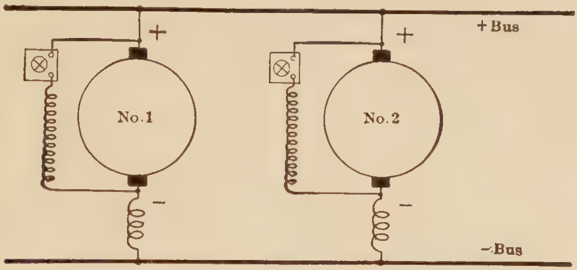


FIG. 366.—Compound generators in parallel.

clearing the machine. Connecting in and removing a machine from service in this manner prevent any shocks or disturbance to the prime mover or to the system.

If the field of one generator be weakened too much, current will be delivered to this generator, which will run as a motor and tend to drive its prime mover.

It is evident that if shunt generators are to divide the load properly at all points, *their characteristics should be similar*, that is, *each should have the same voltage drop from no load to full load.*

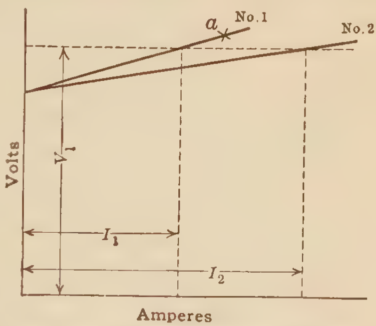


FIG. 367.—Characteristics of compound generators in parallel.

268. Parallel Running of Compound Generators.—Figure 366 shows two over-compounded generators connected to the bus-bars, positive and negative terminals being properly connected as regards polarity. Each generator is taking its proper share of the load.

Assume that for some reason generator No. 1 takes a slightly increased load. The current in its series winding must increase, which strengthens its field and raises its electromotive force,

thus causing it to take still more load. On the other hand, as the system load is assumed to be fixed, generator No. 2 will at the same time drop some of its load, resulting in a weakening of its series field and a consequent further dropping of its load. In a very short time No. 1 will be driving No. 2 as a motor, and ultimately the breaker of at least one of the machines will open.

This condition is again illustrated by Fig. 367, which shows the individual characteristics of the two machines. Assume that the machines are operating at a voltage V_1, which corresponds to the respective currents I_1 and I_2. Assume that No. 1 takes a slightly increased load. Its voltage will then tend to rise to some point a. This increased voltage means that the machine takes still more current and the effect will continue until ultimately the breaker opens.

These compound generators may be considered to be in unstable equilibrium. That is, any action tending to throw the machines out of equilibrium is accentuated by the resulting reactions.

The machines may be made stable by connecting the two series fields in parallel (Fig. 368). This connection, which in Fig. 368 ties the two negative brushes together, is a conductor of low resistance and is called the *equalizer*. Its operation is as follows: Assume that generator No. 1 starts to take more than its proper share of the load. This increased current will pass not only through the field of generator No. 1 but also, by means of the equalizer, some of it will pass through the field of generator No. 2. Therefore, both machines are affected in a similar manner and No. 1 is unable to take the entire load.

To maintain the proportionate division of load from no load to full load, the following conditions must be satisfied:

(a) The regulation of each armature must be the same.

(b) The series-field resistances must be inversely proportional to the machine ratings.

It is not always possible to adjust compound generator characteristics by means of series-field diverters so that they divide the load properly. Suppose (Fig. 368) that the series field of generator No. 1 is shunted by a diverter. If the equalizer and bus-bar have negligible resistance, this diverter shunts the series field of generator No. 2 as well as that of No. 1. Therefore, the diverter

merely drops the characteristic of the entire system but does not affect the *division* of load. The proper load adjustments may be made by means of a very low resistance in series with one of the series fields.

It should be noted that the desired division of load among either shunt or compound generators at any one load may be obtained by adjusting their field rheostats. However, it is usually desirable that this division remain constant at all loads, especially if an operator is not in continuous attendance. There-

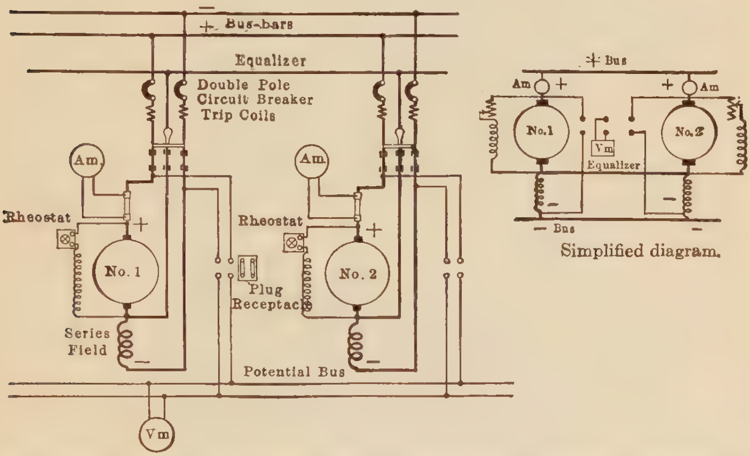


FIG. 368.—Typical connections for two compound generators operating in parallel.

fore, it is desirable that generators operating in parallel have similar characteristics.

A compound generator with a single series field usually has a 3-pole switch, one blade of which connects the equalizer, as shown in Fig. 368. If a 3-wire generator (see p. 457) having two series fields is to be connected, a 4-pole switch is necessary as there are two equalizers (see Fig. 369). The load ammeter in a compound generator should always be connected between the *armature* terminal and the bus-bars. If it is connected in the series-field circuit, the ammeter may not indicate the generator current, due to the fact that some of the generator current may be passing through the equalizer.

Compound generators are put in service and taken out of service in the same manner as shunt generators, that is, the load is adjusted and shifted by means of the shunt-field rheostat.

269. Circuit Breakers.—Generators, motors, and electric circuits in general require protection from short-circuits and overloads. The sudden load imposed by a short-circuit may injure the generator or its prime mover. Wires may overheat under the short-circuit current, resulting in fire hazard. Two common devices are used for opening short-circuits and overloads, the *fuse* and the *circuit breaker.* The fuse has a much lower first cost and occupies less space. On the other hand, it is worthless

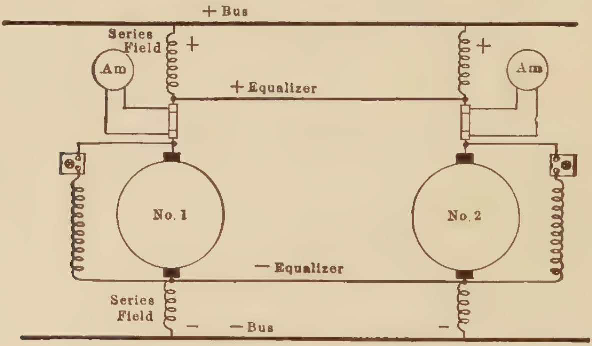


FIG. 369.—Compound generators requiring two equalizers.

after being blown (unless it is of the refillable type) and considerable inconvenience often results from not having spare fuses at hand. The circuit breaker has a higher first cost and requires more space. On the other hand, it operates an indefinitely great number of times without injury and is readily re-set. The action of a breaker is faster than that of a fuse. That is, it opens the circuit more quickly.

Practically all breakers operate on the same principle. The mechanism which presses the breaker contacts together is held by a trigger. This trigger is actuated by a solenoid plunger, the turns of the solenoid itself being in series with the circuit. When the current becomes excessive, the plunger is raised and so trips the trigger, allowing the breaker to spring open. Many breakers also have shunt solenoids, which allow them to be tripped from remote points.

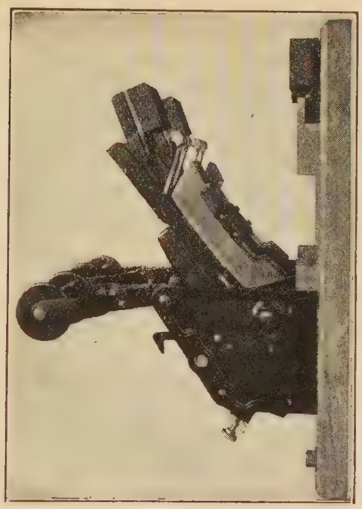


FIG. 370.—Two-pole, 2,000-ampere circuit breaker (*Condit*).

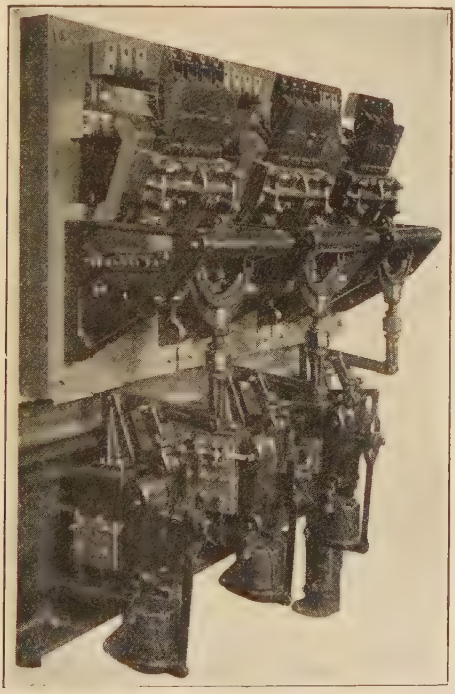


FIG. 371.—6,000-ampere, electrically-operated circuit breaker (*Condit*).

The current in the breaker is usually carried by copper laminations which bridge the copper blocks, as shown in Fig. 370. On closing the switch, these laminations press on the blocks making a wiping contact. The carbon blocks parallel these copper contacts. They ordinarily carry a negligible portion of the current, but when the breaker opens they break contact later than the copper, and so interrupt the arc, which would otherwise burn the copper. The carbon contacts are cheap and easily renewed. Circuit breakers of large capacities are more or less complicated mechanisms, as is illustrated by the 6,000-amp. breaker shown in Fig. 371.

Circuit breakers should always be mounted at the top of the switchboard. If they are placed at the bottom, the arc which rises may cause personal injury or may damage the switchboard equipment.

CHAPTER XIV

TRANSMISSION AND DISTRIBUTION OF POWER

270. Power Distribution Systems.—Under modern conditions, most central stations generate power on a large scale as alternating current and transmit this power as alternating current. The reason for using alternating current in transmitting the power is that the voltage may be efficiently raised and lowered by means of transformers. Much less copper is required to transmit power at high voltages. The Thury system does transmit power as direct current at high voltage (see Par. 232), but is not used in this country.

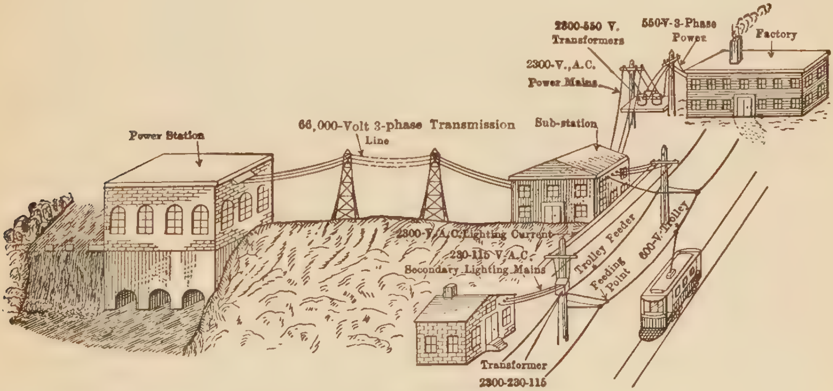


FIG. 372.—Typical power system.

Power is ordinarily utilized at comparatively low voltages (110, 220, 600 volts), but it cannot be economically transmitted to any considerable distance at these voltages. In fact, direct current for commercial use can be economically transmitted and distributed only in the most congested districts of large cities. Some of its advantages for city distribution are that a storage-battery reserve can be readily maintained, which is a very important consideration; direct-current motors are admirably suited to

elevators and printing presses which form an important part of the power load in a city. The absence of inductive and capacitive effects, which are present with alternating current, and the absence of eddy current losses in cables are further considerations. Figure 372 shows the general method of power distribution. Power is generated at the power station, is transmitted as alternating current at high voltage to the substation (66,000 volts is shown; the transmission voltage is seldom less than 6,600 volts). At the substation it is either transformed to 2,300 volts alternating current by transformers or to 600 volts or 230 volts direct current by motor-generator sets or synchronous converters. (Figure 372 shows the substation supplying a trolley with 600 volts direct current; a 2,300-volt alternating-current circuit supplies power for lighting, the voltage being transformed near the consumer's premises to a 230 to 115-volt 3-wire system; a 3-phase 2,300-volt alternating-current power line supplies a factory, the voltage being transformed to 550 volts, 3-phase, by transformers. These systems are discussed more fully in Chap. XIII, Vol. II.) The substation receives the power in large amounts and distributes it to the various consumers in smaller quantities. It bears the same relation to the power system as the middleman or retailer does to an industrial system.

271. Voltage and Weight of Conductor.—*The weight of conductor varies inversely as the square of the voltage,* when the power transmitted, the distance and the loss are fixed.

Let it be required to transmit the power P at the voltage V_1 and current I_1 over wires having a resistance R_1.

The current

$$I_1 = \frac{P}{V_1}.$$

The power loss

$$P_1 = I_1^2 R_1.$$

Assume that the voltage is raised to V_2, the power, the loss and the distance remaining fixed.

The current

$$I_2 = \frac{P}{V_2}.$$

The power loss

$$P_2 = I_2^2 R_2 = P_1.$$

Therefore,

$$I_1^2R_1 = I_2^2R_2.$$

$$\frac{R_1}{R_2} = \left(\frac{I_2}{I_1}\right)^2 = \frac{(P/V_2)^2}{(P/V_1)^2} = \frac{V_1^2}{V_2^2}.$$

That is, the conductor resistance varies *directly* as the square of the voltage. But the volume or the weight of a conductor of given length varies inversely as the resistance.

Let the weight of copper in the two cases be W_1 and W_2, respectively.

$$\frac{W_1}{W_2} = \frac{V_2^2}{V_1^2}. \tag{143}$$

Therefore, the conductor weight varies inversely as the square of the voltage, when the power, the loss and the distance are fixed.

If the voltage of a system is doubled, the weight of the copper is quartered, other conditions being the same.

Example.—50 kw. are delivered at a distance of 500 ft. at 110 volts over a 400,000-C.M. feeder. (*a*) What is the power loss? (*b*) Repeat for 220 volts.

(*a*) The current

$$I_1 = \frac{50{,}000}{110} = 454 \text{ amp.}$$

If the cable had 454,000 C.M. (see Par. 72) the loss would be 454,000 $\times$ 1,000 $\times$ 10^{-5} watts = 4,540 watts. Actually the loss is $\left(\frac{454}{400}\right)^2$ $\times$ 1,000 $\times$ 10^{-5} $\times$ 400,000 = 5,150 watts. *Ans.*

$$(b)\ I_2 = \frac{50{,}000}{220} = 227 \text{ amp.}$$

The loss is

$$\left(\frac{227}{400}\right)^2 4{,}000 = 1{,}290 \text{ watts.} \quad \textit{Ans.}$$

The loss in (*b*) is one-fourth that in (*a*). Therefore, a 100,000-C.M. feeder, having just one-fourth the weight of the feeder in (*a*), would transmit the same power, the same distance, with the *same loss*.

272. Size of Conductors.—In transmitting or distributing power by direct current, four factors must be considered in determining the size of conductor.

1. The wires must be able to carry the required current without overheating.

This is particularly important with inside wiring where fire risk exists. Tables of the permissible current-carrying capacity of wires are given in the Appendix, page 479.

2. The voltage drop to the load must be kept within reasonable limits. This is particularly important when incandescent lamps constitute the load.

(3) The wires must be of sufficient mechanical strength. This is important when the wires are strung on poles. It is not advisable to use wires smaller than No. 8 A.W.G. for pole lines.

(4) The economics of the problem must be considered. Increasing the size of conductor means higher investment costs but less energy loss in transmission. That size of conductor should be chosen which makes the cost of the energy loss plus the interest on the investment a minimum. This may be modified in view of the considerations stated in (1), (2) and (3).

CONSTANT-POTENTIAL DISTRIBUTION

273. Distribution Voltages.—About 110 volts has been found to be the most convenient voltage for incandescent lighting.

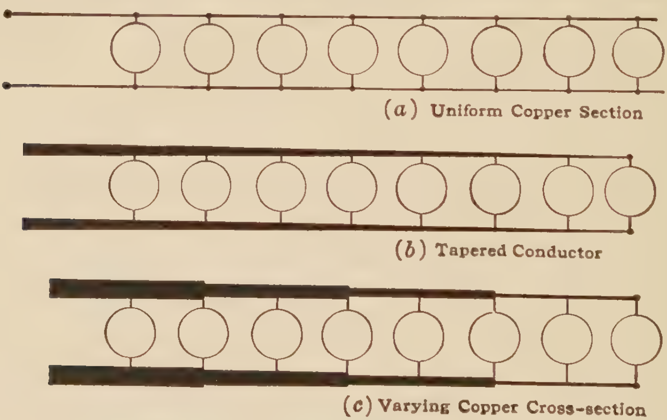


FIG. 373.—Copper cross-section of distributing system or of mains.

It is not so high as to be dangerous to persons. Incandescent lamp filaments for voltages in excess of 110 volts become so long and of so small a cross-section that they are fragile. An even lower voltage than this would be desirable from the standpoint of the filament, but a lower voltage would be accompanied by an increase in the required weight of copper. Therefore, 110 to 115 volts has been standardized for lighting and for domestic use as being the most desirable when all

factors are taken into consideration. Six hundred volts is commonly used for trolley distribution, because it is not so high as to give operating difficulties and it saves considerable copper as compared with systems of lower voltage. At the present time, 1,200, 2,400, and even 3,000 volts are used at the trolley in railway electrification, these higher voltages being for trunk line electrification, not for municipal traction.

274. Distributed Loads.—The load on a feeder or main may be concentrated at one or two points, as is generally the case with feeders, or it may be distributed uniformly or non-uniformly

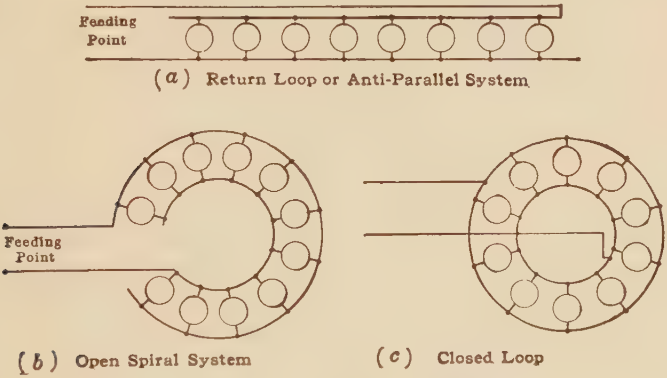


FIG. 374.—System of feeding.

along the conductors, as when lamp loads are located at various points along mains (see Fig. 373).

The conductors may be of uniform cross-section throughout their entire length (Fig. 373 (*a*)). This occurs where the mains are short and the voltage drop is small.

Where the mains are of considerable length, the minimum amount of copper for a given voltage drop is obtained when the mains are uniformly tapered (Fig. 373 (*b*)).

As it is impracticable to have a uniformly tapering conductor, a conductor of constant cross-section is run for a part of the distance, followed by another uniform conductor of lesser cross-section, and so on, as shown in Fig. 373 (*c*). A good rule to remember is that the current *density* in each section should be the same. For example, the first section may consist of a 250,000-C.M. conductor, carrying 200 amp.; assume the second sec-

tion carries 150 amp.; it should be a $^{150}/_{200} \cdot 250{,}000 = 190{,}000$-C.M. conductor. Ordinarily 4/0 wire would be used for this second section.

275. Systems of Feeding.—In order to keep a number of lamps at the same voltage without excessive copper, the *return loop* or *anti-parallel* system shown in Fig. 374 (*a*) is often used. The two feeding wires are connected to opposite ends of the load. This system allows all the lamps to operate at nearly the same voltage and yet the voltage drop in the feeding wires may be large.

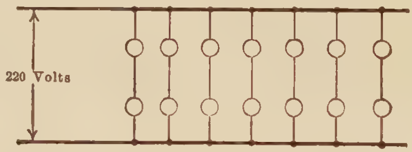


FIG. 375.—Series-parallel system.

The objection to the return loop system is the extra length of wire required. This objection is often overcome by arranging the loads in the manner shown in Fig. 374 (*b*), called the *open-spiral* system. Where large groups of lamps are switched off and on at the same time, as in theaters and auditoriums, it is often possible to arrange the lamps in this way.

The open spiral may be closed at its ends, resulting in the *closed-loop* system of Fig. 374 (*c*).

276. Series-parallel System.—Doubling the voltage of a system results in the weight of required copper being reduced to one-fourth its initial value. If 110-volt lamps be arranged so that two are always in series, as shown in Fig. 375, the system may be operated at 220 volts. The copper section will then be one-fourth that required for straight 110-volt distribution. The obvious disadvantages of the series-parallel system are that lamps can only be switched in groups of two and if one lamp burns out, the lamp to which it is connected ceases to operate. Also, both of the lamps in series must be of the same rating.

THE EDISON 3-WIRE SYSTEM

277. Advantages.—The objections to the series-parallel system may be eliminated by running a third wire, called a *neutral*,

between the two outer wires. This neutral maintains all the lamps at approximately 110 volts. The advantage of a higher voltage in reducing the weight of copper is obtained by the use of this system. If there were no neutral wire, the 220-volt system would require one-fourth the copper of an equivalent 110-volt system. If it be assumed that the neutral of the Edison system is of the same cross-section as the two outer wires, the total copper for the Edison system is ⅜ or 37½ per cent. that for a 110-volt system of the same kilowatt capacity. Therefore the saving in copper is 62½ per cent. In practice, the neutral can be made smaller than the two outer wires so that the saving in copper is even greater than 62½ per cent.

The general plan of the system is shown in Fig. 376. Two wires A and B have 220 volts maintained between them, A being the positive and B the negative. A third wire N is maintained at a difference of potential of 110 volts from each of the other two wires. Therefore N must be negative with respect to A and positive with respect to B. That is, current tends to flow from A to N, and from N to B.

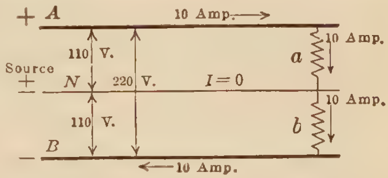


FIG. 376.—Edison 3-wire system—balanced loads.

Figure 376 shows the conditions which exist when the load on each side of the system is the same. Each of the loads a and b takes 10 amp. The 10 amp. taken by load a passes through to load b and then back through wire B to the source. This is equivalent to a series-parallel system as both loads are equal and are in series. Under these conditions the current in the neutral wire is zero and the loads are said to be balanced.

Figure 377 (a) shows the conditions existing when the load a on the positive side of the system is 10 amp., and the load b on the negative side is 5 amp. Under these conditions the extra 5 amp. taken by load a must *flow back* through the neutral to the generator or source. Therefore there are 5 amp. in the neutral returning to the generator. In Fig. 377 (b) the load b is now 10 amp. and load a is 5 amp. Under these conditions the extra 5 amp. must *flow out* to the load through the neutral. It will be observed that the current in the neutral may flow in

either direction, depending upon which load is the greater. Therefore, if an ammeter is used in a neutral it should be of the zero-center type. Moreover, it will be observed that the neutral carries the *difference* of the currents taken by the two loads. In practice the loads are usually so disposed that they are nearly balanced. Ten per cent. unbalancing (that is, a neutral current which is 10 per cent. that in the outer wires) is usually allowed

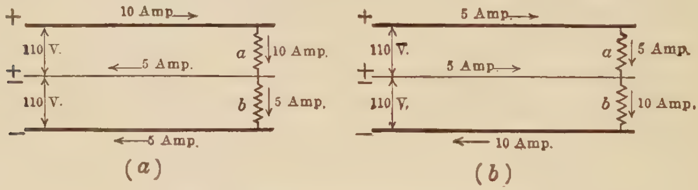


FIG. 377.—Unbalanced 3-wire systems.

in the larger systems, while in a system of small capacity the unbalance may reach 25 per cent. at times. The current in the neutral is ordinarily much smaller than that in the outers and a much smaller conductor can be safely used. In practice the neutral is usually grounded.

Effect of Opening the Neutral.—In practice, it is very desirable to keep the neutral of the 3-wire system closed under *all* conditions. The reason for this is illustrated by the following example.

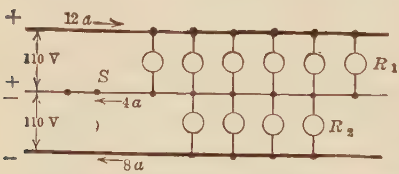


FIG. 378.—Effect upon the balancing of a 3-wire system of opening the neutral.

Figure 378 shows two lamp loads on a 3-wire system. The load on the positive side consists of six lamps each taking 2 amp., making a total of 12 amp. The load on the negative side consists of four lamps each taking 2 amp., making this load 8 amp. The voltage across each load is 110 volts so the resistance R_1 of the positive load is

$$R_1 = {}^{110}\!/\!_{12} = 9.17 \text{ ohms.}$$

The resistance of the negative load is

$$R_2 = {}^{110}\!/\!_{8} = 13.75 \text{ ohms.}$$

If the neutral now be opened at the point S, the two loads R_1 and R_2 are in series and therefore each must take the same current. The total resistance R is $R_1 + R_2 = 22.92$ ohms.

There is now 220 volts across these two loads in series, so that the current

$$I = \frac{220}{22.92} = 9.60 \text{ amp.}$$

The voltage V_1 across load R_1

$$V_1 = 9.60 \times 9.17 = 88.0 \text{ volts.}$$

The voltage V_2 across load R_2

$$V_2 = 9.60 \times 13.75 = 132 \text{ volts.}$$

This assumes that the resistance of the lamp filaments does not change. It will be observed, however, that the larger bank of lamps is operating at a much reduced voltage, resulting in a material decrease of candlepower, and that the smaller bank is operating considerably above rated voltage, which would result in the lamps burning out in a short time.

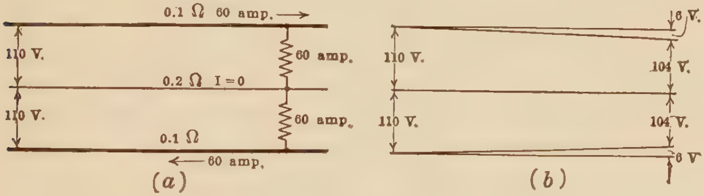


FIG. 379.—Voltage drop in a 3-wire system having balanced loads.

For the above reason the neutral of the 3-wire system is usually grounded and one rarely sees circuit breakers in the neutral wire of power plants.

278. Voltage Unbalancing.—The voltage on the two sides of a 3-wire system may become considerably unbalanced if the loads on the two sides of the system become unequal, as shown in Fig. 379.

In Fig. 379 (*a*) a load of 60 amp. exists on each side of the system. Each outer wire has a resistance of 0.1 ohm and the neutral has a resistance of 0.2 ohm. The generator voltage is 220 volts across the two outer wires.

As the two loads are equal, there is no current in the neutral wire. Therefore, the voltage drop per wire for the outers is

$$e = 60 \times 0.1 = 6.0 \text{ volts.}$$

The voltage across each load is 104 volts. There is no voltage drop along the neutral, as it carries no current. Figure 379 (*b*) shows a plot of the voltage distribution.

Assume that the loads are as shown in Fig. 380, 100 amp. on one side of the system and 20 amp. on the other side. This represents the same total amperes as in Fig. 379.

The drop in the positive wire

$$e_1 = 100 \times 0.1 = 10 \text{ volts.}$$

The drop in the neutral

$$e_2 = 80 \times 0.2 = 16 \text{ volts.}$$

Voltage across positive load

$$V_1 = 110 - 26 = 84 \text{ volts.} \quad \textit{Ans.}$$

The drop in the negative wire

$$e_2 = 20 \times 0.1 = 2 \text{ volts.}$$

Voltage across negative load

$$V_2 = 110 - 2 + 16 = 124 \text{ volts.} \quad \textit{Ans.}$$

There is now 40 volts difference between the voltages on the two sides of the system.

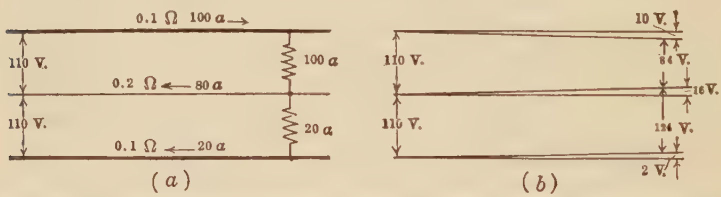


FIG. 380.—Voltage unbalancing in a 3-wire system having unbalanced loads

Under these conditions, the voltage across the load on the negative side is *greater* than the voltage on the negative side of the system at the power station. This rise in voltage from power station to load is obtained at the expense of the drop in the neutral. Figure 380 (*b*) shows these conditions graphically.

When motor loads are to be connected to a 3-wire system they are usually connected between the two outer wires rather than between an outer wire and neutral so that they will not produce any voltage unbalancing. In fact some power companies will not permit motor loads exceeding one horsepower to be connected to neutral.

METHODS OF OBTAINING A 3-WIRE SYSTEM

There are various methods of obtaining a 3-wire system which are as follows:

279. Two-generator Method.—Two shunt generators may be connected in series as shown in Fig. 381. The positive terminal of one should be connected to the negative terminal of the other; that is, the generators are in series between the outers. Both generators may be driven by the same prime mover. When

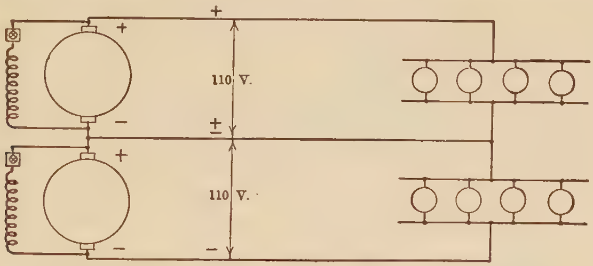


FIG. 381.—Two generators supplying a 3-wire system.

connected in this manner, each machine supplies only the load on its own side of the line. The obvious objection to this method is that two separate machines are required.

280. Storage Battery.—*A storage battery* may be floated across the line as shown in Fig. 382. The neutral wire is connected to

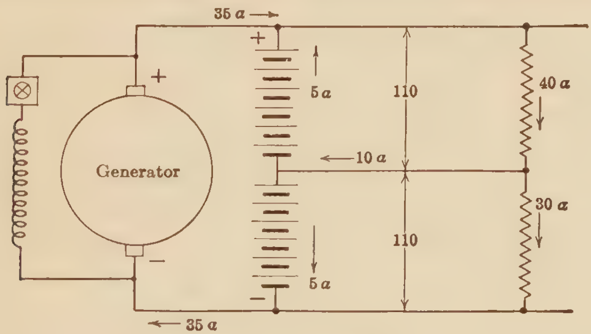


FIG. 382.—Storage battery giving neutral in a 3-wire system.

the middle point of the battery. When the load is unbalanced, that half of the battery on the more heavily loaded side will discharge and the other half will be charged. Figure 382 shows an unbalancing of 10 amp. In this particular case the upper half of the battery supplies 5 amp., and the other 5 amp. in the neutral go to charge the lower half of the battery. The objec-

tions to this method of obtaining a neutral are the high maintenance cost of a storage battery and the difficulty of maintaining both halves of the battery at the same condition of charge.

281. Balancer Set.—A balancer set is a very common method of obtaining the neutral. This set consists of a motor and a generator mechanically coupled together. They are connected in series across the outer wires and the neutral is brought to their common terminal, as shown in Fig. 383.

The action of this set may best be illustrated by the hydraulic analogy shown in Fig. 384. Water is supplied by the canal *A*. This water falls over a weir into canal *B* and may be made to do

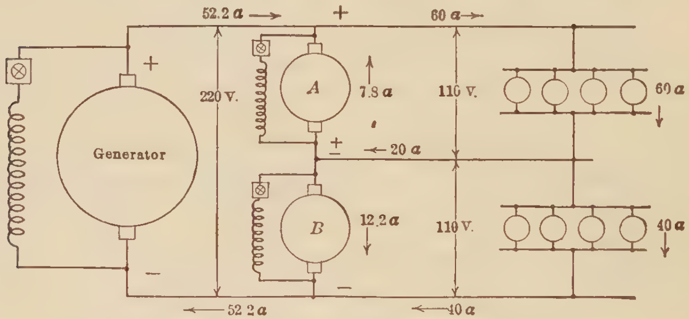


FIG. 383.—Balancer set giving neutral in a 3-wire system.

useful work in so doing. All this water is not needed at the point of utilization *D* between the canal *B* and the tail race *C*. Some of the water which is not needed at *D* passes to *C* through the water wheel shown in the figure. This water wheel is belted to a centrifugal pump operating between *B* and *A*. In virtue of the water passing through the water wheel some of the water in the canal *B* is pumped back to *A* by the pump, where it may be utilized again. The water wheel corresponds to the motor or machine *B* (Fig. 383) and the centrifugal pump to the generator or machine *A*.

If in Fig. 384 more water is required between canals *B* and *C* than can be supplied by the weir at *A*, the centrifugal pump may act as a water wheel and the water wheel as a pump. Some of the extra water required in *B* will be supplied through the upper machine operating as a water wheel and discharging into *B*. In so doing the upper machine drives the lower machine as a pump.

The lower machine then pumps water from C back to B. This condition corresponds to an excess of load on the negative side of the system of Fig. 383.

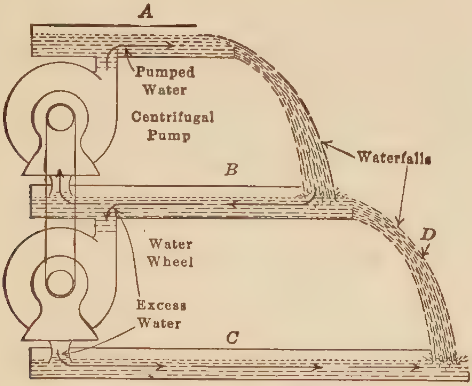


FIG. 384.—Water-wheel analogy of balancer set.

If in Fig. 383 there is an excess of load on the positive side of the system, as represented by 20 amp. in the neutral, 12.2 amp. of this 20 amp. flows through the motor and in dropping through 110 volts gives up its energy. The motor then causes the generator to pump 7.8 amp. back to the positive side of the line. This current distribution is determined in the following manner:

Each of the machines A and B is assumed to have 80 per cent. efficiency. Let I_1 be the generator current in machine A and I_2 be the motor current in machine B. The generator output will be $0.8 \times 0.8 = 0.64$ times the motor input. Assuming that the voltages are equal, actually they will be slightly unbalanced,

$$110I_2 \times 0.64 = 110I_1.$$

$$I_1 + I_2 = 20.$$

Solving

$$I_1 = 7.8 \text{ amp.}$$

$$I_2 = 12.2 \text{ amp.}$$

The machines will respond more readily to unbalanced loads if their fields are crossed, that is, if the motor field is across the generator side of the line and the generator field is across the motor side of the line. In order that a generator may supply additional current, either its terminal voltage must drop or its

induced voltage must rise. In order that a motor may take additional load, either its terminal voltage must rise or its induced voltage must drop. The excess load on the positive side of the system (Fig. 383) tends to reduce the field of machine A and to increase that of machine B. These effects are the reverse of what is desired. If the generator field is across the motor side of the line, the increased voltage is now across the generator field and will raise the generator induced voltage. Therefore, its terminal voltage need not drop so much to take care of unbalanced currents. The same result may be obtained by compounding the two machines. The series fields should be so connected that the machine acting as a generator is cumulatively com-

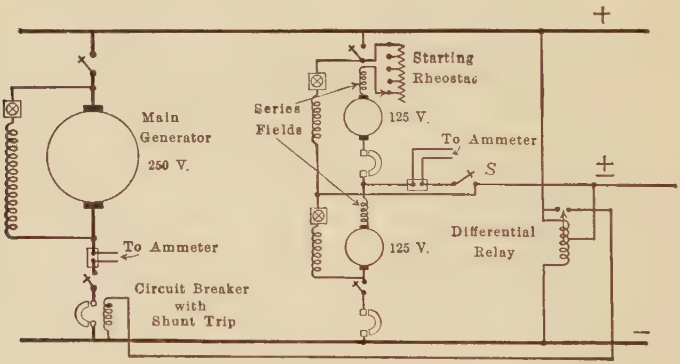


Fig. 385.—Connections of a 3-wire system using a balancer set.

pounded, and that acting as a motor is differentially compounded. If the compounding is properly adjusted, the set may take care of the neutral current with practically no change in the voltages across the terminals of the two machines.

Figure 385 shows standard connections for a balancer set with series fields. The machines are started in series, with the neutral switches open and the shunt fields in series across the line. When the machines are up to speed the neutral switch S is closed. If the voltages on the two sides of the system become widely different, the currents in the two halves of the differential relay become unbalanced. This relay then closes the tripping-coil circuit of the main generator breaker, resulting in the main generator circuit opening, even though *its* load is not excessive.

282. Three-wire Generator.—The 3-wire generator or Dobrowolsky method is a very efficient method of obtaining a neutral. The details of the method can be understood better after alternating currents and the synchronous converter have been studied.[1] The principle of the method is as follows: Alternating current is generated within a direct-current armature as has already been shown. If slip-rings be employed, alternating current can be obtained from the machine. A coil wound on an iron core which therefore has high inductance and offers a high impedance to this alternating current is connected across

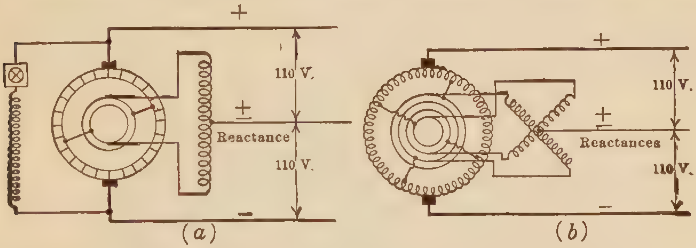


FIG. 386.—Three-wire generator connections (Dobrowolsky method).

the slip-rings. The center of this reactance coil is at the center of gravity of the voltages generated within the armature. Further, this inductance coil offers very little resistance to the flow of direct current. Therefore, if the neutral be connected to the center of this coil, the voltage to either brush from the neutral will be the same. Moreover, any current flowing back through the neutral can readily flow back into the armature through this reactance. The connections of such a generator are shown in Fig. 386 (*a*). Sometimes, to obtain better balancing, two and even three reactances are employed. All have their neutrals tied together, as shown in Fig. 386 (*b*). Occasionally the reactances are placed within the armature. This arrangement requires but one slip-ring, but increases the weight of the armature.

The Edison 3-wire system may be extended to 4, 5, 6, and 7-wire systems. (See Fig. 339, page 398.) The complications and number of wires prevent these multi-wire systems being extensively used.

[1] See Vol. II (revised), Chap. XII.

283. Feeders and Mains.—In congested districts the mains form an underground network. This network is supplied at various points, called centers, by feeders connected to the direct-current bus-bars at the power station. It requires a careful study of the various loads, amount of copper, etc., in order to determine the most advantageous feeding points or centers. Two or more substations may simultaneously feed the same centers. In order that the voltages at these centers may be determined and so maintained at the desired values, pilot or pressure wires run back to the station voltmeter. By means of a dial switch the operator is able to read the voltages at the various centers. Figure 387 shows the cross-section of a concentric 1,000,000-C.M. cable. The outer and inner conductors

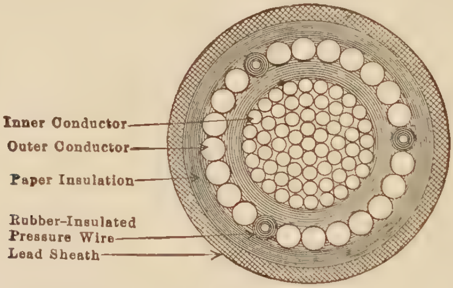


Fig. 387.—Cross-section of a 220-volt, 1,000,000-C.M. concentric cable.

are the outer wires of the Edison 3-wire system. The neutral is usually a separate wire of much smaller cross-section, or there may be one large neutral common to several feeders and mains. The three pilot or pressure wires are connected, one to each outer wire and one to the neutral at the feeding point. If the operator finds that the voltage is too low at the feeding point, he throws a feeder to a bus-bar of higher voltage. A large voltage drop can exist in such feeders, as no loads are taken off at intermediate points.

In practice the following are the percentage drops usually allowed: In feeders, 5 to 10 per cent.; in the distribution mains, 3 per cent.

The services are usually taken directly from the distribution mains and occasionally, extra large loads may be taken directly from junction boxes. Junction boxes are circular iron castings

containing a set of insulated bus-bars, to which either the distribution mains or the feeders are connected. Distribution mains are connected, through fuses, to suitable terminals already installed in the junction boxes. A junction box thus provides a convenient method of connecting the single feeding wires to the several distribution wires. The mains are almost always fused, but only disconnecting links are used for the feeders, it being

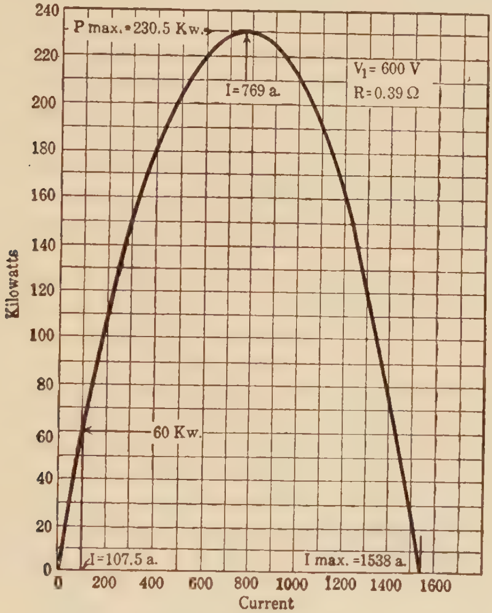


FIG. 388.—Power and current over a feeder.

deemed advisable to allow the feeders to burn themselves clear of any short-circuits.

284. Maximum Power over a Feeder.—In Par. 75, page 79, it was shown that the maximum power which a battery can deliver occurs when the external resistance is equal to the internal resistance of the battery. Under these conditions the voltage drop within the battery is equal to its terminal voltage. This same rule applies to any condition where a constant-voltage source is supplying a concentrated load through a fixed resistance. Consider a direct-current feeder or transmission line which is fed

from constant-voltage bus-bars. The line resistance is constant. As load is applied to the end of the line, the current I increases and the voltage V at the load decreases, as was shown in Par. 69, page 71. The power delivered is equal to VI. This power will increase so long as I increases faster than V decreases. When the load voltage V is equal to one-half the sending-end voltage, the product VI is a maximum. Further decrease in the load resistance results in V decreasing faster than I increases and the power therefore decreases with further increase in I. This relationship is shown in Fig. 388. The following example is given to illustrate these conditions.

Example.—A power station supplies 60 kw. to a load, over 2,500 ft. of 000 copper feeder whose resistance is 0.078 ohm per thousand feet. The bus-bar voltage is maintained constant at 600 volts. Determine: (*a*) the current; (*b*) the voltage at the load; (*c*) the efficiency of transmission; (*d*) the maximum power which can be transmitted; (*e*) the maximum current which can be supplied.

(*a*) Total resistance, $R = 5 \times 0.078 = 0.39$ ohm.

Let I be the current and V the voltage at the load.

$$60{,}000 = VI \qquad \text{(I)}$$

$$V = 600 - 0.39I \qquad \text{(II)}$$

Substituting V from Eq. (II) into Eq. (I),

$$60{,}000 = (600 - 0.39I)I$$

$$0.39I^2 - 600I = -60{,}000 \qquad \text{(III)}$$

Dividing Eq. (III) by 0.39

$$I^2 - 1{,}538I = -153{,}800 \qquad \text{(IV)}$$

This is a quadratic equation and must have two roots. Hence, two values of current will satisfy the given conditions. Completing the square and solving,

$$I^2 - 1{,}538I + (769)^2 = -153{,}800 + (769)^2.$$

$$(I - 769)^2 = 437{,}500.$$

$$I - 769 = \pm\sqrt{437{,}500}.$$

$$I = +769 \pm 661.5.$$

$$= 1430.5 \text{ or } 107.5 \text{ amp.}$$

Both values of current satisfy Eqs. (I) and (II), but if the larger value of current were used, the efficiency would be too low to be practicable. Hence the current

$$I = 107.5 \text{ amp.} \quad \textit{Ans.}$$

(*b*) $V = 600 - 0.39 \times 107.5 = 600 - 42 = 558$ volts. *Ans.*

$P = 558 \times 107.5 = 60$ kw. (check).

(*c*) $\eta = \dfrac{558}{600} = 0.93$ *Ans.*

(*d*) Repeating Eqs. (I), (II), (III) and (IV) with the power represented by P,

$$P = VI = (600 - 0.39I)I \qquad \text{(V)}$$

$$0.39I^2 - 600I = -P$$

Dividing by 0.39 and completing the square ($591{,}000 = \overline{769}^2$.).

$$I^2 - 1{,}538I + 591{,}000 = 591{,}000 - 2.564P.$$

$$I - 769 = \pm\sqrt{2.564}\,\sqrt{230{,}500 - P}.$$

$$I = 769 \pm 1.6\sqrt{230{,}500 - P}.$$

If the power P exceeds 230,500 watts, the expression under the radical becomes negative, the radical an imaginary quantity and the current becomes the sum of a real and an imaginary quantity, an impossible condition, practically. Hence the maximum value which the power P can have is equal to 230,500 watts, or 230.5 kw.

Since, under these conditions, the radical is equal to zero, the two roots of the equation are equal. Hence, the current I has but a single value and is equal to 769 amp. (see Fig. 388).

The voltage at the load,

$$V = 600 - 769 \times 0.39 = 600 - 300 \text{ or } 300 \text{ volts.}$$

Under these conditions, one-half the power is lost in the line, and the efficiency is 50 per cent. This corresponds to the similar conditions for batteries.

(*e*) The maximum current occurs when the load is short-circuited. That is,

$$I = \frac{600}{0.39} = 1{,}538 \text{ amp.} \quad \textit{Ans.}$$

The power under these conditions must be zero since the voltage at the load is zero.

To illustrate further these relations between power and voltage, the curve (Fig. 388) is given. Current is plotted as abscissas and power as ordinates. Different values of current are assumed, substituted in Eq. (V), and the equation solved for the power P. The power must be zero when the current is zero (voltage V a maximum) and when the voltage V is zero (current a maximum). It is clear that for any given value of power, except the maximum, there are two different values of current. The maximum power is equal to 230.5 kw. and occurs when the current is equal to 769 amp. At this point both roots of the quadratic equation are equal. In practice the power transmitted is usually a small proportion of the maximum power, as can be seen by the position of the 60-kw. point on the curve (Fig. 388).

285. Electric Railway Distribution.—Electric railway generators are generally compounded, the series field being on the negative side. The negative terminal is usually connected directly to ground or to the rail through a switch. The positive terminal feeds the trolley through an ammeter, a switch, and a circuit breaker.

On short lines, with light traffic, the trolley alone may suffice to carry the current to the car, as shown in Fig. 389 (*a*). Except

in small installations, the trolley is of insufficient cross-section to supply the required power and at the same time to keep the voltage drop within the necessary limits. At the size of the trolley wire is limited by the trolley wheel, it cannot be conveniently

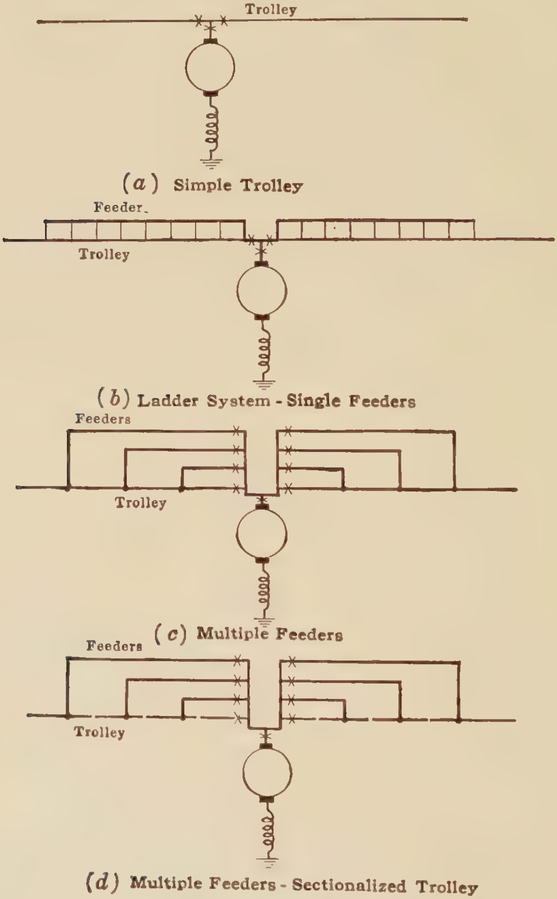


FIG. 389.—Methods of feeding a trolley system.

increased. The same effect as increasing the size of the trolley may be obtained by running a feeder in parallel with the trolley and connecting the feeder to the trolley at short intervals, as shown in Fig. 389 (b). This is called the *ladder system* of feeding.

The trolley and feeder together may be considered as forming a single conductor.

Where the density of traffic requires several feeders, the best results are obtained by connecting the feeders in the manner shown in Fig. 389 (*c*). Each feeder is protected by a circuit breaker.

The objections to the preceding methods of feeding are that trouble, due to a ground, for example, at any point on the trolley, involves the entire system. In cities where traffic is particularly dense, it is not permissible to take chances of having the entire system shut down due to a ground at one point only. Therefore, the trolley is sectionalized as shown in Fig. 389 (*d*). In this method the trolley is divided into insulated sections, each of which is supplied by a separate feeder. Trouble in one section is not readily communicated to the other sections. This

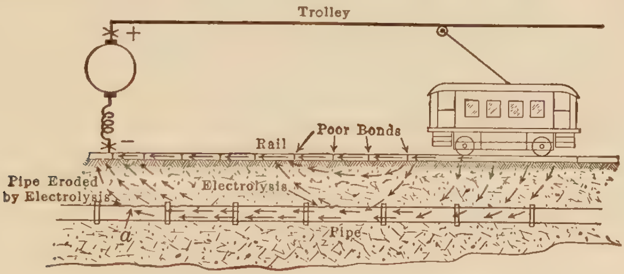


FIG. 390.—Electrolysis by earth currents.

increased reliability is obtained at the expense of a less efficient use of the copper, as the feeders are unable to assist one another. In the preceding systems this mutual help is obtained.

286. Electrolysis.—Most trolley systems use the track as the return conductor for the current taken by the car. The return currents not only pass through the tracks themselves, but seek the paths of least resistance by which they may return to the negative terminal of the station generator. Such currents in spreading through the earth follow such low-resistance conductors as water pipes, gas pipes, cable sheaths, etc., as shown in Fig. 390. The fact that the current *enters* and flows along these conductors in itself does no harm. However, it is obvious that such currents must ultimately leave these pipes as at (*a*) (Fig.

390). In so doing they tend to carry the metal of the pipe into electrolytic solution, which ultimately results in the pipe being eaten away. To decrease the effects of electrolysis several expedients have been devised. The two most successful methods are the following: (*a*) provide as good a return path through the track as is practicable. This is done by good bonding and by using insulated negative feeders, that is, heavy copper feeders that are run back to the negative bus from various points along the track. Figure 390 shows how poor rail bonds may cause the current to leave the track and enter the pipe. In some cities the total permissible drop in the ground return circuit must not exceed from 10 to 15 volts. (*b*) Discourage the entering of the current into the pipes by inserting occasional insulating joints in the pipes.

In testing for electrolysis the usual method is to measure the voltage existing between the track and the water pipes (as at a hydrant). The magnitude of this voltage indicates roughly the magnitude of the current which must be flowing from one to the other. The polarity shows which way the current is flowing. For example, if the track is positive to the pipe, current must be flowing from the track to the pipe.

The electrolysis situation is still in an unsettled state, both as regards its mitigation and as to the ultimate responsibility for the damage resulting.

STORAGE BATTERY SYSTEMS

287. Central Station Batteries.—Figure 391 shows a typical load curve of a central station. Between 11.00 p.m. and 5.00 a.m. the load is comparatively small, consisting of street lights and a few all-night commercial loads.

This portion of the load curve is called a "valley." The load increases rapidly from 5.00 to 7.00 a.m. due to commercial power loads, lights and perhaps to the beginning of street-car service. The morning peak occurs about 8.00 a.m. The load drops off gradually until noon.

The valley between 12.00 and 1.00 is due to the shutting down of the commercial loads because of the luncheon hour. The evening peak, which is usually the largest, occurs between 5.00 and 6.00 p.m. This peak may hold up for an hour, after which

it drops to the evening load, which consists mostly of lighting. This load gradually diminishes to the all-night value.

Obviously the power company must have sufficient station and distributing capacity to carry the peak. Even though this apparatus is in use only one hour a day, the investment charges are in effect 24 hr. a day.

The ratio of the average load to the maximum load of a station is called the *load factor*.

Example.—A station delivers 192,000 kw.-hr. in a day and its peak load is 20,000 kw. What is the daily load factor?

$$\text{The average load} = \frac{192,000}{24} = 8,000 \text{ kw.}$$

$$\text{The load factor} = \frac{8,000}{20,000} = 40 \text{ per cent.} \quad \textit{Ans.}$$

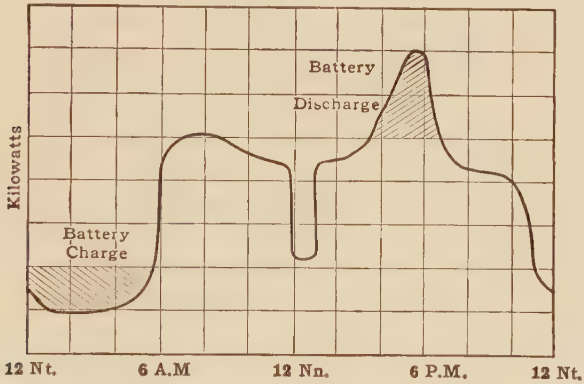


FIG. 391.—Battery smoothing out central station load curve.

Obviously a *high* load factor is very desirable. In fact power companies welcome any loads that will fill in the valleys of the curve and are usually prepared to offer attractive rates for such loads in order to improve their load factors and thus to utilize apparatus at times when it would otherwise be idle.

The load curve of a direct-current station may be smoothed out by the use of a storage battery. The battery may be charged at night and early morning and so fill in the valley of the load curve and then be discharged on the peak of the load curve, as shown in Fig. 391. This equalizes the load on the station and increases its load factor.

As a rule, batteries are not installed for the purpose of smoothing out the load curve. A storage battery operating under the best conditions is good for only a limited number of complete charges and discharges. Therefore, the battery maintenance is usually found to more than offset the economies effected by taking some of the load off the peak. Such batteries may be very useful in office buildings and other isolated plants, because it is often possible to shut down the entire lighting plant and run on the batteries at night, thus eliminating considerable labor charge.

Batteries are commonly installed as reserve in large central station systems. They are placed, therefore, near the center of the load. In case of a shut-down in the generating system or in the transmission system, the battery can help maintain service. For this reason pasted plate batteries are more often used, because of their high overload capacity (see page 116, Par. 102).

Storage batteries are also useful in taking care of unexpected loads. For example, a thunder-storm may result in a sudden demand which could not be foreseen and so cannot be met immediately by the generating station, as it takes time to get up steam and put a generator on the line. A battery may be put on the line immediately and so carry the sudden load increase until boilers and turbines can be brought into service. If the battery is already floating across the line it takes the load increase automatically.

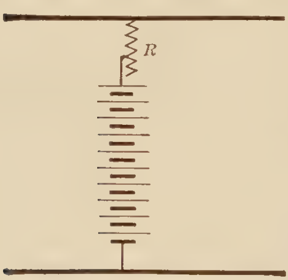


FIG. 392.—Resistance control of battery discharge.

288. Resistance Control.—In order to control the load taken by a generator connected to the bus-bars, it is necessary to change its induced voltage by adjusting the field current. It is not possible to adjust the voltage of a storage battery in this manner. One method of controlling the battery load is to have the battery voltage several volts higher than the bus-bar voltage and to insert resistance in series with the battery, as shown in Fig. 392. By adjusting this resistance, the load delivered by the battery may be controlled. The disadvantage of this method is the loss of power in the resistance and the voltage drop in the resistance,

which depends upon the load. Even with constant load the resistance must be adjusted occasionally to compensate for the drop of battery voltage during discharge.

Example.—It is desired to discharge a storage battery, consisting of 115 cells each having an electromotive force of 2.1 volts and an internal resistance of 0.001 ohm, into 220-volt bus-bars so that the battery delivers 100 amp. To what value must the series resistance be adjusted?

The total battery electromotive force

$$E = 115 \times 2.1 = 242 \text{ volts.}$$

The bus-bar voltage

$$V = 220 \text{ volts.}$$

The battery resistance

$$r = 115 \times 0.001 = 0.115 \text{ ohm.}$$

Let R = the added external resistance

$$100 = \frac{242 - 220}{0.115 + R}.$$

$$100R = 22 - 11.5 = 10.5.$$

$$R = 0.105 \text{ ohm.} \quad \textit{Ans.}$$

289. Counter Electromotive Force Cells.—If an electric current be sent through two plain lead plates immersed in dilute sulphuric acid, a simple storage battery is formed which immediately develops a counter electromotive force of about 2.0 volts (see page 110, Par. 99). Neglecting the small IR drop in such a cell, the counter electromotive force is practically independent of the current. This principle is utilized in controlling the current delivered by a battery.

Plain lead plates are immersed in dilute acid and are connected in series with the battery. If it is desired to decrease the discharge rate of the battery more of these cells are cut in. To do this an end cell switch, similar to that shown in Fig. 393, is used. The advantage of this method over the resistance control is that the opposing or control electromotive force is independent of the load.

290. End-cell Control.—A battery usually consists of a sufficient number of cells to give an electromotive force exceeding that of the bus-bars by an ample margin. To charge such a battery a booster may be used (see page 127, Par. 111). The electromotive force of the battery, and hence its load, may be controlled by cutting in or out the cells at the end of the battery.

It is essential to do this without opening the circuit. For this purpose a switch similar to that shown in Fig. 393 is used.

The main contact is connected to the auxiliary contact by a resistance R. When sliding from one battery contact to the next the auxiliary contact maintains the circuit connections through the resistance R. Were there zero resistance between the main contact and its auxiliary contact the individual cells would be dead short-circuited during the transition period. The resistance R is usually so chosen as to allow the normal battery current to flow during the transition period. The end-cell switches become rather massive in large battery installations and are often operated by a motor-driven worm. This also permits remote control.

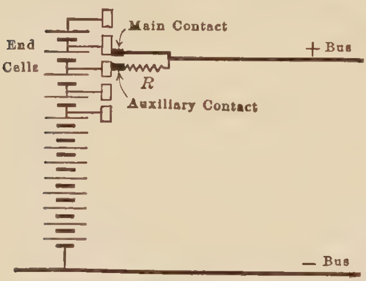


FIG. 393.—End-cell control of storage battery.

The end cells, not being in continuous service, are discharged to a lesser degree than the others. Therefore they require individual attention on charging.

291. Floating Battery.—A battery is occasionally used to equalize sudden fluctuations of load such as commonly occur in railway systems. The battery voltage should be such that with an average load on the station it is just equal to the bus-bar voltage. The battery is then delivering no current and is merely "floating."

When a sudden load comes on the station, the bus-bar voltage drops. The battery then discharges and assists the generators. On the other hand, if the load drops to a low value, the bus-bar voltage rises and the battery charges.

As a rule the bus-bar voltage does not change enough to cause the battery to respond sufficiently to the load changes. In fact with over-compounded railway generators the reverse action might well occur. There are several methods of causing the battery to charge and discharge at the proper time. One typical method is shown in Fig. 394. The battery is connected to the bus in series with a motor-driven booster. Two carbon rheostats R_1 and R_2 are connected in series across the battery. The booster field is connected from their common point to the middle of the battery. If $R_1 = R_2$, the booster field is connected

across two points of equal potential, the field current is zero and no voltage is induced in the armature. An increase of load, however, causes solenoid P to pull down on the lever. This compresses R_1 and releases the pressure on R_2. The resistances R_1 and R_2 now differ considerably so that the booster field is no longer across points of equal potential. A current now flows through the booster field causing the booster to generate an electromotive force of such polarity as to assist the battery to discharge. S is a spring.

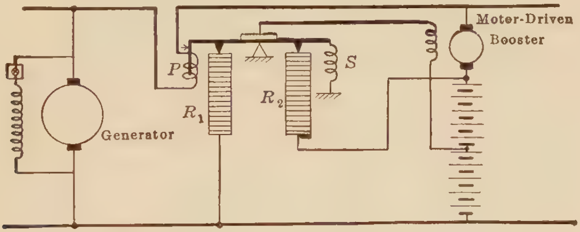


FIG. 394.—Regulating storage battery discharge with a booster set.

In order to reduce the current flowing through R_1 and R_2 and the battery, the change of booster excitation is often accomplished through an intermediary exciter, whose field is connected in the same manner as that of the booster of Fig. 394.

Battery at End of Line.—Very poor voltage regulation may occur at the end of a trolley line, due to insufficient copper. Rather than to install more copper, it may be more economical to install a storage battery at the end of the line. This battery not only steadies the trolley voltage but tends to reduce violent fluctuation of the power station load as well.

As the voltage at the end of the line requiring a battery undergoes fluctuations of considerable magnitude, the battery is usually self regulating both as regards charge and discharge. With little load on the line, the voltage at the battery should be high enough to charge it. On the other hand, when a car is near the battery, the line voltage should drop to such a value as to allow the battery to discharge and assist the power station.

Example.—The bus-bar voltage (Fig. 395) at the station is maintained constant at 600 volts. A 4/0 trolley having a resistance of 0.26 ohm per mile extends out 4 mil. from the station. The resistance of ground and track is 0.05 ohm per mile. At the end of the line a storage battery consisting of

240 cells is "floated." Each cell has an average electromotive force of 2.0 volts and a resistance of 0.002 ohm. At what rate will the battery charge when there is no load on the line? When the load at the battery is 150 amp., how much current does the battery supply and how much does the station supply?

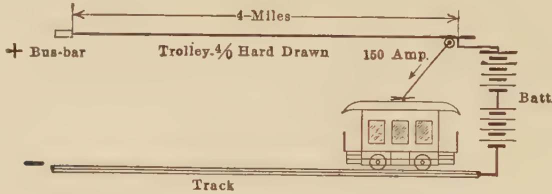


FIG. 395.—Battery floating at end of trolley line.

The total resistance of the trolley and track

$$R = 4(0.26 + 0.05) = 1.24 \text{ ohms.}$$

The battery resistance

$$R_2 = 240 \times 0.002 = 0.48 \text{ ohm.}$$

The total battery electromotive force

$$E = 240 \times 2.0 = 480 \text{ volts.}$$

When there is no load between the station and the battery the current to the battery

$$I_1 = \frac{600 - 480}{1.24 + 0.48} = \frac{120}{1.72} = 70 \text{ amp.} \quad \textit{Ans.}$$

To find the division of the 150-amp. load at the battery, first find the current at which the battery will just "float."

$$I' = \frac{600 - 480}{1.24} = \frac{120}{1.24} = 96.8 \text{ amp.}$$

That is, with a load of 96.8 amp. at the battery the line drop from the power station to the battery will be 120 volts, making 480 volts at the battery. Under these conditions the battery will neither charge nor discharge but will "float."

The remaining 53.2 amp. will be divided inversely as the line and battery resistance. (See Par. 81.)

Let I_L be the line current and I_B the battery current.

$$\frac{I_B}{I_L} = \frac{1.24}{0.48}.$$

$$I_B + I_L = 53.2.$$

Solving

$$I_B = 38.4 \text{ amp.}$$
$$I_L = 14.8 \text{ amp.}$$

The station is already supplying 96.8 amp.

The total station current is then 96.8 + 14.8 = 111.6 amp. and the battery current is 38.4 amp. *Ans.*

This may be checked by calculating the voltage at the battery.

$$600 - (111.6 \times 1.24) = 461.6 \text{ volts.}$$
$$480 - (38.4 \times 0.48) = 461.6 \text{ volts.} \quad \text{(Check)}$$

292. Series Distribution.—In the parallel system of distribution, the loads are all independent of one another. That is, a load applied at any one point does not affect any of the other loads, provided the voltage does not change. In the series system the loads are all in series with one another so that the same current passes through each. Therefore, if the circuit of any one load be opened, the current to all the other loads will be interrupted. As this is not permissible in practice, a load must be *short-circuited* when it is desired to remove it from service.

Power is usually supplied to a constant-current system by one of two methods; the series generator, of which the Brush arc and Thomson-Houston machines are examples, and the constant-current transformer operating in conjunction with the mercury-

FIG. 396.—Open loop series circuit.

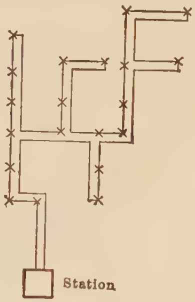


FIG. 397.—Parallel loop series circuit.

arc rectifier (see Chap. VIII, Vol. II). Both of these methods tend to maintain constant current under all conditions of load. Therefore, if the circuit be opened and a very high resistance thus introduced, a constant current is maintained across a high resistance and a very high voltage results. For this reason, the lamps used on a constant-current system are protected by having a thin disc of paper between the lamp terminals (film cut-out). If the lamp burns out, the high voltage across this paper punctures it and so prevents the circuit being opened.

The advantage of the series system is the small amount of copper required. This is due to the fact that the copper carries only the current of any single load. As the loads are in series, the resulting voltage is high. Therefore, this system is applicable only to outside work, such as street lighting, because it would ordinarily be dangerous to have such high voltages in buildings.

There are two general methods of connecting such series loads. In the open loop system, shown in Fig. 396, the circuit is connected to the loads without reference to the separation of the two conductors. This system is economical of copper.

In the parallel loop system the outgoing and return conductors are always kept near each other, as shown in Fig. 397. This system requires more copper than the other but facilitates testing for faults and reduces inductive disturbances.

APPENDIX A

Relations of Units

Length

1 inch	= 2.54 centimeters
1 foot	= 30.48 centimeters
1 mile	= 1.609 kilometers

Area

1 circular mil	= 0.7854 square mil
1 circular mil	= 0.000507 square millimeter
1 square inch	= 6.452 square centimeters
1 square meter	= 10.76 square feet

Volume

1 cubic inch	= 16.39 cubic centimeters
1 liter	= 1,000 cubic centimeters
	= 0.2642 gallon
1 gallon	= 231 cubic inches

Weight

1 gram	= 981 dynes
1 ounce (av.)	= 28.35 grams
1 kilogram	= 2.205 pounds
1 ton	= 2,000 pounds
1 long ton	= 2,240 pounds
1 metric ton	= 1,000 kilograms
	= 2,205 pounds

Work

1 joule (watt-second)	= 10,000,000 ergs
1 gram degree Centigrade (gram calorie)	= 4.183 joules
1 pound degree Fahrenheit (British thermal unit)	= 252.1 gram degree Centigrade (gram calorie)
	= 777.5 foot-pounds

1 kilogram-meter	= 9.81 joules
	= 7.233 foot-pounds
1 foot-pound	= 1.356 joules
1 horse-power-second	= 178.3 gram degree Centigrade (gram calorie)
	= 0.7074 lb. degree Fahrenheit (British thermal unit)
	= 550 foot pounds

Pressure

1 atmosphere	= 14.70 pounds on square inch
	= 29.92 inches of mercury at 32° F.
	= 760.0 millimeters of mercury at 32° F.
	= 33.94 feet of water at 60° F.
1 pound on square inch	= 702.9 kilograms on square meter

APPENDIX B

Relations among Electrical Units

Quantity	Practical system	C.G.S. magnetic system	C.G.S. electrostatic system
Electromotive force	1 volt	10^8 abvolts	$1/300$ statvolt
Current	1 ampere	$1/10$ absampere	3×10^9 statamperes
Quantity	1 coulomb	$1/10$ abcoulomb	3×10^9 statcoulombs
Resistance	1 ohm	10^9 absohms	$1/(9 \times 10^{11})$ statohm
Capacitance	1 farad	$1/10^9$ abfarad	9×10^{11} statfarads
Capacitance	1 microfarad	$1/10^{15}$ abfarad	9×10^5 statfarads
Inductance	1 henry	10^9 abhenrys	$1/(9 \times 10^{11})$ stathenry
Energy	1 joule	10^7 abjoules or ergs	10^7 statjoules or ergs
Power	1 watt	10^7 abwatts or ergs per second	10^7 statwatts or ergs per second

APPENDIX C

Specific Gravities

Aluminum	2.67	Mercury	13.60
Copper	8.89	Nickel	7.83
Gold	19.26	Platinum	20.30
Iron, bar	7.48	Silver	10.55
Iron, wrought	7.79	Tin	7.29
Steel	7.85	Zinc	6.86
Lead	11.45		

1 *cu. ft. of water weighs* 62.5 *lb.*

APPENDIX D

Greek Alphabet

Name	Large	Small	Quantity commonly used to designate
Alpha	A	α	angles; coefficients
Beta	B	β	flux density; angles; coefficients
Gamma	Γ	γ	(small); conductivity
Delta	Δ	δ	variation; density
Epsilon	E	ϵ	logarithmic base
Zeta	Z	ζ	coefficients
Eta	H	η	(small) hysteresis coefficient; efficiency
Theta	Θ	θ	temperature; angle of phase displacement
Iota	I	ι	
Kappa	K	κ	dielectric constant
Lambda	Λ	λ	(small) wave-length
Mu	M	μ	(small) permeability; micro; amplification factor
Nu	N	ν	reluctivity
Xi	Ξ	ξ	output coefficient
Omicron	O	o	
Pi	Π	π	circumference $\div$ diameter $= 3.1416$
Rho	P	ρ	(small) resistivity
Sigma	Σ	σ	(cap.) summation; (small) surface density; mutual conductance
Tau	T	τ	time constant; time-phase
Upsilon	Υ	υ	displacement
Phi	Φ	ϕ, φ	magnetic flux
Chi	X	χ	
Psi	Ψ	ψ	angular velocity in time; dielectric flux
Omega	Ω	ω	(cap.) ohms; (small) angular velocity

APPENDIX E

Table of Turns per Square Inch; Solid Layer Winding[1]

The Acme Wire Company

Size, A.W.G.	Single-cotton covered	Enamel and cotton	Single-silk covered	Enamel and silk	Enamel
10	87.5	84.5	.....	.....	92.5
11	109	105	.....	.....	117
12	136	130	.....	.....	147
13	169	161	.....	.....	184
14	210	199	.....	.....	231
15	260	248	.....	.....	292
16	321	304	.....	.....	366
17	396	374	.....	.....	458
18	488	456	.....	.....	572
19	598	556	.....	.....	715
20	772	722	865	807	907
21	947	890	1,075	1,010	1,150
22	1,155	1,075	1,330	1,230	1,425
23	1,410	1,303	1,650	1,510	1,780
24	1,720	1,575	2,045	1,860	2,220
25	2,080	1,910	2,520	2,290	2,800
26	2,500	2,310	3,090	2,830	3,540
27	3,020	2,770	3,810	3,460	4,440
28	3,630	3,300	4,690	4,220	5,570
29	4,270	3,910	5,650	5,100	6,950
30	5,100	4,630	6,950	6,200	8,730
31	5,920	5,330	8,410	7,300	10,650
32	6,950	6,300	10,000	8,900	13,500
33	8,120	7,300	12,080	10,650	16,900
34	9,430	8,410	14,500	12,600	21,000
35	10,850	9,610	17,300	14,900	26,000
36	12,350	10,850	20,400	17,300	31,900
37	.....	.....	23,700	20,400	40,000
38	.....	.....	27,800	23,700	49,300

[1] "Standard Handbook," 5th Ed. Sec. 5, Par. 102.

APPENDIX F

Allowable Carrying Capacities of Wires

(National Electrical Code)

American wire gage	Diameter of solid wires, mils	Area, circular mils	Table A, rubber insulation, amperes	Table B, varnished cloth insulation, amperes	Table C, other insulation amperes
18	40.3	1,624	3	.....	5
16	50.8	2,583	6	.....	10
14	64.1	4,107	15	18	20
12	80.8	6,530	20	25	25
10	101.9	10,380	25	30	30
8	128.5	16,510	35	40	50
6	162.0	26,250	50	60	70
5	181.9	33,100	55	65	80
4	204.3	41,740	70	85	90
3	229.4	52,630	80	95	100
2	257.6	66,370	90	110	125
1	289.3	83,690	100	120	150
0	325	105,500	125	150	200
00	364.8	133,100	150	180	225
000	409.6	167,800	175	210	275
		200,000	200	240	300
0000	460	211,600	225	270	325
		250,000	250	300	350
		300,000	275	330	400
		350,000	300	360	450
		400,000	325	390	500
		500,000	400	480	600
		600,000	450	540	680
		700,000	500	600	760
		800,000	550	660	840
		900,000	600	720	920
		1,000,000	650	780	1,000
		1,100,000	690	830	1,080
		1,200,000	730	880	1,150
		1,300,000	770	920	1,220
		1,400,000	810	970	1,290
		1,500,000	850	1,020	1,360
		1,600,000	890	1,070	1,430
		1,700,000	930	1,120	1,490
		1,800,000	970	1,160	1,550
		1,900,000	1,010	1,210	1,610
		2,000,000	1,050	1,260	1,670

APPENDIX G

Carrying Capacity in Amperes of Lead-covered Cables

(Simplex Wire and Cable Company)

Size	Rubber			Cambric-paper		
	Temperature rise—30° C. = 54° F. maximum temperature, 125° F.			Temperature rise—37.5° C. = 67.5° F. maximum temperature, 150° F.		
	Single conductor	Two conductors	Three conductors	Single conductor	Two conductors	Three conductors
10 B. & S.	30	26	23	34	29	26
8 B. & S.	45	39	34	50	44	38
6 B. & S.	60	52	45	67	58	50
4 B. & S.	78	68	59	87	76	65
2 B. & S.	122	106	92	136	118	102
1 B. & S.	146	127	110	163	142	122
0 B. & S.	169	147	127	189	165	142
00 B. & S.	192	167	144	215	187	161
000 B. & S.	245	213	184	274	238	206
0000 B. & S.	285	248	214	319	278	239
250,000 C. M.	320	278	240	358	311	269
300,000 C. M.	370	322	278	414	360	311
350,000 C. M.	415	361	311	464	404	348
400,000 C. M.	460	400	345	514	447	386
500,000 C. M.	550	479	412	615	535	461
600,000 C. M.	630	548	473	705	613	529
700,000 C. M.	710	618	532	794	691	596
750,000 C. M.	750	653	563	838	730	630
800,000 C. M.	790	688	593	883	770	665
900,000 C. M.	840	732	627	940	818	700
1,000,000 C. M.	900	783	675	1,005	875	755
1,250,000 C. M.	1,055	916	790	1,180	1,028	885
1,500,000 C. M.	1,200	1,045	900	1,343	1,168	1,007
1,750,000 C. M.	1,340	1,165	1,005	1,500	1,305	1,126
2,000,000 C. M.	1,400	1,220	1,050	1,565	1,362	1,175

This table should be used only under ordinary conditions, and does not apply in special cases.

QUESTIONS ON CHAPTER I

1. What metal is the most useful for magnetic purposes? Why? What other substances show magnetic properties?

2. Distinguish between a natural magnet and an artificial magnet. Under what conditions should soft iron be used for magnetic purposes? Hardened steel?

3. What is a magnetic field? What are lines of induction and do such lines actually exist? Distinguish between a north-seeking pole and a north pole. In what way does the magnetic circuit differ from the magnetic field?

4. What is the effect of breaking a bar magnet near the neutral zone? Explain how the newly created poles come into existence.

5. What is Weber's molecular theory of magnetism? How does it explain the phenomenon that occurs when a bar magnet is broken?

6. What is meant by consequent poles? Distinguish between consequent poles and poles obtained by breaking a bar magnet.

7. When a freely suspended south pole is brought into the presence of a north pole, what effect is noted? What effect is noted if the freely suspended south pole is brought into the presence of a south pole? What is the general law governing attraction and repulsion between like poles, and between unlike poles?

8. What is a unit pole? How is pole strength determined? What is the law governing the force of attraction and the force of repulsion between magnetic poles?

9. Distinguish between lines of force and lines of induction. Are both closed lines? In what way are lines of force and lines of induction similar? At what part of the magnetic circuit is the magnetic force quite distinct from the lines of induction?

10. What is unit field intensity? How are the lines of force related to field intensity? What relation exists in the magnetic field between field intensity and "lines of force per square centimeter?"

11. How many lines of force emanate from a unit pole? From a pole of strength m units? If B is the flux density near the center of a long steel rod of 1 sq. cm. cross-section, what is the pole strength at the end of the rod?

12. Of what does a compass needle consist? How is it used in practice to determine the correct polarity of motors and of generators? How is it possible to obtain accurate indications from a compass when it is used upon steel ships? How may a compass needle be used to map out an electrical field in the vicinity of a magnet?

13. How may the flux distribution in a certain region be determined by iron filings? Explain the relation of pole attraction and repulsion to the distribution of the magnetic lines existing near the poles.

14. What is magnetic induction? What is the relation between the inducing and the induced pole? How does magnetic induction explain the attraction of soft iron to magnetic poles? How may a compass become reversed? What is the use of a "keeper" in connection with a horseshoe magnet?

15. What general law governs the path taken by the lines of induction? How does this law explain the attraction of iron to the poles of a magnet?

16. What is the objection to the use of the bar magnet in practical work? What advantages have the ring and the horseshoe magnet over the bar magnet?

17. What is the principle underlying the compound or laminated magnet? Where are laminated magnets used in practical work?

18. How may sensitive instruments be shielded from stray magnetic fields that may exist in their vicinity?

19. How may a steel bar be magnetized by means of a bar magnet? By means of two bar magnets? In practice, how may magnets be magnetized by the use of electromagnets and also by means of electric current?

20. State why the compass needle does not point to the true north and the true south in most places on the earth's surface. What information is necessary in order to determine the true north from the indication of the compass needle? What is the dip of the needle?

PROBLEMS ON CHAPTER I

1. Sketch the magnetic field about the two bar magnets arranged as shown in Fig. 1*A*.

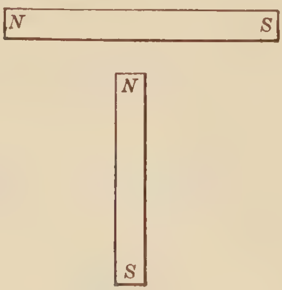


FIG. 1*A*.

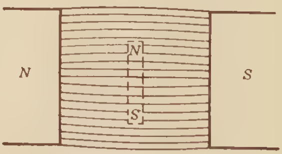


FIG. 4*A*.

2. Sketch the magnetic field about the two bar magnets (Fig. 1*A*) when the lower magnet is reversed end for end.

3. A uniform field is produced between two parallel polar surfaces of a magnet. A bar magnet is inserted in this field parallel to the lines of induction and with its north pole pointing to the north pole of the magnet:

(*a*) sketch the resulting field; (*b*) show the ultimate effect upon the magnetic flux distribution of increasing the strength of the field due to the large magnet; (*c*) what change ultimately occurs in the bar magnet?

4. In Fig. 4*A* is shown a uniform magnetic field between pole-faces of large area *N* and *S*. Sketch the resulting magnetic field when a bar magnet is placed in this field and at right angles to it, in the position shown dotted. The north pole of the bar magnet is upwards.

5. Two north poles of strength $m = 1{,}200$ unit poles and $m' = 900$ unit poles are spaced 12 cm. apart in air. (*a*) What is the force in dynes acting on the poles. (*b*) In what direction does this force act? (*c*) Find the force in grams.

6. If each of the two poles of a horseshoe magnet has a strength of $m = 3{,}000$ unit poles and the centers of the poles are a distance of 1.4 in. apart, find the force in pounds tending to pull the two poles together. Assume the poles to be concentrated at their centers.

7. A north pole $m = 600$ unit poles and a south pole $m' = 800$ unit poles are spaced 4 cm. apart in air. (*a*) With what force in dvnes does the north pole act on a unit pole midway between these two poles? (*b*) With what force in dynes does the south pole act on a unit pole midway between these two poles? (*c*) What is the total force acting on this unit pole? (*d*) What is the field intensity at the point midway between the two poles?

8. A bar magnet (Fig. 8*A*) is 20 cm. long and has poles at the ends of strength $m = 800$ unit poles. (*a*) Find the magnitude and direction of the force which the north pole exerts on a unit north pole at point *P*, where the angle *aPb* is 90° and point *P* is equidistant from both the north and the south pole. (*b*) Find the magnitude and direction of the force which the south pole exerts on the unit north pole at point *P*. (*c*) Find the magnitude and direction of the resultant force acting on the unit pole at point *P*. (*d*) What is the field intensity and the direction of the magnetic field at point *P*?

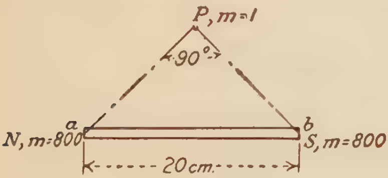


FIG. 8*A*.

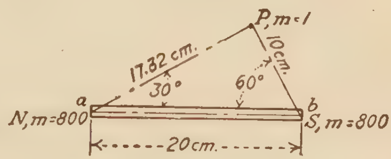


FIG. 9*A*.

9. Repeat Prob. 8 when point *P* is such that angle *aPb* is still 90°, but angle *baP* is 30° and angle *Pba* is 60°. This makes side *aP* 17.32 cm. and side *Pb* 10 cm. (see Fig. 9*A*).

10. The force acting on a unit pole at point *P*, 8 cm. in air from a north pole, is 0.012 g. What is the pole strength of the north pole?

11. A bar magnet 20 cm. long, whose poles have a strength of $m = 200$ unit poles, is placed in a uniform magnetic field and at right angles to its direction. The turning couple acting on the bar magnet is 1,600 gm.-cm. What is the intensity of the field?

12. A magnetic field of 160,000 lines and having the shape of a truncated cone exists between two parallel, plane surfaces having areas of 50 and 75 sq. cm., respectively. What force is exerted upon a unit N-pole if placed near the first surface? Near the second? Explain.

13. An N-pole has a strength of 200 units. (*a*) How many lines of force emanate from this pole in air? (*b*) What force does it exert on a unit pole 8 cm. away? (*c*) What is the density of the lines of force 1 cm. away? (*d*) 8 cm. away?

14. A north pole having a strength $m = 400$ unit poles and a south pole having a strength $m' = 600$ unit poles are 16 cm. apart in air. (*a*) How many lines of force emanate from each? (*b*) What is the density of the lines of force emanating from the north pole at a point 6 cm. away? (*c*) What is the density of the lines of force emanating from the south pole at a point 10 cm. away? (*d*) What resultant force do the two poles exert on a unit pole on their common axis and 6 cm. from the north pole?

15. Repeat Prob. 14 substituting a north pole for the south pole.

16. A long steel rod with a circular cross-section has a diameter of 0.5 in. The flux density in the cross-section taken at the center of the rod is 15,000 lines per square inch. What is the approximate strength of the poles at the ends of the rod?

QUESTIONS ON CHAPTER II

1. What is the nature and general shape of the magnetic field about a cylindrical conductor carrying an electric current? What relation exists between the direction of the current and the direction of the field produced about the conductor?

2. How may the above relations be shown experimentally? What two simple rules enable one to remember the relation which exists between the current direction and the direction of the magnetic field?

3. The current in a conductor flows from left to right. In what direction will the north end of a compass needle point if held over the wire? If held beneath the wire?

4. If two parallel conductors carry current in the same direction, do these wires tend to separate or come together? Give two reasons for the answer. Repeat for two conductors carrying current in opposite directions.

5. A single loop of wire lying in the plane of the paper carries a current in a clockwise direction. What effect will be noticed if a compass is placed within this loop? Has this loop any properties in common with those of a bar magnet?

6. Show how several loops similar to the one mentioned in Par. 5 may be combined to form a long solenoid.

7. Give three methods whereby the poles at the ends of a solenoid may be determined, provided the direction of the current through the solenoid turns be known.

8. What are commercial uses of the solenoid? Name seven such uses.

9. Explain by the fundamental laws of magnetism why the plunger is drawn into a solenoid when current flows in the solenoid winding.

10. Plot the relation between the pull on the plunger and the position of the plunger in the solenoid.

11. What effect does "iron-cladding" have upon the pulling characteristic of the plunger? State one practical application of the simple solenoid; of the iron-clad solenoid. What effect does the stop have upon the solenoid characteristic? State a practical use of this type of solenoid.

12. Show the principle whereby a U-shaped solenoid attracts an armature. Explain the principle of operation of the telegraph relay; the ordinary electric door-bell.

13. Sketch a lifting magnet, showing its general construction. Where are such magnets used commercially, and in what way are they more economical than the older methods of handling material? Does the magnet itself do appreciable work when it is being used to handle iron and steel?

14. What is the disadvantage of the early types of magnetic circuits of dynamos, as represented by the Edison bipolar type? How has the design of the magnetic circuits of the more modern generators overcome some of the disadvantages of the earlier ones? What should be the approximate ratio of the cross-section of the field cores of a multipolar generator to the cross-section of the yoke? What general rule should be followed in the placing of the exciting ampere-turns upon a magnetic circuit? Does magnetic leakage between the poles of a generator represent a direct loss of power?

PROBLEMS ON CHAPTER II

17. A portion of a direct-current feeder is shown in Fig. 17*A*. When a compass is held above the feeder the needle deflects as shown. In what direction does the current in the feeder flow, in or out of the duct?

FIG. 17*A*.

FIG. 18*A*.

18. Fig. 18*A* shows two positive feeders of a trolley system running upon a pole line and carrying current in the same direction. If the trolley wire drops upon the track, causing an enormous current to flow in the feeders for an instant, in what direction will these conductors tend to move and what is the direction of the force acting upon the insulators?

19. Fig. 19*A* shows a bipolar electromagnet. Connect the two exciting coils so that the left-hand pole is north and the right-hand pole is south.

Mark the polarity of the magnet terminals. Sketch the magnetic field between the poles.

20. Fig. 20*A* shows in cross-section a lifting magnet about to pick up a heavy iron sphere known as a "skull cracker" (used in breaking up scrap

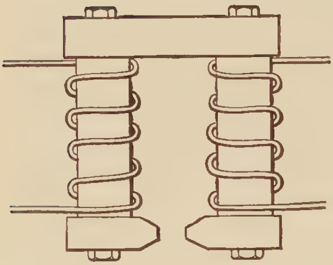


Fig. 19*A*.

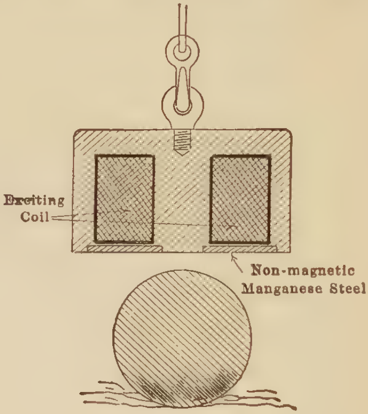


Fig. 20*A*.

iron). Sketch the magnetic lines and mark the poles existing under these conditions. The horizontal section of the magnet is circular. Assume that the current enters the coil in the right-hand section of the exciting coil.

21. In Fig. 21*A* is shown the principle upon which one type of electric hammer operates. Two coils *C* and *C*′ are connected in series and in the

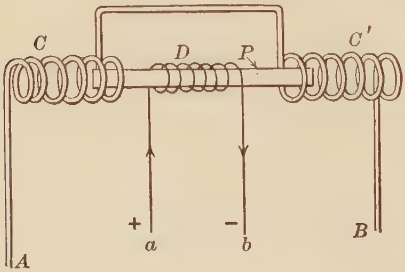


Fig. 21*A*.

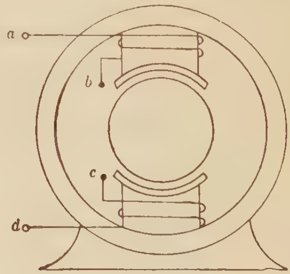


Fig. 22*A*.

positions shown. *P*, a soft-iron plunger running in guides, actuates the hammering device. *A* coil *D*, encircling the plunger *P*, is excited continuously with direct current. If the terminals *a* and *b* of the coil *D* are of the polarity shown, indicate the polarity of the ends of the plunger *P*. If terminal *A* is + and terminal *B* is −, in what direction will the plunger *P* tend to move? If the polarity of terminals *A* and *B* is reversed, in what direction does the plunger tend to move?

22. Fig. 22*A* shows a sectional view of a bipolar dynamo with its field coils *ab* and *cd*. Connect the terminals of the coils in such a manner that they both act in conjunction on the magnetic circuit of the dynamo. Let *a* be one of the line terminals. What should be the polarity of *a* in order that the upper pole may be north and the lower pole south?

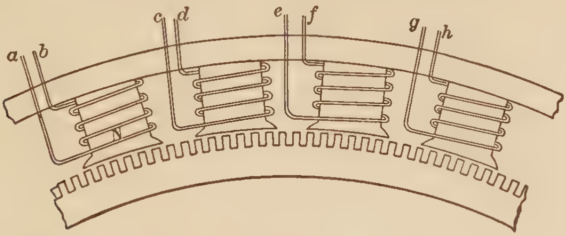


Fig. 23*A*.

23. Connect the coils *ab*, *cd*, *ef*, *gh*, in the multipolar machine shown in Fig. 23*A*, so that the proper sequence of poles is obtained. Make the left-hand pole a north as shown. Sketch the paths of the magnetic lines.

QUESTIONS ON CHAPTER III

1. In what ways does electrical resistance in an electrical circuit manifest itself? What is the mechanical analogue of resistance?

2. What is the unit of resistance? How is it defined?

3. Distinguish between insulating materials and conductors. What is a "megohm?" A "microhm?"

4. May two conductors, each of the same material and of equal volume, have different resistances? Explain.

5. How does the resistance of a homogeneous material vary with its length and with its cross-section? What is specific resistance or resistivity?

6. If the volume of a substance is fixed, how does its resistance vary with its length? With its cross-section? If the volume is fixed and the length doubled, how is the resistance affected?

7. What is conductance and how does it vary with length and with cross-section? Distinguish between conductance and conductivity. What is the general meaning of "per cent. conductivity?"

8. What is the relation of the total resistance of a circuit to the resistances of its individual parts when these latter are connected in series?

9. What is the relation of the total conductance of a circuit to the conductances of its individual parts when these latter are connected in parallel? From this relation show how resistances connected in parallel may be combined into an equivalent resistance.

10. What is the meaning of the term "mil?" What is a square mil? A circular mil? What relation does one bear to the other? Where is the circular mil usually chosen as the unit of cross-section? What are its advantages over such units as the square mil and the square inch? What relation

does the number of circular mils in a circular cross-section bear to its diameter?

11. What is a cir.-mil-foot? What is its approximate resistance for copper? How may the resistance of a copper wire be determined if its length in feet and its cross-section in cir. mils be known?

12. Define a circular-mil-inch. What is its resistance at 60° C.? For what types of resistance calculations is this unit of resistivity very convenient?

13. For what purposes are resistor materials used? Enumerate some of the more common alloys, with their advantages and uses.

14. How is the resistance of most of the unalloyed metals affected by temperature? What is the "temperature coefficient of resistance?" How is it used? How is the temperature coefficient of resistance at any initial temperature determined?

15. At what temperature would the resistance of copper be zero if the resistance decreased at the same rate that it decreases within ordinary ranges of temperature? How may this principle be used to solve problems involving resistance and temperature?

16. What is the basis of the American Wire Gage (A.W.G.)? What relations do the diameters of successive gage numbers bear to one another?

17. What relation do the cross-sections of the wires in the A.W.G. bear to one another? How does this relation enable one to determine readily the resistance and weight of any given size of wire? What is the resistance of 1,000 ft. of No. 10 wire? What is the weight of 1,000 ft. of No. 10 wire? Of No. 2 wire?

18. What are the best conductors among the metals? Which is most commonly used and why? Compared with copper what are the advantages and the disadvantages of aluminum as a conductor? When are iron and steel used as conductors? Explain.

PROBLEMS ON CHAPTER III

24. Two conductors, A and B, of the same material, have the same length, but the cross-section of A is twice that of B. If the resistance of A is 25 ohms, what is that of B?

25. Two conductors, C and D, of the same material, have the same length, but the diameter of C is twice that of D. If the resistance of C is 25 ohms, what is that of D?

26. If the resistance of copper is 1.724 microhms per centimeter-cube at 20° C., what is the resistance of an inch cube at the same temperature?

27. Nichrome (iron-nickel-chromium alloy) has a resistivity of 109 microhms per centimeter cube and its temperature coefficient is so small as to be negligible over ordinary ranges of temperature. Find the resistance in microhms between each pair of opposite edges of a rectangular sheet 10 mils thick, 10 in. on one side and 18 in. on the other.

28. It is desired to make nichrome ribbon resistor elements having a resistance of 0.8 ohm each (see Prob. 27). The width of the ribbon must be

$\frac{1}{4}$ in. and the thickness 12 mils, in order that it may carry the current. What length of the ribbon will be required for each element?

29. Copper has a volume resistivity of 1.724 microhm-centimeters at 20° C. Find the resistance at 20° C. of a cylindrical copper rod 20 ft. long and $\frac{1}{2}$ in. diameter.

30. Determine the resistance at 20° C. of a copper bus-bar 30 ft. long and having a cross-section 3 in. by $\frac{1}{2}$ in. Copper weighs 0.32 lb. per cubic inch. What is the weight of the bus-bar in pounds? In kilograms?

31. Repeat Prob. 30 for an aluminum bus-bar of the same dimensions. Aluminum has a volume resistivity of 2.828 microhms-centimeter at 20° C. The density of copper is 8.89 and that of aluminum 2.70 (see Appendix C).

32. Determine the cross-section of an aluminum bus-bar which will have the same length and the same total resistance as the copper bus-bar (Prob. 30). What is the ratio of the weight of aluminum to the weight of copper in the two cases? Copper has a density of 8.89 and aluminum has a density of 2.70.

33. A 1,000-ft. length of No. 10 A.W.G. copper wire has a resistance of 1.02 ohms at 25° C. What would be the resistance of this same mass of copper if it were drawn down to one-half the cross-section?

34. A third rail has a cross-section of 6.9 sq. in. and the steel has a resistivity of 21.5 microhms-centimeter. Neglecting the effect of joints, determine the resistance of 1 mi. of such rail.

35. Ideal wire, a copper-nickel alloy, has a resistivity of 50 microhm-centimeters. Determine the resistance of a 100-ft. length of such wire having a diameter of 51 mils.

36. A copper rod 5 m. long has a resistance of 8.5 microhms at 20° C. This rod is drawn into a wire having a diameter of 1.015 mm. What is the resistance of the resulting wire? Assume constant temperature and also assume that due to successive annealings, the resistivity does not change during the drawing process. The resistivity of this copper is 1.744 microhm-centimeters at 20° C.

37. A 4-ft. sample of 0000 hard-drawn trolley wire having a diameter of 460 mils is tested for its per cent. conductivity. Measurements at 20° C. between points 44 in. apart give an average resistance of 0.0001795 ohm. Determine the per cent. conductivity of this trolley wire in terms of the International Annealed Copper Standard.

38. A sample of conducting material 6 ft. long has a square cross-section 0.25 in. on a side. Its resistance is measured and found to be 0.02248 ohm at 20° C. (*a*) Find its resistivity in ohms per cm. cube. (*b*) What is its conductivity in terms of that of the International Annealed Copper Standard?

39. Aluminum has a conductivity of 353,600 mhos per centimeter-cube at 20° C. Find the conductance and the resistance at 20° C. of an aluminum bus-bar 6 in. by 0.5 in. by 20 ft. long. What is its weight in pounds? In kilograms? The density of aluminum is 2.70.

40. The resistance of a 5-ft. sample of No. 24 A.W.G. annealed copper wire is measured at 20° C. and found to be 0.1315 ohm. The diameter is 20.1 mils. Determine its per cent. conductivity.

41. A copper bar ⅜ in. by 1 in. and 3.5 ft. long rolled from electrolytic copper is found to have a resistance of 0.0000755 ohm at 20° C. What is its per cent. conductivity?

42. A resistance of 8.6 ohms is connected in series with resistances of 12.5, 20, and 25 ohms all in series. What is the resistance of the combination?

43. Three resistances, each of 5.6 ohms, are in series and are in turn connected in series with a series grouping of two resistances of 8.4 ohms each. What is the total resistance of the combination?

44. A resistance of 8.2 ohms is connected in parallel with a resistance of 18.4 ohms. What is the resistance of the combination?

45. Three conductances of 0.2, 0.4, and 0.5 mho are connected in parallel. (*a*) What is the total conductance of the combination? (*b*) The total resistance? (*c*) Determine the resistance resulting from connecting the three conductances in series. (*d*) Determine the resulting conductance.

46. Determine the resistance of the combination shown in Fig. 46*A*.

47. Determine the resistance of the combination shown in Fig. 47*A*.

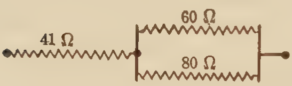


FIG. 46*A*.

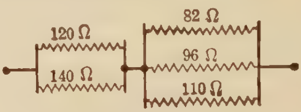


FIG. 47*A*.

48. A resistance of 8.6 ohms, a parallel combination of two resistances of 4.0 and 5.0 ohms, and a parallel combination of three resistances of 12.5, 20, and 25 ohms, are all connected in series. (*a*) What is the resistance of the entire combination? (*b*) What is the conductance of each of the three portions of the circuit? (*c*) What is the conductance of the entire combination?

49. A galvanometer having a resistance of 459 ohms is shunted by 51 ohms (see p. 141). What is the combined resistance of galvanometer and shunt?

50. A 0000 trolley wire (hard drawn) having a resistance of 0.0523 ohm per 1,000 ft. extends 5 miles from the power station. It is paralleled for 3

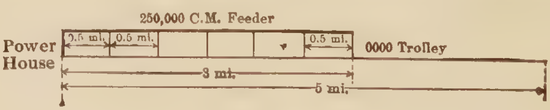


FIG. 50*A*.

miles by a 250,000-C.M. cable, having a resistance of 0.0431 ohm per 1,000 ft., the feeder and the trolley being connected every half mile by taps (Fig. 50*A*). What is the total resistance of the overhead circuit from the power house to the end of the trolley line?

51. The resistance of each rail of the trolley system of Prob. 50 is 0.10 ohm per 1,000 ft. including bonding. An insulated negative feeder con-

sisting of a 0000 stranded cable of resistance 0.0509 ohm per 1,000 ft. runs from the power house and is bonded to both tracks 2½ miles out. What is the total resistance of the return circuit, neglecting any conductance of the earth itself?

52. A cylindrical copper rod has a diameter of 0.25 in. What is its cross-section in circular-mils? Repeat for 0.50 in.; 0.75 in.; 1.0 in.

53. No. 22 A.W.G. wire has a diameter of 0.0253 in. What is its circular milage?

54. No. 00 A.W.G. wire has a cross-section of 133,000 circular-mils. What is the diameter in inches of solid No. 00 wire?

55. What are the diameters in mils of the solid cylindrical conductors having the following cross-sections: 212,000 C.M.; 13,100 C.M.; 810 C.M.; 31.5 C.M.?

56. A 7-strand, No. 2 cable has a total cross-section of 66,400 circular-mils. What is the diameter of its individual strands?

57. What is the total resistance of a 2-conductor, 1,800-ft. length of 1,000,000-C.M. feeder, assuming the resistance of a C.M.-foot to be 11.0 ohms at the operating temperature of the feeder? (Include the resistance of the outgoing and return conductor).

58. A telephone cable is made up of 240 pairs of No. 19 A.W.G. copper conductor. What is the resistance of a 50-mile length of each loop (consisting of a pair), using 10.37 as the circular-mil-ft. resistivity of the copper? The diameter of No. 19 wire is 36.0 mils.

59. Hard-drawn copper wire is used for trolleys, because of its toughness. The resistance of the circular-mil-foot standard of annealed copper is 10.37 ohms at 20° C. The conductivity of hard-drawn copper is approximately 97.3 per cent. that of the annealed standard at the same temperature. Determine the resistance of a 5-mile length of 0000 hard-drawn trolley wire at a temperature of 20° C. Its diameter is 460 mils.

60. Steel reinforced, 0000 (00 copper equivalent) hard-drawn aluminum cable is used for high-voltage transmission-line conductor. The cable consists of six strands of aluminum and a solid steel core. The diameter of the individual aluminum strands and of the steel core is 0.1880 in. Aluminum has a resistivity of 17.02 ohms per circular-mil-foot at 20° C. and the steel a resistivity of 78 ohms per circular-mil-foot. What is the resistance of a single, 100-mile length of such conductor; (*a*) neglecting the conductance of the steel core; (*b*) including the conductance of the steel core?

61. A generator field coil has 600 turns of No. 12 d.c.c. magnet wire. The cross-section of the pole core is a circle whose diameter is 5 in. Between the core and the bottom layer of the winding is a ⅜-in. layer of insulation. The axial length of the field coil is 6 in., and its radial depth is 1 in. Assume that at its operating temperature of 60° C. the copper has a resistivity of 1 ohm per circular-mil-in. Determine: (*a*) The mean length of turn; (*b*) the resistance of the coil at 20° C.; (*c*) the resistance of the coil at its operating temperature, 60° C.

62. The temperature coefficient of resistance of copper at 0° C. is 0.00427 (see p. 44). If a coil of copper magnet-wire has a resistance of 6.84 ohms at 22° C., what is its resistance at 0° C.?

63. A 4-mile length of hard-drawn 000 trolley wire has a resistance of
957 Ω 1.085 ohm at 20° C. What is its resistance at the minimum winter tempera-
.171 Ω ture of −10° C.? At the maximum summer temperature of 40° C.?

64. The field coils of a shunt generator have a measured resistance of 142 ohms after the machine has been standing for some time in a room whose temperature is 21° C. After the machine has been in operation for 3 hr., the resistance is again measured and found to be 154 ohms. (*a*) What is the average temperature of the field coils at this time? (*b*) What is the temperature rise?

65. The resistance of an armature, excluding brush and contact resistance, is 0.0842 ohm at a room temperature of 22° C. What is its resistance at 0° C. and at its allowable operating temperature, which is 40° C. above room temperature?

66. The resistance of the copper circuit of a direct-current armature is measured at a room temperature of 24° C. and found to be 0.032 ohm. After the machine has been in operation 4 hr., the resistance of the same circuit is again measured and found to be 0.037 ohm. What is the rise in temperature during the 4 hr.?

67. Tungsten wire has a temperature coefficient of resistance of 0.0050 at 0° C. If the hot resistance of a 100-watt lamp is 120 ohms at its operating temperature of 2200° C., what is its cold resistance at a room temperature of 20° C.?

68. Determine the temperature coefficient of resistance for copper at reference temperatures of 18° C. and 27° C.

The following problems should be solved without consulting the wire tables.

69. Determine the approximate resistance and cross-section in circular-mils of a 1,000-ft. length of No. 13 A.W.G. copper wire; of No. 16; of No. 7.

70. What is the approximate diameter of each of the wires, Prob. 69? What is the weight in pounds of a 1,000-ft. length of each?

71. Determine approximately the cross-section, the diameter, the resistance, and the weight of 1,800 ft. of 000 A.W.G. annealed copper wire.

72. Find the approximate resistance, weight, cross-section, and diameter of a 1,200-ft. length of No. 36 A.W.G. copper wire.

73. Repeat Prob. 72 for No. 28 A.W.G. copper wire.

74. Find the resistance and weight of a 4-mile length of 0000 trolley wire.

QUESTIONS ON CHAPTER IV

1. Upon what fundamental relationship is the electrostatic system of units based? Where is this system used?

2. Upon what fundamental relationship is the electromagnetic c.g.s., or absolute system of electrical units based? Where is this system generally used?

3. What is the basis of the "practical" electrical system? How is the ampere, the unit of current, defined so that it may be determined experimentally? How is the unit of quantity related to the unit of current?

4. Give the fundamental definition of potential difference. How is unit potential difference related to current and resistance? Name some mechanical analogues of potential difference.

5. How is the international ohm defined?

6. State briefly the meaning of "absolute potential." Why is it usually not important to know absolute potential?

7. What is the nature of voltage drop in a line? Compare it with pressure drop in a pipe. Show that it is impossible to supply power over a simple direct-current power line and have the voltage at the load equal to the voltage at the sending end of the line. Show that there must be a drop in potential in the return wire to the generator as well as in the outgoing wire. Can potential exist without a current flowing? Illustrate.

8. What is meant by "difference of potential?" Show that it is possible to have two or more e.m.fs. within a circuit and yet have no difference of potential between the circuit terminals.

9. How should a voltmeter ordinarily be connected in a circuit? Show that an ammeter must not be connected in the same way as a voltmeter. Why should an ammeter never be connected across a line?

10. What fundamental relation does Ohm's law express? In what three forms may the law be expressed? Name the conditions under which it is most convenient to use each of these expressions.

11. How are series-connected resistances combined to give an equivalent resistance? How are parallel-connected resistances combined to give an equivalent resistance? In a two-branch parallel circuit, what is the relation of each current to the total current? In a parallel circuit of three branches, what is the relation of each current to the total current?

12. What is the unit of electrical power, and how is it defined? How may it be expressed in terms of volts, amperes, and ohms, taken two at a time? Differentiate carefully between power and energy. What is the practical unit of electrical energy and what relation does it bear to the unit of power? What is the unit of mechanical horsepower? What relation does it bear to the units of electrical power?

13. How is energy related to power? What is the fundamental unit of energy? What is the practical unit of energy and how is it defined? Discuss the various forms in which energy is stored or in which energy may appear. Describe the energy cycle involved in a steam-driven electrical power plant. In what forms does the energy appear ultimately? Approximately what is the over-all efficiency of a modern power system?

14. How is a British thermal unit defined? A gram-calorie? What is the relation between a gram-calorie and a watt-second?

15. What simple relation exists between the voltages at the sending and receiving ends of a power feeder with a single concentrated load, and the efficiency of transmission?

16. Under what conditions is the voltage-drop in each foot of wire independent of the total current? How is this principle utilized in solving electrical problems? Can this method be applied to obtaining the power loss? Explain.

PROBLEMS ON CHAPTER IV

75. A storage cell has a constant potential difference of 2.05 volts between its two terminals. What current flows in a resistance of 3.0 ohms connected across the terminals of the cell?

76. An electric toaster has a resistance of 25 ohms. What current does it take when connected across 122-volt mains?

77. A carbon filament incandescent lamp has a cold resistance of 330 ohms and a hot resistance of 240 ohms. What current does it take when it is first connected to 115-volt mains? At what current does it operate?

78. A 110-volt, 25-watt tungsten lamp has a cold resistance of 40 ohms and a hot resistance of 480 ohms. What current does it take when it is first switched to 110-volt mains and what current does it take when it has attained normal operating conditions?

79. A solenoid has a resistance of 53.4 ohms. What current does it take from 110-volt bus-bars?

80. The resistance of the shunt field of a 550-volt generator is 320 ohms. (*a*) Find the field current when the terminal voltage of the generator is 550 volts and the field rheostat is adjusted so that its resistance is 92 ohms. (*b*) To what value must the rheostat be adjusted to give a current of 1.4 amp. in the field circuit? (See Fig. 80*A*.)

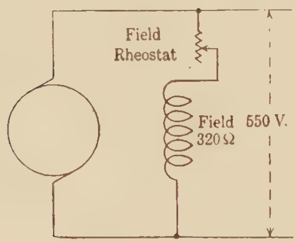


FIG. 80*A*.

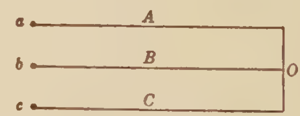


FIG. 84*A*.

81. An incandescent lamp takes 1.2 amp. at 32 volts. What resistance must be connected in series with it in order that the lamp may operate across 112-volt mains?

82. An ammeter shunt has a resistance of 0.00062 ohm. What is the voltage-drop across its terminals when the rated current of 75 amp. flows in it?

83. The field of a 220-volt shunt generator has a resistance of 36.4 ohms and the field current is 4.6 amp. when the generator terminal voltage is 220 volts. (*a*) What is the voltage drop across the field? (*b*) Across the rheostat? (*c*) What is the resistance of the rheostat? (See Fig. 80*A*.)

84. Three wires *A*, *B*, *C* (Fig. 84*A*) of a 3-wire power system are short-circuited at their far end *O* by jumpers. Resistance measurements are made at the home ends *a*, *b* and *c*. The resistance between ends *a* and *b* is found to be 2.7 ohms; that between ends *b* and *c*, 2.8 ohms; and that between

ends a and c, 1.9 ohms. What is the resistance of each of the wires A, B, and C?

85. A telegraph relay which has a resistance of 160 ohms and operates with 60 milliamps is connected at the far end of a telegraph line. What voltage is it necessary to impress at the sending end of the line which has a loop or circuit resistance of 32 ohms?

86. The heating units of a small electric furnace consist of six chrome-alloy resistance units in series. Each unit has a resistance of 1.2 ohms. What current does the furnace take from a 115-volt source of supply?

87. What voltage must a generator develop to supply 25 amp. to an electric oven in which the heating coils have a resistance of 7.5 ohms and the connecting wires a total resistance of 0.25 ohm?

5400 **88.** A series lighting system consists of 97 lamps, each having a resistance of 7.4 ohms and requiring 6.6 amp. If the line resistance is 100 ohms, what is the voltage of the generator supplying this system?

89. The shunt-field circuit of a generator takes 2.7 amp. when the generator terminal voltage is 116 volts. What is the resistance of the field circuit?

90. When a space heater is connected across 112-volt mains it takes 2.8 amp. What is its resistance?

91. In order to measure the resistance of an armature with 115 volts as the source of supply, a rheostat in series with the armature is necessary to limit the current (see p. 412). When the current is 23.4 amp. the voltage across the armature terminals is 7.2 volts. (*a*) What is the resistance of the armature? (*b*) What is the resistance of the series-connected rheostat?

92. When a copper bus-bar carries 1,580 amp., the voltage-drop across a 6-ft. length is found to be 1.26 millivolts. What is the resistance per foot of the bus-bar?

93. The voltage-drop across the series field of a compound generator delivering 350 amp. is 0.7 volt. What is the resistance of the series field?

94. When a lamp load takes 120 amp., the voltage at the lamps is 112 volts. If the connecting leads from the load to the bus-bars have a total resistance of 0.05 ohm, what is the voltage at the bus-bars? (See Fig. 96A.)

95. A direct-current multiple arc lamp takes 6.2 amp. at 110 volts. If the voltage-drop across the arc is 68 volts, what is the resistance of the "ballast?"

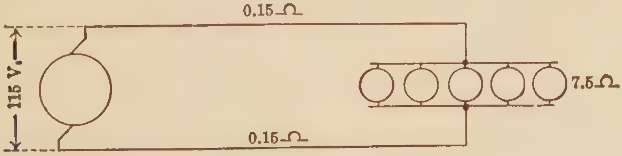


FIG. 96A.

96. Fig. 96A shows a lamp bank, having a total resistance of 7.5 ohms, being supplied from a 115-volt generator over connecting wires having a resistance of 0.15 ohm per wire. What current does the lamp bank receive? What is the voltage across the lamp bank?

97. One of the heating units of an enamel baking oven takes 10 amp. at 115 volts. It is desired to reduce the current to 8 amp. How much resistance must be connected in series? What is the voltage across this resistance and across the unit?

98. It is desired to operate a projection incandescent lamp which is rated at 32 volts and 4 amp. from 115-volt direct-current mains. What series resistance is necessary?

99. Three resistances of 8.1, 5.6, and 10.5 ohms are connected in parallel. (*a*) What is the equivalent resistance of the combination? (*b*) If the current in the 8.1-ohm resistance is 4.94 amp., what is the voltage across the combination? (*c*) What is the current in the other two resistances?

100. The series-field winding of a compound generator has a resistance of 0.0054 ohm and is shunted by a diverter which has a resistance of 0.018 ohm (see p. 351). A current of 500 amp. is delivered by the generator, which divides, part going through the series field and the remainder through the diverter. Determine: (*a*) the current in the series field; (*b*) the current in the diverter; (*c*) the voltage across the two.

101. Four resistance units, having resistances of 20, 25, 32, and 40 ohms, are connected in parallel across 110-volt mains. (*a*) What is the equivalent resistance of the combination? (*b*) What current does the combination take? (*c*) What current does each unit take?

102. Four selective relays connected in parallel are supplied by a common wire shown in Fig. 102*A*. If their resistances are 20, 25, 31, and 37 ohms,

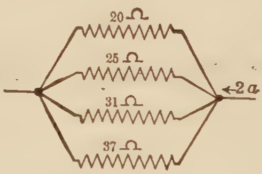


Fig. 102*A*.

respectively, what is the voltage across their terminals when 2 amp. are supplied by the common wire?

103. In Prob. 102 how will the 2 amp. divide among the four relays?

104. Three incandescent lamps, having hot resistances of 55 ohms, 108 ohms, and 160 ohms, are all connected in parallel across direct-current mains and an ammeter in circuit indicates a total current of 3.87 amp. (*a*) What single resistance will take the same current as the combination? (*b*) What current does each resistance take? (*c*) What is the voltage across the combination?

105. A resistance of 10 ohms is connected in series with a combination of two resistances of 24 and 32 ohms in parallel. This entire combination is connected across 110-volt mains. Find: (*a*) the equivalent resistance of the entire circuit; (*b*) the total current; (*c*) the voltage across the 10-ohm resistance; (*d*) the voltage across the parallel combination; (*e*) the current in the 10-, the 24-, and the 32-ohm resistances.

106. A series-parallel circuit shown in Fig. 106*A* is connected across a 150-volt supply. Find: (*a*) the equivalent resistance of the entire combination; (*b*) the total current; (*c*) the voltage between points *ab* and *bc*; (*d*) the current in each resistance.

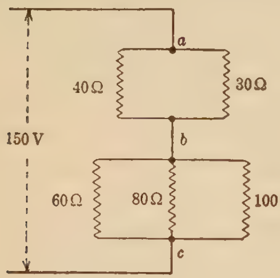


FIG. 106*A*.

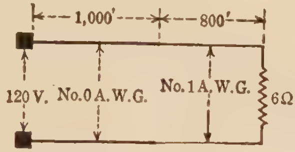


FIG. 107*A*.

107. A lamp load having a total hot resistance of 6 ohms (Fig. 107*A*) is supplied from a 120-volt service by conductors consisting of a 1,000-ft. length of No. 0 A.W.G. copper wire having a cross-section of 106,000 C.M. each, and an 800-ft. length of No. 1 A.W.G. having a cross-section of 83,700 C.M. Using 10.8 ohms as the resistance of a circular-mil-ft. of copper, find the current taken by the system.

108. A 220-volt shunt generator is rated at 250 kw. What is its ampere rating?

109. A shunt motor takes 42 amp. at 220 volts and delivers 10.3 hp. at its pulley. (*a*) What is the watt output of the motor? (*b*) Determine the efficiency of the motor. 7680 Watts 83.2 %

110. The field circuit of the motor (Prob. 109) takes 1.27 amp. and the field coils, exclusive of the rheostat have a total resistance of 108 ohms. (*a*) How much power is taken by the entire field circuit? (*b*) What percentage of the power taken by the motor is the power in (*a*)? (*c*) How much power is taken by the field coils themselves? (*d*) How much power is dissipated by the rheostat?

111. The armature of the motor (Prob. 109) has a resistance between terminals of 0.37 ohm. The armature current is equal to the total current of 42 amp. less the field current. (*a*) How much power is lost in heating the armature? (*b*) What percentage of the total power input is this?

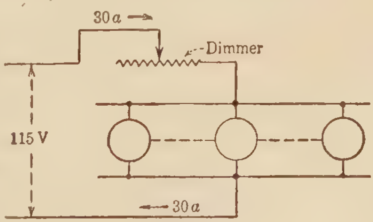


FIG. 112*A*.

112. A theater dimmer (Fig. 112*A*) is in series with a combination of stage lamps, and the power is supplied at 115 volts. When the lamps are taking 30 amp., their voltage is 62 volts. Find: (*a*) the resistance in the dimmer; (*b*) the power lost in the dimmer; (*c*) the percentage of the total power supplied which is lost in the dimmer.

113. Fig. 113*A* shows a series-parallel combination of five resistances a, b, c, d, and e connected across a 200-volt source. Find: (a) the equivalent resistance of the entire combination; (b) the current taken by the entire system; (c) the voltage across each parallel combination of resistances; (d) the power consumed by each of the resistances a, b, c, d, and e.

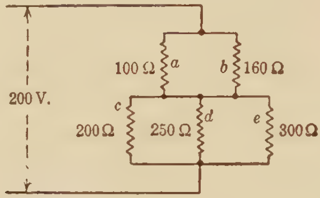


FIG. 113*A*.

114. Five space heaters, each having a resistance of 53 ohms, are connected in series across a 500-volt street-car circuit. What total power do they take?

115. A 2,000-amp. shunt has a resistance of 0.000025 ohm. What power is lost in this shunt when carrying its rated current?

116. A tungsten lamp having a hot resistance of 202 ohms is connected across 110-volt mains. What is its watt rating?

117. A 150-volt voltmeter has a resistance of 17,200 ohms. How much power does it take when it indicates 118 volts?

1.4 kw **118.** How much current does a 100-watt, 115-volt, Mazda C lamp take?

12.7 amp **119.** The energy to an electric oven which is operating under steady load is registered by a watthour meter and shows 2,260 kw.-hr. over a period of 72 hr. (a) What average power does the oven take? (b) If the voltage is 220 volts, what is the average current?

120. A 115-volt, 5-hp., blower motor delivers a steady load of 4.8 hp. at an efficiency of 80.2 per cent. At 3.2 cts. per kilowatt-hour what is the monthly cost of energy if the motor operates steadily 24 hr. per day for 30 days?

121. At a cost of $2\frac{1}{2}$ cts. per kilowatt-hour, what is the monthly cost of operating the space heaters (Prob. 114) if they are in service 12 hr. per day and 25 days per month?

122. At $8\frac{1}{2}$ cts. per kilowatt-hour, how much does it cost to raise the temperature of a quart of water in an electric kettle from 18 to 100° C.? Neglect losses. 1 qt. = 0.946 l. (liter). What is the cost for a month of 30 days, if the kettle is operated twice each day?

1,166 **123.** Energy costs $7\frac{1}{2}$ cts. per kilowatt-hour. Determine the cost of heating $1\frac{1}{2}$ qt. of water at room temperature of 25° C. to the boiling point (100° C.). Assume that the efficiency of the heater is 80 per cent. (water weighs 8.35 lb. per gallon).

124. A water-barrel rheostat contains 40 gal. of water. How long must 60 amp. at 230 volts flow through the rheostat before the temperature of the water is raised to 200° F. from a room temperature of 70° F.? (1 gal. water weighs 8.35 lb.) Neglect losses.

125. Fig. 125*A* shows a drop wire, used for regulating the field current of a generator from zero to its maximum value. The total resistance of the drop wire *ab* is 12 ohms and that of the field is 30 ohms. If the line voltage is 120 volts, what current does the generator field take when the contact *x* is ¼ the distance from *a* to *b*? ½ the distance? ¾ the distance? In each case determine the percentage of the total power received by the field.

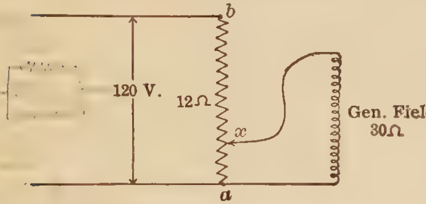


Fig. 125*A*.

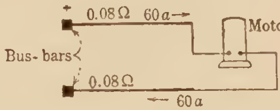


Fig. 126*A*.

126. In Fig. 126*A* is shown a 230-volt motor which takes 60 amp. over a feeder, each conductor of which has a resistance of 0.08 ohm. The bus-bar voltage is 236 volts. Determine: (*a*) the voltage at the motor; (*b*) the efficiency of power transmission.

127. An electric railway is fed by a 5-mile trolley line of 0000 hard-drawn copper. A 250,000-C.M. feeder parallels this trolley for 3 miles, being tapped in every half mile (Fig. 127*A*). The resistance of the ground return

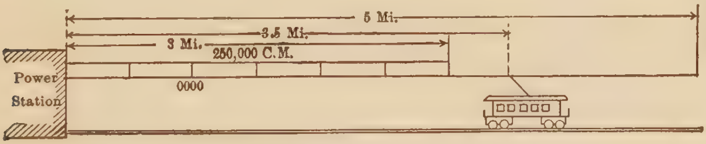


Fig. 127*A*.

may be considered as 0.03 ohm per mile. If the station voltage is 600, what is the voltage at the car when it is 3½ miles from the station and is taking 110 amp.? What is the voltage at the end of the line at this time? What is the efficiency of transmission?

128. Find the voltage at the car (Prob. 127) when it is at the end of the line and starting, the starting current being 120 amp. What is the efficiency of transmission at this time?

129. A 100-kw. load is situated 1,700 ft. from 230-volt bus-bars. It is desired that the voltage at the load be not less than 218 volts when the load is 100 kw. (*a*) What is the resistance of each conductor of the feeder? (*b*) Taking 10.8 ohms as the resistance of a circular-mil-foot of copper, what should be the size, in circular-mils, of the feeder? (*c*) What is the efficiency of transmission? (*d*) What is the weight in pounds of the copper?

130. Repeat Prob. 129 with the bus-bar and load voltages each halved. The load, the distance, and the efficiency of transmission remain unchanged.

Does it appear economical to transmit 100 kw. a distance of 1,700 ft. at 115 volts?

131. A feeder from a direct-current substation supplies two concentrated loads as shown in Fig. 131*A*. The first load is 800 ft., in terms of length of feeder, from the bus-bars, and the feeder to this load consists of a two-conductor, 1,500,000-C.M. cable. (The outgoing and the return conductor each has a cross-section of 1,500,000 C.M.) At a distance 600 ft. further on is a 500-amp. load which is fed by a two-conductor, 1,000,000-C.M. cable. The voltage at the 500-amp. load is 225 volts. The operating temperature is 25° C. Determine: (*a*) the voltage at the 700-amp. load; (*b*) the voltage at the bus-bars; (*c*) the total power transmitted; (*d*) the total power loss; (*e*) the efficiency of transmission. (The circular mileage is for each conductor.)

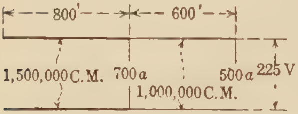


Fig. 131*A*.

132. A 1,000-ft. feeder, consisting of two (one positive and one negative) 1,000,000-C.M. copper conductors, supplies two loads, one of 450 amp. 600 ft. from its sending end and the second a load of 400 amp. at its far end. If the sending end is maintained at 240 volts, determine; (*a*) the voltage at each load; (*b*) the total power which is transmitted; (*c*) the total power loss; (*d*) the efficiency of transmission.

133. In the car system, shown in Fig. 127*A*, determine the voltage at each of two cars, one of which is 2 miles from the station and taking 60 amp. and the other 4 miles from the station and taking 75 amp. What is the efficiency of transmission to the cars?

In the following problems assume that a circular-mil-foot of copper has a resistance of 10 ohms.

134. A 1,200-ft. feeder consisting of two (one positive and one negative) 250,000-C.M. copper conductors is supplying a single load at its far end. What is the total voltage drop in the feeder if it is operating at the *normal density* of 1,000 circular-mils per ampere? If the sending-end voltage is 250 volts, what is the efficiency of transmission?

135. If the voltage-drop (Prob. 134) is limited to 20 volts, what size feeder must be used? What is the current density in circular-mils per ampere and what is the efficiency of transmission?

136. A 400-amp. load is situated 1,500 ft. from 230-volt bus-bars and it is desired that the voltage at this load shall not be less than 220 volts. (*a*) If the feeder is of such size that it is operated at the *normal* current density, what is the voltage-drop to the load? (*b*) What size of feeder must be used to keep the voltage-drop within the required limits? (*c*) What is the power loss per circular-mil-foot? (*d*) What is the efficiency of transmission?

137. A 0000 hard-drawn trolley wire is paralleled for 5 miles with a 300,000 circular-mil feeder. The circular milage of the trolley wire is 211,000. If this parallel system is operated at the *normal* current density, what is the total voltage-drop in feeder and trolley? If it is desired to keep the voltage-drop within 50 volts, what is the maximum current density at which the system may operate? What is the total power loss in the overhead system?

138. It is desired to supply power to a 60-hp. motor, 2,000 ft. from 600-volt bus-bars, the minimum permissible voltage at the motor being 550 volts. The motor efficiency at rated load is 0.89. (*a*) What size of copper conductor is necessary? (*b*) What is the total power loss? (*c*) What is the efficiency of transmission to the motor?

QUESTIONS ON CHAPTER V

1. What is the effect upon the terminal voltage of a battery of applying a load to its terminals? Explain. Why does the electromotive force of a cell differ from the terminal voltage? Under what conditions are they the same?

2. Is it possible to make a *direct* measurement of the internal voltage of a cell when it is delivering current? How may this internal voltage be calculated if the battery resistance be known?

3. To what is the internal resistance of a battery due? Is this resistance a constant quantity?

4. If the electromotive force and the resistance of a battery be known, how may the current delivered to an external resistance be calculated? If the battery becomes short-circuited what current does it deliver? What becomes of the energy that the cell develops under these conditions?

5. When a battery has constant e.m.f. and constant internal resistance, for what value of external resistance is the power delivered a maximum? What is the efficiency under these conditions? Is it usually advisable to operate a battery so that it delivers maximum power? Explain.

6. Under what conditions may a battery be made to receive electrical energy? What relation does the direction of current flow bear to its direction when the battery delivers energy? If a generator has a voltage equal to that of the battery, what effects are noted when the generator is connected to the battery, terminals of like polarity being connected together? What effect is noted when the generator voltage is raised above this value? What is meant by the battery "floating?"

7. Before current can be sent into a battery, what voltage must first be applied? Explain why the voltage in excess of that of the battery alone is effective in causing the flow of current. What is a very common illustration of a battery receiving energy?

8. If several cells are connected in series, what is the resultant electromotive force of the combination? What is the resultant resistance of the combination? How may the current be found if the external resistance be known?

9. Under what conditions do batteries operate most satisfactorily in parallel? What is the electromotive force of the combination under these conditions? What is the relation between the external current and the current in the individual cells? What is the relation between the total battery resistance and the resistances of the individual cells? If the resistances of the individual cells are not equal, how may the resistance of the entire battery be found? What relation does the current delivered by each

cell bear to the resistance of the cell? What relation exists among the terminal voltages of individual cells connected in parallel?

10. What is a series-parallel grouping of cells? What is the voltage of the entire battery? How may the resistance of the battery be found if the resistance of the individual cells be known? How may the current in an external circuit be found if the external resistance, the electromotive forces and resistances of the individual cells and their arrangement be known?

11. In general, how should cells be grouped to obtain the best economy? How should cells be arranged to obtain the maximum power output?

12. Show how the division of current between two batteries in parallel, having unequal e.m.fs. and unequal resistances, may be determined by first finding the no-load circulatory current and then superposing on it the load current.

13. What two fundamental principles are stated in Kirchhoff's laws? If several currents meet at a junction, how should their direction of flow be taken into account?

14. How should a rise in potential be represented? A drop in potential? When passing from a $-$ to a $+$ terminal of a battery, what should be the sign of the potential change and why? When passing from $+$ to $-$? When passing through a resistance in the direction of the current does a rise or a drop in potential occur? What then should be the proper sign to use? When passing through the resistance in opposition to the current what sign should be used? Why?

15. If the assumed direction of a current in a network is in error, how is this fact indicated in the result?

16. In applying Kirchhoff's first law to a network, what rule regarding every current must be followed? In applying Kirchhoff's second law to a network, what rule regarding each path through the network must be followed? How may the number of equations for any network be reduced?

PROBLEMS ON CHAPTER V

139. The open-circuit voltage of a dry cell is 1.35 volts. When the cell delivers a current of 0.5 amp., its terminal voltage drops to 1.30 volts. What is its apparent internal resistance?

140. The terminal voltage of a Le Clanché cell is 1.08 volts when it delivers a current of 2.4 amp. If its internal e.m.f. is 1.34 volts what is its apparent internal resistance?

141. A starting battery consists of three lead cells connected in series. On open circuit, the e.m.f. of the battery is 6.3 volts. When it delivers a current of 85 amp., its terminal voltage drops to 5.0 volts. What is its internal resistance?

142. A battery of six dry cells in series has an open-circuit e.m.f. of 6.6 volts and a total internal resistance of 0.54 ohm. When an external load is applied, its terminal voltage drops to 6.1 volts. What current is the battery delivering?

143. When the starting battery (Prob. 141) delivers its lighting load of 15 amp., what is the value of its terminal voltage?

144. A dry cell has an e.m.f. of 1.30 volts and an internal resistance of 0.08 ohm. (*a*) For what value of external resistance will the power delivered by the cell be a maximum? (*b*) What is the value of the power delivered? (*c*) What power is lost within the cell?

145. What is the maximum power which the battery in Prob. 141 can deliver to an external resistance?

146. A starting battery has a total e.m.f. of 6.2 volts and each of the three cells has an internal resistance of 0.007 ohm. (*a*) What is the maximum power which this battery can deliver to the starting motor, including the resistance of the leads? (*b*) What is the value of current under these conditions? (*c*) What is the terminal voltage of the battery?

147. What voltage must be applied to the terminals of the starting battery (Prob. 141) in order to charge it at the 15-amp. rate?

148. A storage battery consists of 116 cells, all connected in series. Each cell has an e.m.f. of 2.15 volts and an internal resistance of 0.0022 ohm. What voltage is necessary to charge the battery at a 50-amp. rate?

149. What is the terminal voltage of the battery (Prob. 148) when it delivers a current of 40 amp.?

150. A storage battery consists of 60 cells, each of which has an e.m.f. of 2.20 volts and an internal resistance of 0.0018 ohm. It is desired to charge this battery at the 75-amp. rate from 230-volt bus-bars. (*a*) What is the terminal voltage of the battery under these conditions? (*b*) What resistance is it necessary to connect in series with the battery?

151. Two dry cells, having e.m.fs. of 1.35 and 1.28 volts, are connected in series aiding and in series with an external resistance of 20 ohms. The cells have resistances of 0.10 and 0.12 ohm. (*a*) What is the value of current in the circuit? (*b*) What is the terminal voltage across each cell? (*c*) What internal power does each cell develop? (*d*) What is the internal loss in each cell? (*e*) What power is delivered to the 20-ohm resistance?

152. If the two batteries in series (Prob. 151) were short-circuited; (*a*) what current would flow; (*b*) how much power would each cell deliver; (*c*) how much power would be dissipated in each cell?

153. A station battery consists of 110 storage cells in series, each of which has an e.m.f. of 2.2 volts and an internal resistance of 0.0005 ohm. In order to test the battery a resistance of 0.81 ohm is connected across its terminals. (*a*) What current does the battery deliver? (*b*) What power does it deliver? (*c*) What is its loss in internal heating?

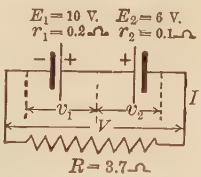


FIG. 154*A*.

154. In Fig. 154*A* are shown two cells connected in series and in series with a 3.7-ohm resistance. Determine the current I, the power p_1 and p_2 developed in each cell, the power P_1 and P_2 delivered by each cell, the power lost in each cell, the voltage v_1 and v_2 across each cell and the voltage V across the resistance. (If a cell is absorbing energy, the power developed is negative.)

155. Five dry cells have the following e.m.fs. and internal resistances: 1.40, 1.35, 1.35, 1.30, 1.20 volts; 0.08, 0.10, 0.11, 0.12, and 0.15 ohm. If all

are connected in series aiding, what current will the battery deliver to an external resistance of 12 ohms? (*a*) What is the terminal voltage of the combination under these conditions? (*b*) What is the power loss in each cell?

156. Repeat Prob. 155 when the third cell, whose e.m.f. is 1.35 volts and whose internal resistance is 0.11 ohm, is reversed.

157. Determine the terminal voltage of each cell (Prob. 156).

158. Four dry cells, each of which has an e.m.f. of 1.3 volts and an internal resistance of 0.1 ohm, are connected in parallel with terminals of like polarity together. What current does this battery deliver to an external resistance of 1.2 ohms? What is the terminal voltage of the battery?

159. What is the maximum power which the battery (Prob. 158) can deliver? What is the terminal voltage of the battery under these conditions?

160. Six batteries, each of which has an e.m.f. of 10 volts and an internal resistance of 1 ohm, are connected in parallel with terminals of like polarity connected together. (*a*) Determine the current which this battery delivers to an external circuit whose resistance is 20 ohms. (*b*) What is the terminal voltage of the battery under these conditions? (*c*) How much power is lost in each of the batteries?

161. Each of two batteries has an e.m.f. of 12 volts, but one has an internal resistance of 1.2 ohms and the other an internal resistance of 1.0 ohm. These batteries are connected in parallel with terminals of like polarity together. When the external current is 4 amp. determine: (*a*) the current delivered by each battery; (*b*) the terminal voltage of the battery; (*c*) the value of the external resistance; (*d*) the e.m.f. and resistance of a single battery which would replace these two in parallel.

162. Each of four dry cells has an e.m.f. of 1.30 volts, but their internal resistances are 0.12, 0.10, 0.09, and 0.08 ohm, respectively. These cells are all connected in parallel with terminals of like polarity together. Determine; (*a*) the e.m.f. and internal resistance of a single cell that would replace this battery; (*b*) the current which this battery delivers to an external circuit whose resistance is 0.75 ohm; (*c*) the current delivered by each cell.

163. Each of two starting batteries has an electromotive force of 6.3 volts; one has an internal resistance of 0.007 ohm and the other a resistance of 0.010 ohm. What is the equivalent resistance of the battery consisting of the two connected in parallel? What is the terminal voltage of the combined battery when it delivers 150 amp.? How does this current divide between the two individual batteries?

164. A battery consists of four storage cells all connected in parallel. The internal resistances of these cells are 0.008, 0.006, 0.004, and 0.0025 ohm respectively. If the electromotive force of each is 2.1 volts, what current does the battery deliver when its terminal voltage is 1.9 volts?

165. Twenty-four dry cells are arranged in rows of six in series and the four rows in parallel. The electromotive force of each cell is 1.4 volts and the resistance of each is 0.1 ohm. What is the total battery voltage and what is

its total resistance? If an external resistance of 2.0 ohms is connected across its terminals, what current flows?

166. Arrange the cells of Prob. 165 so that the maximum amount of power may be supplied to a load resistance of 0.6 ohm. Under these conditions how much power is absorbed by the resistance and how much is lost in the battery?

167. A certain load is such that the potential difference at its terminals must not be less than 6 volts. Twelve storage cells, each having an electromotive force of 2.1 volts and a resistance of 0.002 ohm, are available. How should these be connected so that the maximum efficiency is obtained? When the load requires 100 amp., what is the battery terminal voltage? What is the load resistance? What is the battery efficiency?

168. Arrange the cells in Prob. 167 so that the maximum amount of current will be delivered to the load resistance. What is the efficiency of the battery under these conditions?

169. A telegraph circuit has a total loop resistance of 20 ohms and the relay has a resistance of 80 ohms. Forty gravity cells are available. Each has an e.m.f. of 1.1 volts and an internal resistance of 0.25 ohm. (*a*) How should these cells be connected in order that they may deliver the maximum power to the circuit? (*b*) What is the battery efficiency? (*c*) The efficiency of the telegraph line? (*d*) How should they be connected in order to deliver the maximum current?

170. In Fig. 170*A* are shown two batteries, *A* and *B*, in parallel, with like terminals connected together. *A* has an e.m.f. of 6 volts and an internal resistance of 0.5 ohm; *B* has an e.m.f. of 4 volts and an internal resistance of 0.3 ohm. (*a*) With the switch *S* open, what is the circulatory current? (*b*) What current, with the proper sign, do *A* and *B* each deliver? (*c*) When the switch *S* is closed, the resistance *R* takes 4 amp. How many amperes of *this current* does *A* deliver? *B*? (*d*) What is the *total* current delivered by both *A* and *B*? (*e*) What is the voltage across *ab*? (*f*) What is the resistance of *R*?

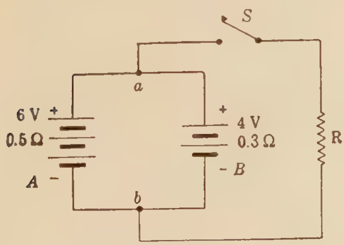


FIG. 170*A*.

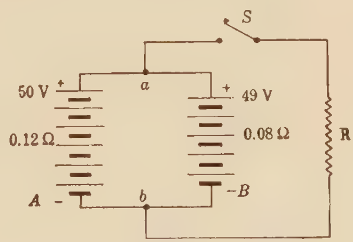


FIG. 171*A*.

171. Two batteries *A* and *B* (Fig. 171*A*), having e.m.fs. of 50 and 49 volts, and internal resistances of 0.12 and 0.08 ohm, are connected in parallel with terminals of like polarity connected together. The current taken by *R* when the switch *S* is closed is 18 amp. Repeat (*a*) to (*f*) inclusive, Prob. 170.

172. Two storage cells, having e.m.fs. of 2.1 and 2.2 volts and internal resistances of 0.02 and 0.03 ohm, are connected in parallel with terminals of like polarity connected together. Find: (*a*) the no-load circulatory current; (*b*) the total current delivered by each cell when a load of 10 amp. is connected across their terminals; (*c*) the power lost in each cell; (*d*) the power delivered by each cell.

173. Two batteries *A* and *B* (Fig. 173*A*), having electromotive forces of 4 and 3 volts and resistances of 1.2 and 1.0 ohm respectively, are connected in parallel, positive terminal to positive terminal. What current flows through a 2-ohm resistance connected across the battery terminals? What is the battery terminal voltage and how much current does each battery deliver?

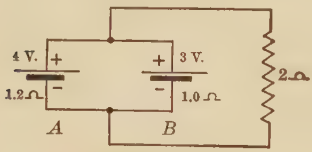


Fig. 173*A*.

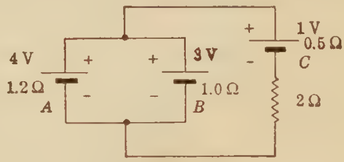


Fig. 174*A*.

174. In Fig. 174*A* is shown the circuit of Fig. 173*A* with a third battery *C* whose e.m.f. is 1.0 volt and whose resistance is 0.5 ohm, introduced in series with the 2-ohm resistance. Find the current in each branch of the circuit and the terminal voltage across batteries *A*, *B*, and *C*.

175. In Fig. 175*A* is shown an electric network consisting of two batteries and three resistances. Find the currents I_1, I_2, and I_3 with their proper signs.

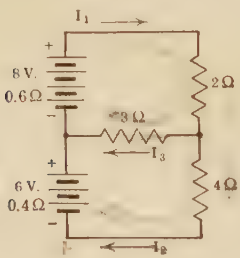


Fig. 175*A*.

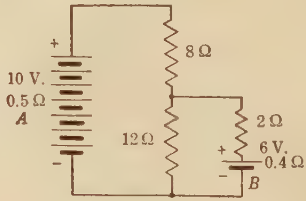


Fig. 176*A*.

176. A 20-ohm resistance is connected across the battery *A* (Fig. 176*A*) whose e.m.f. is 10 volts and whose internal resistance is 0.5 ohm. A second battery *B*, whose e.m.f. is 6 volts and whose internal resistance is 0.4 ohm, is connected with its negative terminal to the negative terminal of *A*, and its positive terminal is connected through a 2-ohm resistance across 12 ohms of the 20-ohm resistance. Find the currents in *A* and *B* and the terminal voltages across batteries *A* and *B*.

177. Repeat Prob. 176, with the 6-volt battery reversed.

178. Across how many ohms of the 20-ohm resistance (Prob. 176) should the 6-volt battery and its circuit be connected in order that the current in the 6-volt battery may be zero?

179. In Fig. 179A is shown an electric network consisting of batteries and resistances. Three currents I_1, I_2, and I_3, are shown flowing toward the junction a. Determine I_1, I_2, and I_3 and indicate the actual direction of each current with respect to the junction a.

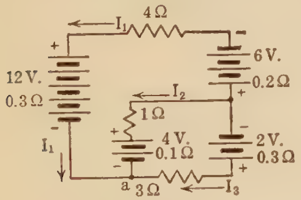


FIG. 179A.

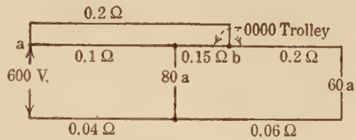


FIG. 180A.

180. In Fig. 180A is shown a trolley system. The 0000 trolley is fed by a feeder whose resistance is 0.2 ohm, and which is connected between points a and b. There are two cars in the system which take 80 and 60 amp. The resistances of the various sections of trolley and rail are given. Find the voltage at each car.

181. Two substations A and B feed into the same distributing center. The voltage at the bus-bars of station A is maintained constant at 600 volts and that at station B is maintained at 580 volts. Station A feeds a distance of 2,000 ft. through 400,000-C.M. cable and station B a distance of 1,000 ft.

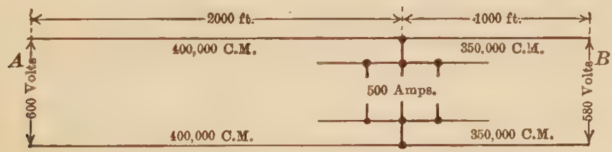


FIG. 181A.

through 350,000-C.M. cable (see Fig. 181A). When the load at the distributing center is 500 amp., how much does each station supply? How much power does each station supply, and how much is received at the distributing center?

182. Fig. 182A shows a distribution system. The voltage at the substation A is maintained constant at 240 volts. A radial feeder extends from A to each of the distributing centers B, C and D. The feeder to B is 2,300 ft. long and 2,000,000 C.M. equivalent; that to C is 1,800 ft. long and 2,500,000 C.M. equivalent; that to D is 2,000 ft. long and 2,000,000 C.M. equivalent (per wire in every case). A tie line 1,100 ft. long and of 500,000 C.M.

connects B and C and another similar line connects C and D. At B is a load of 1,000 amperes; at C a load of 500 amperes; and at D a load of 800 amperes. Find the voltage at each of the distributing centers B, C, and D.

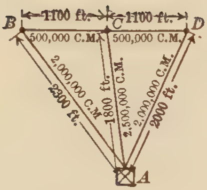


FIG. 182A.

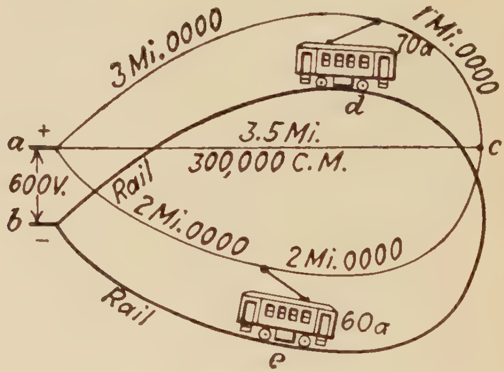


FIG. 183A.

183. In Fig. 183A is shown the wiring diagram of a loop trolley system with a single 300,000-C.M. feeder. A potential difference of 600 volts is maintained between trolley and rail at the bus-bars ab. The 0000 trolley has a resistance of 0.265 ohm per mile and the 300,000 C.M. feeder has a resistance of 0.187 ohm per mile, and is connected from a to c, a distance of 3.5 miles. A car at d, 3 miles from the station, takes 70 amp. and a car at e, 2 miles from the station on the other side of the loop, takes 60 amp. Neglecting the resistance of the rail, determine: (a) the voltage at the car d; (b) the voltage at the car e.

184. Repeat Prob. 183 when the car at e has reached c and car at d has moved two miles toward a. The currents remain unchanged.

185. Solve Prob. 183, assuming the resistance of the rail to be 0.08 ohm per mile.

186. Solve Prob. 184, assuming the resistance of the rail to be 0.08 ohm per mile.

QUESTIONS ON CHAPTER VI

1. What occurs if two copper strips be immersed in a dilute sulphuric acid solution and a voltmeter connected between them? If the two copper strips be replaced by two zinc strips? By two lead strips? Under what conditions may a voltage between the strips be obtained?

With dissimilar metals as electrodes would a voltage exist if the sulphuric acid were replaced by some other type of solution? Name three other such solutions.

2. What is meant by one metal being electrochemically positive to another? If metal A is electrochemically positive to metal B, what will be the direction of the current flow between them within the cell? What will

be the direction of the current flow between them through the external circuit?

What is an electrode? What is the cathode? The anode?

3. In what form is the energy stored within the cell? What changes take place in the electrodes when the cell delivers current? Distinguish between a primary cell and a secondary cell.

4. What are the four requirements for a satisfactory primary cell?

5. What is the nature of the internal resistance of a cell? In what manner may this resistance be reduced? In what way does increasing the size of the elements of a cell increase its current capacity? Its electromotive force?

6. What voltage does a voltmeter indicate when it is connected to the terminals of a cell which is open-circuited? If the circuit is suddenly closed, to what is the initial voltage drop due? To what is the excess drop over this initial drop due? Explain the part that hydrogen plays in polarization. Describe two general methods of reducing polarization.

7. Describe the construction of the Daniell cell. What electrodes and what electrolytes are used? For what type of work is it designed? What is the electromotive force of this cell?

8. In what way does the gravity cell differ from the Daniell cell? Which electrode requires replacing? What changes occur in the other electrode? What is the cell electromotive force and for what type of work is the gravity cell designed?

9. Describe the Edison-Lalande cell. What electrolyte and what electrodes are used? In what way does its electrolyte differ from those in the cells already described? What is the chief advantage of this type of cell? What is its electromotive force and what is its terminal voltage when delivering a current?

10. What materials are used for the positive and for the negative electrodes in the Le Clanché cell? What is the electrolyte? What is its electromotive force? When planning to use the cell commercially, what voltage per cell should be allowed? What materials are introduced in the cell to reduce polarization? How is the cell renewed? For what type of work is this cell best suited?

11. What is the function of a Weston cell in distinction to the uses made of other types of cells? In practice what two common electrical quantities are most easily reproduced and maintained? What must be the characteristics of a standard cell? How is the Weston cell constructed and how is its permanency insured? In what way does the saturated cell differ from the normal cell? Why cannot the voltage of the Weston cell be measured with an ordinary voltmeter?

12. In what way does a dry cell resemble a common type of wet cell? Is a dry cell really "dry?" Of what is the positive electrode composed? The negative? What is the electrolyte and how is it placed in the cell? What materials are placed between the carbon and the zinc and what are their functions?

13. What is the electromotive force of a dry cell when new? After it has stood idle for some time? What is the magnitude of the internal resistance

when new and is it subject to change? How does the polarization effect compare with the internal resistance effect? How much current should a good cell deliver upon short-circuit? What is the terminal voltage when a cell delivers current?

14. To what cause is the cell's becoming exhausted principally due? Can this cell be temporarily revived by any means? Name some of the commercial applications of dry cells.

15. In what way is a storage cell renewed when it becomes discharged? What condition concerning the materials of the cell is necessary for proper functioning of the cell? What two general types of storage cells are in commercial use?

16. Describe a very elementary experiment which illustrates the underlying principle of the lead cell. State the change that occurs in each of the lead strips. What voltage is observed to exist at different times in the experiment? What gases are evolved and from which plate does each emanate?

17. Even although both of its plates are of lead, show that the existence of an e.m.f. in a lead storage cell does not in any way violate the principle governing the e.m.f. of electric cells in general. When the cell is approaching discharge what changes in the materials would account for the approach of the voltage to zero? Account for the 2.5 volts per cell utilized in the process of charging.

18. What change in the electrolyte during the charge and the discharge of a cell is shown by the chemical equation? Why is a cell composed of plain lead plates not useful in practice? Give two reasons. Describe briefly the Planté process and describe two plates that are formed by this process.

19. Describe the Faure or pasted process for making battery plates. What are the advantages and the disadvantages of pasted plates over the Planté plates? What commercial conditions demand a pasted plate and why? How does the life of a pasted plate compare with that of a Planté plate?

20. Describe briefly the construction of the "Exide Ironclad" cell and its principal use in practice.

21. What are the two general classes into which storage batteries may be divided? What types of plate are best suited for regulating duty and for emergency duty in stationary batteries? Why?

22. What two types of containing tanks are used for stationary batteries? Under what conditions is each used and why? In what manner should the joints and seams in lead-lined tanks be made non-leakable? How are the plates suspended in the lead tank? What factors must be considered in designing and installing a lead-lined wooden tank?

23. What three types of separators are in general use? Name the advantages and the disadvantages of each type. For what type of battery is each kind commonly used? What one precaution must be taken in handling wood separators? Why? Describe a method of reinforcing the wood separators, which is used in the better types of starting batteries.

24. What should be the specific gravity of a fully-charged battery having Planté plates? Pasted plates? What precaution should be taken in diluting sulphuric acid for storage battery use? What simple device is used for determining specific gravity? How is this device adapted for use with vehicle and portable batteries?

25. What change takes place in the electrolyte during the charging period? What is the effect of gassing on the specific gravity? What change takes place in the specific gravity after the charging has ceased? Explain. How does the specific gravity of the electrolyte change during discharge? What practical use is made of these changes of specific gravity?

26. When a battery is received, what special attention should be given to the wood separators? In what manner should the jars be installed? How should the plates be placed in position? Why is an initial charge necessary and what should be its duration?

27. What happens to the active material in a cell if it is allowed to stand idle over long periods? In what way may injury to the battery from this cause be avoided? If it is desired to withdraw a battery from service for an indefinite period, what procedure should be followed?

28. What are the requirements of a vehicle battery that make its design different from that of a stationary battery? What changes are made in the plates? Separators? Specific gravity of the electrolyte? How is a battery made up? In what way does a vehicle battery differ from a stationary battery in the manner of shipment? What special attention should be paid to the electrolyte?

29. In what manner is the rating of a storage battery expressed? What is meant by the 8-hr. rate? Can as many ampere-hours be extracted from a cell at the 3-hr. rate as at the 8-hr. rate? To what is this difference due? If a cell is apparently exhausted after discharging at the 3-hr. rate, would it be possible later to extract any further current from it? What can be said of the overload capacity of a storage battery?

30. What general principle governs the charging rate of a battery? When does it become necessary to reduce this current? What are the objections to pronounced gassing in a cell? What is the "finishing rate?"

31. Name a very common example of constant-current method of charging. What care should be taken in the connecting up of the battery? Describe a simple test by which the determination of the correct terminal polarity may be ascertained.

32. What is the one great advantage of the constant-potential method of charging? About what voltage per cell is necessary in this method? Why is the use of some resistance desirable?

33. When a battery is just floating on a bus-bar and it is desired to charge it, in what manner may the necessary excess potential for charging be obtained? Does the generator employed supply the entire energy necessary for charging?

34. What change occurs in the electromotive force of a cell during the charging period? What corresponding changes occur in the terminal voltage? To what is the discrepancy between the cell electromotive force

and the terminal voltage due? Can it be said that the voltage characteristic of a storage battery is such that its use upon lighting circuits is practicable?

35. What is lost by a lead storage battery during its period of service? With what should this loss be replaced except in rare instances? What circumstances justify the addition of acid to a cell? What care should be taken in the selection of water for use with storage batteries?

36. In what manner can the freezing of the electrolyte in a storage battery be absolutely prevented? How does a rise of temperature affect the rating of a storage battery?

37. Compare roughly the kilowatts per pound of plate for a given cell at different discharge rates. Repeat for kilowatts per pound of cell. Compare the above factors for three different types of cell, stating the type of service for which each type is best adapted.

38. Of what is the positive plate, the negative plate and the electrolyte composed, in an Edison cell? In the chemical reaction that takes place both on charge and on discharge, what part does the electrolyte play? How does its specific gravity change during charge and discharge?

39. Describe briefly the mechanical construction of the Edison cell, stating the method of holding the plates and connecting them with the binding posts. What kind of a tank is used for this cell? What is the advantage of this type of construction? For what purpose is the valve necessary and what care does the valve require? How is the battery mounted?

40. In what way does the normal rating of an Edison cell differ from that of a lead cell? What is the voltage per cell? Is it possible to tell accurately the condition of charge by readings of either voltage or of specific gravity? How can complete charge be assured?

41. What should be used to replace evaporation of the electrolyte? Is any greater care required in the selection of water for the Edison battery than for the lead battery? Explain.

42. State the advantages of the Edison battery over other types of storage batteries. What are some of the commercial applications of the battery and what factors limit the applications of the battery? Compare the weights per kw. with similar weights for the lead cell.

43. In what terms is the efficiency of a storage battery expressed? Is the ampere-hour efficiency a true indicator of efficiency?

44. State the reason why the ratio of the kilowatt-hours of discharge at the 3-hr. rate to those of charge at the 8-hr. rate does not give the true efficiency. Give some of the factors which determine the efficiency of a battery.

45. What is the order of magnitude of the kilowatt-hour efficiency of a lead storage battery? The ampere-hour? Why do the two differ? In what manner does the cycle of operation of a storage battery affect the efficiency?

46. What is the approximate kilowatt-hour and the ampere-hour efficiency of an Edison battery?

47. State some of the factors which govern the selection of a storage battery for any particular purpose.

48. State a simple method of producing copper plating upon a carbon brush such as is used with generators. Which electrode is connected to the positive terminal of the supply and which is connected to the negative terminal? When copper is used in connection with a copper sulphate solution, is there any marked change in the electrolyte? Explain.

49. Can copper be plated from a solution in which neither terminal is copper? What voltages in the plating bath must the supply voltage overcome? How are these voltages reduced to a minimum? Is electroplating considered a high voltage or a low voltage process? In what way are plating baths connected, when possible?

50. Show how the gravity cell is an electroplating bath which supplies its own electroplating current.

51. Describe briefly the process of electrotyping.

PROBLEMS ON CHAPTER VI

187. A Daniell cell has an electromotive force of 1.08 volts and an internal resistance of 0.3 ohm. (*a*) What is the maximum current which it can deliver? The size of the cell is increased in such a manner that the plate area is doubled. (*b*) What is the new electromotive force? (*c*) What is the approximate maximum current that the cell can now deliver? (*d*) Determine the maximum power which this cell can deliver.

188. Two gravity cells have electrodes of the same materials and solutions of the same kind, concentration, etc., but one cell has each linear dimension twice that of the other, making its volume eight times greater. The two cells are connected with terminals of like polarity together. The electromotive force of the smaller cell is 1.0 volt and its internal resistance 0.4 ohm; the internal resistance of the larger cell is 0.1 ohm. (*a*) How much current flows between the two cells? Give reasons for the answer. (*b*) What is the short-circuit current of the larger cell? (*c*) What is the maximum power that the combination can deliver to a resistance connected across the cells in parallel?

189. A Le Clanché cell has an electromotive force of 1.47 volts on open-circuit. A load of 2 amp. is suddenly applied and the terminal voltage drops to 1.25 volts almost instantly. After a lapse of some time it drops to 1.06 volts. What is the actual internal resistance of the cell and what is the "electromotive force of polarization?" What is the total apparent cell resistance?

190. It is desired to use Le Clanché cells, similar to those of Prob. 189, to operate a watchman's signal system. A current of 0.25 amp. is required and the total circuit resistance is 67.6 ohms. What is the minimum number of cells that can be used and how should they be connected? The current delivered is intermittent so that the polarization effect can be neglected.

191. Assume that gravity cells are to be used in operating the signal system (Prob. 190). Each cell has an e.m.f. of 1.08 volts and an internal resistance of 0.14 ohm. How many cells should be used, and how should they be connected? What should be the value of the external resistance in order that the maximum power may be drawn from the battery?

192. There are four series relays in a certain telegraph system, each of which requires 50 milliamps for its operation. Each relay has a resistance of 120 ohms and the total line resistance is 378 ohms. How many gravity cells, similar to those of problem 191, are necessary? How should they be connected?

193. A certain type of electric gong requires 1.2 amp. for satisfactory operation, and the resistance is 8.4 ohms. It is to be operated by Le Clanché cells, each having an e.m.f. of 1.3 volts and an internal resistance of 0.1 ohm. How should they be connected for the most satisfactory operation?

194. A certain signal motor requires 3 amp. at 10 volts at its terminals for satisfactory operation. The leads from the battery to the motor have a total resistance of 1.1 ohms. How many Edison-Lalande cells would be necessary to operate this system? (See Par. 94.)

195. Three Le Clanché cells, connected in series, are used to operate a door opener which has a resistance of 1 ohm. Each cell has an e.m.f. of 1.2 volts and a resistance of 0.09 ohm. The resistance of the connecting wires is 0.5 ohm. What is the current taken by the door opener?

196. A Weston cell having an electromotive force of 1.0183 volts is connected to a potentiometer wire as shown in Fig. 196*A*, in order to calibrate the wire *AC*. Between *A* and *B* is a resistance of 0.915 ohm and 10 coils

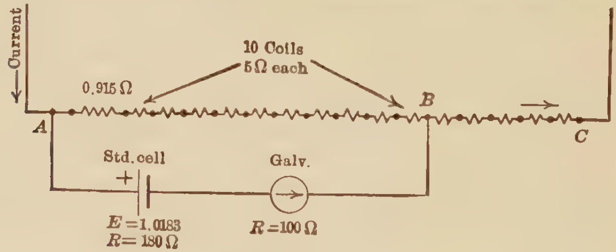


FIG. 196*A*.

having a resistance of 5 ohms each. The cell has a resistance of 180 ohms and the galvanometer a resistance of 100 ohms. (*a*) When the current in *AB* is 0.021 amp., how much current passes through the galvanometer and what is its direction? (*b*) For what value of current in the wire *AC* will the current through the cell and galvanometer be zero? (*c*) Under these conditions what will be the voltage across each of the 5-ohm coils?

197. (*a*) When the current in the wire *AC* is 0.019 amp. (Prob. 196), what is the current in the galvanometer? In the process of adjustment this current is increased 2 per cent. (*b*) What is the percentage change in the galvanometer current? (Note that this arrangement gives a very sensitive method of adjustment.)

198. A voltmeter having a resistance of 1,000 ohms is used in an attempt to measure the electromotive force of the Weston cell in problem 196. What will the voltmeter read? Is this a practicable method of using the Weston cell as a standard?

199. The ignition system of a gasoline engine requires a voltage somewhat in excess of 6 volts for its operation. (*a*) How many dry cells, connected in series, should be used? (*b*) If each cell has an e.m.f. of 1.2 volts and an internal resistance of 0.1 ohm, how much current can the battery deliver without its terminal voltage falling below 6 volts? (*c*) What should be the resistance of the ignition coil if it is to operate at this value of current and at 6 volts?

200. A dry cell shows an open-circuit e.m.f. of 1.1 volts and a short-circuit current of 5 amp. What is its internal resistance? What does this test show as regards the condition of this cell?

201. Each of five dry cells of a 6-cell ignition battery has an e.m.f. of 1.2 volts and an internal resistance of 0.06 ohm. The cells are connected all in series conjunction. One cell, which has become exhausted, has an e.m.f. of 0.90 volt and an internal resistance of 1.0 ohm. (*a*) When the battery delivers 50 milliamps, what is the voltage across the defective cell? (*b*) When the battery is short-circuited, what current does the battery deliver? (*c*) What is the voltage, with its proper sign, across the defective cell under the conditions of (*b*)?

202. A certain flashlight has a 3 cp. lamp which is operated by three small dry cells, each of which has an e.m.f. of 1.1 volts and an internal resistance of 0.2 ohm. Assuming that the efficiency of the lamp is 1.2 watts per candlepower, what current does it take, and what is the terminal voltage of the battery when the lamp is delivering its rated candlepower? (Note: Two values of current can be found to satisfy these conditions, but the more rational value should be used.)

203. It is desired to dilute a quart of concentrated sulphuric acid (sp. gr. = 1.84) to make acid having a specific gravity of 1.20. How much water is needed and what is the total volume of acid when the solution is mixed? State the procedure that should be followed in mixing the liquids.

204. A certain carboy contains 5 l. of concentrated sulphuric acid (sp. gr. = 1.84). (*a*) How many kilograms does the acid weigh? (*b*) How many liters of dilute sulphuric acid (sp. gr. = 1.200) can be made from it? (*c*) To how many gallons is this equivalent? (See Appendix *A*.)

205. Immediately after a certain battery has been completely charged, the specific gravity is found to be 1.210. Each battery jar then contains 1.8 l. of electrolyte. Due to evaporation, the specific gravity in time becomes 1.240. How much water by volume (liters) must now be added to each cell to bring the specific gravity back to 1.210?

206. The specific gravity of a normal 640 amp.-hr., stationary battery, as shown by a hydrometer in the pilot cell, is 1.190. How many more ampere-hours are necessary before it is completely charged? (See Fig. 103, p. 120.)

207. After the battery (Prob. 206) has been completely charged, it is discharged until its specific gravity is 1.180. How many ampere-hours remain in the cell?

208. The specific gravity of a vehicle battery when completely discharged is 1.185 and when completely charged as 1.280. The variation of the specific

gravity may be considered along a straight line between these limits, when the charge and discharge rates are uniform. After the battery has been charging at the 30-amp. rate for 4.5 hr., the specific gravity is 1.235. How many hours longer must it charge at this rate before it is completely charged?

209. How many ampere-hours remain in the cell (Prob. 208) when it is discharged until its specific gravity is 1.220?

210. A certain battery with Planté plates, after being completely charged, is able to deliver 30 amp. for 8 hr. before being completely discharged. How many ampere-hours would it deliver under similar conditions, except that the rate is adjusted so that the battery is apparently discharged in 5 hr.? What is the discharge rate under these conditions? Repeat for complete discharge in 3 hr. (see p. 125).

211. Repeat Prob. 210 for pasted plates.

212. A 6-volt lead storage battery, consisting of three cells in series, is charged from 110-volt mains, the connections being similar to those shown in Fig. 107, page 126. The terminal voltage of each cell is 2.4 volts. The charging rate is 15 amp. (*a*) Of the total power taken from the circuit, what percentage is delivered to the battery? (*b*) How many ohms series resistance are necessary?

213. Repeat Prob. 212 with two 6-volt batteries of this same type in series.

214. A storage battery, consisting of 16 cells in series, is charged at the 30-amp. rate from 110-volt mains. The e.m.f. of each cell is 2.2 volts and the internal resistance of each cell is 0.007 ohm. (*a*) Of the total energy taken from the mains, what percentage is effective in producing chemical reactions within the cell? (*b*) What percentage of the energy taken from the mains is delivered to the battery? (*c*) What value of series resistance is necessary?

215. If the battery (Prob. 214) were connected with the polarity reversed, the same resistance being retained, what current would it take?

216. A battery having a normal rating of 40 amp. at the 8-hr. rate is just received new. How many ampere-hours charge should be given it before it is ready for active service (see page 122).

217. The average charging voltage per cell in problem 216 is 2.3 volts. There are 40 cells in series and a 0.5-ohm resistance in series with the battery. At 4 cents per kilowatt-hour, what is the energy cost of getting the battery ready for service?

218. A booster set (see Fig. 108, p. 127) is used to charge a 60-cell, 110-volt storage battery from 110-volt mains. The battery requires a maximum charging current of 50 amp. and when charging at this rate the terminal voltage per cell is 2.45 volts. (*a*) What is the voltage and current rating of the booster? (*b*) If the generator of the booster set has an efficiency of 0.72 and the motor an efficiency of 0.74, how much power does the booster set take from the mains?

219. A booster set is used to charge a 120-cell lead storage battery from 220-volt mains. When the battery is being charged at the normal 60-amp. rate, the terminal voltage per cell is 2.5 volts. (*a*) What should be the rating of the booster generator? (*b*) If the generator has an efficiency of 0.73 and the motor an efficiency of 0.76, what power does the booster set take from the

mains? (*c*) If energy costs 3 cts. per kilowatt-hour, how much is the energy cost of charging the battery at the 60-amp. rate for a period of 8 hr.? Assume that the battery terminal voltage has an average value of 2.5 volts per cell during the charging period.

220. A 320-amp. hour storage battery, consisting of 55 cells in series, after being practically discharged is being charged at the 8-hr. rate. At the end of 5 hr. what is the voltage across its terminals and how much power is it taking? (See Fig. 109, p. 128.)

221. If the battery (Prob. 220) is completely charged and then discharged at the 5-hr. rate, how much power does it deliver at the end of 2 hr.? At the end of 5 hr.? (See Fig. 109, p. 128.)

222. A storage cell has an 8-hr. rating of 25 amp. This rate is maintained constant for the 8 hr. of charge. During this period the voltage rises according to the curve shown in Fig. 109, page 128. How many ampere-hours are delivered to the cell? How many watt-hours? (Note: Mark several equally spaced points on the voltage curve and take their average.)

223. If the cell of Problem 222 discharges at the 8-hr. rate and its voltage follows the 8-hr. discharge curve of Fig. 109, page 128, how many watt-hours are discharged?

224. Repeat Prob. 223 for the 3-hr. rate of discharge.

225. A storage battery of 50 cells has a total internal resistance of 0.5 ohm and is charged from 115-volt d.-c. mains. At the beginning of charge its electromotive force is 1.8 volts per cell. (*a*) What current does it take? After charging 4 hr. the electromotive force rises to 2.0 volts per cell. (*b*) What current does it take at this time? (*c*) What must be the electromotive force per cell when the battery ceases taking current? What method of charging is used and is this method a desirable one?

226. The specific gravity in a vehicle battery is found to be 1.240. Is there any possibility of its freezing in the climate of the United States? Give reasons.

227. It is desired to install a 110-cell stationary battery having a capacity of 1.22 kw. per cell at the 4-hr. rate. (*a*) What will be the approximate weight of the plates of this battery? (*b*) Of the total battery? (*c*) What will be the weight of a 24-cell vehicle battery composed of "ironclad" cells, this battery to have a total output of 1.28 kw. at the 8-hr. rate? (See Par. 113.)

228. An electric truck operates with 42 "Ironclad Exide" MV 13 cells in series. Determine: (*a*) the kilowatt rating of the battery; (*b*) the kilowatt-hour rating of the battery; (*c*) the total weight of the plates; (*d*) the weight of the entire battery. (*e*) If the vehicle goes 50 miles per charge, determine the kilowatt-hours per mile. Use the data of Par. 113.

229. An electric yard-locomotive is to be operated with a 50-volt series motor. Of how many Edison cells in series will the locomotive battery consist? If a 300-amp.-hr. battery is required, what is the approximate weight of the battery? (See Par. 116.)

230. A generator is to be installed to charge locomotive batteries similar to those of Prob. 229. The generator capacity should be such that four

batteries in parallel may be charged at the 5-hr. rate. Determine the generator rating (see Fig. 113, p. 132).

231. If each of the batteries (Prob. 229) is discharged for 5 hr. at its normal rate, how many kilowatt-hours does it discharge? The voltage of each cell varies according to the discharge curve (Fig. 113, p. 132).

232. A storage cell, in a fully charged condition, discharges 40 amp. for 11.8 hr. at an average terminal voltage of 1.95 volts. The battery is then put on charge. It requires 50 amp. for 10 hr. at an average terminal voltage of 2.35 volts to bring it back to its original condition. Determine its ampere-hour and its watt-hour efficiencies over this cycle.

233. A lead storage battery, consisting of 55 cells in series in a fully charged condition, is discharged at a 57-amp. rate for 8 hr., when the battery is practically exhausted. The terminal voltage per cell varies according to the 8-hr. discharge curve (Fig. 109, p. 128). The battery is then put on charge for 8 hr. at the 60-amp. rate when it is restored to its original condition. The terminal voltage per cell varies according to the upper curve (Fig. 109). Determine the ampere-hour and the watt-hour efficiencies of the battery over this cycle.

234. Repeat Prob. 233, except that the battery is discharged at the 3-hr. rate. Use the 3-hr. curve (Fig. 109).

235. An Edison battery of 12 cells in series, fully charged, is discharged for 5 hr. at the 28-amp. rate with an average terminal voltage of 14.4 volts. It is then charged for 6 hr. at the 25-amp. rate with an average terminal voltage of 19.8 volts, when it is restored to its original condition. Determine its ampere-hour and its watt-hour efficiencies over this cycle.

236. A 50-cell Edison battery, after being brought to a condition of full charge, is discharged for 5 hr. at the 30-amp. rate, the voltage per cell varying according to the discharge curve (Fig. 113, p. 132). The battery is then brought back to its original condition of charge in 6 hr. by charging at the 30-amp. rate, the voltage per cell varying according to the discharge curve (Fig. 113). (*a*) How many watt-hours are given to the battery? (*b*) At 4 cts. per kilowatt-hour, what is the cost of the energy delivered to the battery? (*c*) How many watt-hours are delivered by the battery? (*d*) What is the ampere-hour efficiency of the battery over this cycle? (*e*) What is the watt-hour efficiency of the battery over this cycle?

237. One ampere-hour will deposit 1.186 g. of copper upon the cathode in an electroplating bath. If the voltage across a plating bath is 12 volts and the current is 12 amp. and the current is allowed to flow for 6 hr., how many kilograms of copper are deposited and how many kilowatt-hours are utilized in the process?

QUESTIONS ON CHAPTER VII

1. If a coil carrying a current be placed in a magnetic field, what effect is noticed? Explain this effect in two ways, showing that it is based on fundamental laws of the magnetic field. Of what importance is this principle?

2. How is the principle of the moving coil adapted to measuring small currents in the D'Arsonval galvanometer? How is the coil suspended? How is the current led in and out of the coil? Why is a soft-iron core usually placed between the poles?

3. What two common methods are used to read the galvanometer deflection? What is meant by the "damping" of a galvanometer? How may this damping be accomplished?

4. How may a galvanometer be protected from excessive currents? Sketch the connections of two types of shunt. What are the advantages of the Ayrton shunt?

5. What was the underlying principle of the early types of electrical instruments? What two factors caused these instruments to be inaccurate?

6. Show that the movement of a Weston d.-c. instrument is an evolution of the D'Arsonval galvanometer. How is the moving coil pivoted? How is the current led to the coil? What means are used to oppose the motion of the coil? Is the coil damped? Explain. What is meant by a "radial field" and what effect does it have on the calibration of the instrument scale? Why are the top and the bottom springs coiled in opposite directions? Is it possible to utilize the movement of a Weston instrument as a galvanometer?

7. Of what order of magnitude is the current that will give full-scale deflection in a Weston instrument? Is it possible to use the instrument for measuring current in excess of this value? Explain.

8. Describe briefly the construction of a shunt. Why are four posts or terminals necessary? Show that when a Weston instrument is used in connection with a shunt, it is acting as a voltmeter.

9. What law does the current follow in dividing between the shunt and the instrument? Why should the resistance of the shunt and the resistance of the instrument remain constant? What errors may be caused by the heating of the shunt or of the instrument?

10. In what way may an ammeter be made to have several scales? In general, when is an internal shunt used? An external shunt?

11. Does the movement of a voltmeter differ materially from that of an ammeter? In what important respect does the voltmeter differ from the ammeter? How is the current in the coil of a voltmeter limited when the voltmeter is connected across the line?

12. Show that it is possible for a voltmeter to have more than one scale. Explain. What is meant by a multiplier or extension coil?

13. In what manner may the heating effect of an electric current be utilized to measure the value of the current? State some of the advantages and the disadvantages of hot-wire instruments.

14. Show the connections that are used in measuring resistance with a voltmeter and an ammeter. What precaution should be taken in connecting the voltmeter? What special type of voltmeter contact should be used in measuring very low resistances?

15. Show the connections that can be used in measuring resistance by a voltmeter alone. What is the order of magnitude of resistances that can be measured by this method? What special type of voltmeter is often

desirable for this work and why? To what type of resistances is this method especially applicable?

16. Sketch an arrangement of four resistances, a battery and a galvanometer, whereby one of the resistances may be measured. How is the condition of "balance" in the bridge detected? Prove the law of proportionality that exists when this condition of balance has been reached.

17. Describe briefly the procedure which should be followed in obtaining a balance with a plug bridge.

18. What two types of plug bridge are in general use? Compare them with the dial bridge from the standpoint of ease of manipulation; plug-contact resistance; convenience.

19. In what way does the slide-wire bridge resemble the Wheatstone bridge? Compare it with the Wheatstone bridge from the standpoint of simplicity and accuracy.

20. Make a sketch showing the connections of the Kelvin bridge. Under what conditions do the resistances in the four principal arms bear the same relation to one another as in the usual Wheatstone bridge? When is the Kelvin bridge used?

21. Give the connections whereby the slide-wire bridge may be put to practical use in locating an earth fault in a cable. What is the name of this method? Explain why the galvanometer and battery do not occupy the same positions in the slide-wire bridge of Fig. 140 as they do in Fig. 137.

22. Sketch the connections used in the Varley loop. Upon what arm is the balance obtained? What additional factor must be known before the position of the fault can be determined? Was it necessary to know this factor in the Murray loop test? Which is the simpler method? What possible sources of error exist?

23. Why is it desirable in practice to know the insulation resistance of cables? Why is the voltmeter method not always practicable? What is the general principle of the method described in Par. 134?

24. What method is used to obtain readable deflections of the galvanometer under all conditions of circuit resistance? Why is it desirable to keep the 0.1 megohm in circuit continually and does it introduce any appreciable error?

25. What other factor besides the resistance of the insulation affects the value of the current flowing in the circuit? What time of electrification has been adopted as standard in commercial measurements of insulation resistance? What precautions should be observed in the installation of cable testing apparatus?

26. Why is a guard wire sometimes necessary in insulation measurements? Explain the principle of the guard wire, showing the path of the end leakage-current.

27. Upon what standard of electrical quantity do potentiometer measurements primarily rest? Against what is the standard cell balanced? What care as regards polarity must be observed if a balance is to be obtained? Why is a "nul" method the only one which will give satisfactory results when a standard cell is used?

28. Show how a wire may be calibrated and marked in volts, after the standard cell balance has been obtained. Is it possible to measure other electromotive forces with this standardized wire? What method is employed in such measurements?

29. Does the Leeds and Northrup potentiometer whose connections are shown in Fig. 146 differ materially from the simple device sketched in Fig. 145? What minor changes are necessary? Where are the one-tenth volt divisions located and how are they utilized when obtaining a balance? How are the smaller decimal divisions obtained? What resistances are used in each of these units? What is the working current of this potentiometer?

30. What provision is made for the variations in the voltages among standard cells? What protection is afforded the galvanometer during the preliminary adjustments?

31. What is the disadvantage of a potentiometer in which there are no sliding contacts in the circuit of the working current? Show how the number of dials may be increased by the Thomson-Varley method and by the Wolff method. What factor is essential in all such types of potentiometers?

32. What is the maximum voltage ordinarily measurable with a potentiometer alone? By what means can voltages in excess of this be accurately measured? Is the device used for increasing the voltage range of the potentiometer in any way complicated? What is meant by a "drop wire" and how may it be used to vary the voltage when the supply is at constant voltage?

33. Is the potentiometer, as a voltage-measuring device, adapted to measuring currents? What is the principle underlying the measurement of current? What is a standard resistance? Why does it have four terminals? In what units of resistance are standard resistances generally manufactured? Why is it desirable that their temperature remain normal and what means are adopted to accomplish this?

34. What instruments are generally used in measuring the power in a direct-current circuit? Do these instruments take any power themselves? What should be the relative positions of the voltmeter and the ammeter when the power delivered to a high resistance is being measured? When that delivered to a low resistance is being measured?

35. Describe a wattmeter. In what way do the fixed and moving coils differ in construction? In their manner of connection to the circuit? Why are the instrument deflections a function of the power? What care is necessary when using this type of instrument with direct currents?

36. What does a watthour meter measure? Upon what familiar electrical device is it based? From what source are its field coils supplied? Its armature? To what is the torque acting upon the armature proportional?

37. Why is a retarding device necessary and what must be the law of retardation? Upon what principle does this device operate?

38. At what values of meter load does friction produce the greatest error? Explain. How is this friction error practically eliminated?

39. What methods are used to reduce friction in a watthour meter? What are some of the causes of a meter running slow? How is the recording dial of a meter actuated?

40. Why is it usually very important that a watthour meter register accurately? What load and measuring devices are necessary in testing a meter?

41. Give the fundamental relation which exists in some meters between the revolutions of the disc and the energy registered. What measurements are made in checking the meter?

42. What two adjustments are made to change the meter speed? What is the effect of moving the magnets nearer the center of the disc? Nearer the periphery? At what loads is this adjustment made?

43. What adjustment is made to correct the meter registration at light loads? Why is this adjustment made at light rather than at heavy loads?

44. In what general respect does a 3-wire meter differ from a 2-wire meter?

45. Describe in a general way the construction of a meter which makes the meter practically astatic and therefore enables it to be used near bus-bars carrying heavy currents. What two elements in a meter are most likely to be affected by stray fields? How are these elements safeguarded from these effects?

PROBLEMS ON CHAPTER VII

238. A galvanometer has a resistance of 425 ohms. What should be the resistance of a shunt for use with this galvanometer if it be desired that $\frac{1}{10}$ the total current of the line pass through the galvanometer? If it be desired that $\frac{1}{100}$ of the line current pass through the galvanometer?

239. The resistance of a certain galvanometer is 516 ohms. Design a shunt which will allow $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ the line current to pass through the galvanometer.

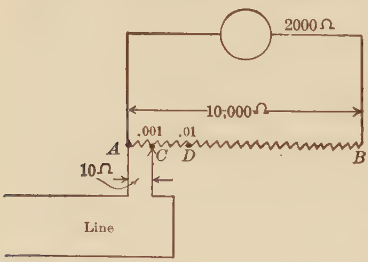


FIG. 240A.

240. An Ayrton shunt (Fig. 240A) has a resistance from A to B of 10,000 ohms. It is used to shunt a galvanometer having a resistance of 2,000 ohms. When the shunt is set at the 0.001 point (the resistance AC = 10 ohms), determine the current through the galvanometer when 4 milliamperes flow in the line.

241. If the line contact be moved to the 0.01 point at D (Fig. 240A), the resistance AD being 100 ohms, determine the current through the galvanometer when the line current is 0.54 milliampere.

242. With 1.0 milliampere in the line, determine the current in the galvanometer (Fig. 240A) under the following conditions: (a) with the line contact at C; (b) with the line contact at D; (c) with the line contact at B. Compare these currents with the settings of the shunt.

243. A 30-scale millivoltmeter has a resistance of 2.5 ohms. It is desired that it measure a current of 75 amp. at full-scale deflection. What should be the resistance of the shunt under these conditions? How much current flows through the instrument and can it be neglected as compared with the current in the shunt?

244. Find the resistances of shunts necessary for measuring currents of 150 amp. and 600 amp., full-scale deflection, with the instrument of problem 243.

245. An instrument of the Weston type has a resistance of 1.04 ohms. It is used to measure a current of 50 amp. The shunt has a resistance of 0.000667 ohm. How much current passes through the instrument? Through the shunt? When the current through the shunt is 75 amp., the instrument needle makes full-scale deflection. What is the rating of the instrument in millivolts?

246. It is desired to measure a current which does not exceed 60 amp. An internal shunt, 5-scale ammeter, which has a resistance of 0.01 ohm, alone is available. What should be the resistance of a shunt to be used with this instrument?

247. A millivoltmeter having a resistance of 1.96 ohms is used in connection with a shunt having a resistance of 0.096 ohm to measure current. When a current of 0.242 amp. flows in the line, determine the current in the shunt and in the millivoltmeter.

248. The shunt of a 15-scale ammeter has a resistance of 0.0032 ohm. If a manganin strip having a resistance of 0.0048 ohm is brazed between the copper terminal blocks of this shunt, what does the range of this instrument become?

249. The resistance of the moving coil of a direct-current voltmeter is 60 ohms and it gives full-scale deflection when 0.54 volt is impressed across the coil alone. (*a*) What resistance in series with the moving coil is necessary if the instrument is to have a 5-volt range? (*b*) 150-volt range?

250. A 5-scale, direct-current voltmeter has a resistance of 520 ohms. (*a*) What additional resistance in series will give this instrument a range of 150 volts? (*b*) 300 volts?

251. It is desired to measure the voltage between trolley and rail of a 1,200-volt railway system. A 300-scale direct-current voltmeter having a resistance of 31,200 ohms, is available. (*a*) What additional resistance in series with this voltmeter is necessary in order that the scale may have a multiplying factor of 5? (*b*) Should this resistance be connected between the instrument and trolley or between the instrument and ground? Explain.

252. A 150-scale voltmeter, having a resistance of 17,200 ohms, reads slightly off scale when connected across a potential somewhat in excess of 150 volts. When a 2,000-ohm resistance is connected in series with it, the instrument reads 146 volts. What is the unknown voltage?

253. A 300-scale voltmeter, having a resistance of 36,400 ohms, in series with its 5:1 multiplier is used to measure the direct current voltage of an electron-tube, plate-circuit generator. It is found that the voltmeter with its multiplier has insufficient range, so the voltmeter is shunted with a

resistance of 70,000 ohms. The voltmeter now reads 254 volts. What is the voltage of the generator?

254. In Prob. 253, what additional resistance in series with the voltmeter and its multiplier would have given the same multiplying power?

255. A 150-scale voltmeter, having a resistance of 18,800 ohms, and a 300-scale voltmeter, having a resistance of 32,000 ohms, are connected in series across a potential difference of 320 volts. What does each voltmeter read?

256. A space heater, when connected across 116-volt d.c. mains, takes 2.8 amp. What is its resistance?

257. When a 50-scale ammeter reads 27.2 amp., a millivoltmeter connected across its terminals reads 32.5 millivolts. What is the resistance of the ammeter?

258. When the armature of a 220-volt, 25-hp. motor is stationary, a current of 44.6 amp. gives a voltage drop across its terminals of 3.8 volts. What is the resistance of the armature?

259. The current in Prob. 258 was taken from d.-c. mains whose potential difference was known to be 115 volts. What resistance was connected in series with the armature?

260. The resistance of a sample of copper bus-bars is measured at 20° C. by the method shown in Fig. 131, page 154. When the ammeter reads 140 amp., the millivoltmeter reads 3.5 millivolts. The bus-bar is 0.5 in. by 2 in. in cross-section and the distance between voltmeter contacts is 3 ft. (*a*) What is the resistance of the sample? (*b*) What is its resistance per centimeter cube? (*c*) What is its per cent. conductivity? (See Par. 39, Chap. III.) Resistance of standard copper = 1.724 microhm-centimeters at 20° C.

261. The series field of a compound generator is shunted by a diverter (see p. 351). It is desired to know the resistance of the series field and of the diverter. With the diverter connected across the series field, a voltmeter across the two reads 4.3 volts when the current is 84 amp. The diverter is then disconnected and the voltmeter reads 5.8 volts, the current remaining unchanged. What is the resistance of the diverter and of the series field?

262. It is desired to obtain the resistance of a 70-lb. rail. A current of 375 amp. is sent through the rail and a millivoltmeter is connected between two contact points on the rail spaced 10.6 ft. apart. The millivoltmeter reads 9.49 millivolts. What is the resistance per foot of the rail?

263. A 300-scale voltmeter having a resistance of 35,000 ohms is connected across d.-c. mains and indicates 230 volts. It is then connected in series with an unknown resistance across these same mains. It now indicates 37 volts. What is the value of the unknown resistance?

264. A special 150-scale, 100,000-ohm voltmeter, when connected across d.-c. mains, reads 115 volts. The iron frame of a generator is connected to one wire of these mains and the copper of the field coil is connected to the other wire through the voltmeter, as shown in Fig. 264*A*. Under these conditions the voltmeter reads 6.7 volts. What is the resistance in megohms of the insulation of the field circuit to the frame of the machine?

265. A measurement of the insulation resistance of the armature of the machine (Prob. 264) is made by transferring the voltmeter lead from the field circuit to the commutator. The line voltage remains unchanged at 115 volts. With the insulation to ground of the armature in series with the voltmeter, it now reads 4.8 volts. What is the value of the insulation resistance to ground of the armature?

266. In a Wheatstone bridge measurement, the unknown resistance is connected at X between one end of the arm M and the arm P (see Fig. 133, p. 156). When a balance is obtained, M = 1,000 ohms; N = 10 ohms; P = 1,334 ohms. What is the value of the unknown resistance?

267. A resistance, whose value is known to be between 30 and 40 ohms, is connected to a Wheatstone bridge at X as shown in Fig. 133, p. 156. What are the best values of M and N to use? If the unknown resistance is 36.21 ohms, what will P read when a balance is obtained?

268. A Wheatstone bridge, similar to that shown in Figs. 133 and 134, pages 156 and 157, is used to measure an unknown resistance X. With M = 100, N = 1,000, an approximate balance is obtained when P = 14 ohms, the galvanometer deflecting decidedly to the right when P = 13 ohms and deflecting to the right to a considerably less degree when P = 14 ohms. What is the apparent value of the unknown resistance? Have the best values of the ratio arms M and N been chosen? Explain. What values of M and N should be chosen to give the resistance of X to four significant figures?

269. Choose the proper values of M and N (Fig. 134, p. 157) to give four significant figures in the value of X when X = 1.683 ohms; when X = 19,430 ohms; when X = 8,632 ohms; when X = 23.72 ohms. Give the value of P for each balance.

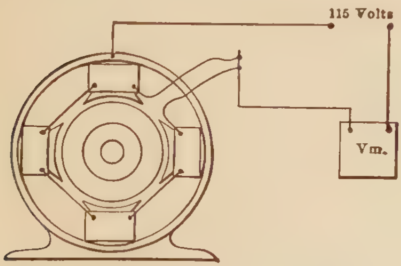


FIG. 264A.

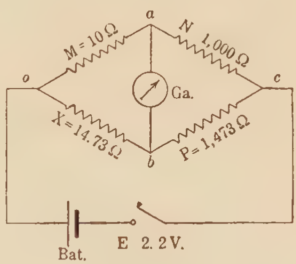


FIG. 270A.

270. In Fig. 270A is shown a Wheatstone bridge in balance. If the battery has an e.m.f. of 2.2 volts and an internal resistance of 0.7 ohm, what current does it deliver to the bridge?

271. Repeat Prob. 270 with the battery and the galvanometer interchanged, that is with the battery connected between points ab and the galvanometer connected between points oc. Which connection appears to be preferable?

272. An unknown resistance X is measured by means of a slide-wire bridge, similar to that shown in Fig. 137, page 161. (*a*) With $R = 10$ ohms, a balance is obtained when the slider reads 83.8 cm. Determine the value of X. (*b*) At what reading of the slider will the bridge balance if a 100-ohm resistance is used at R?

273. An unknown resistance is measured by means of a 100-cm. slide-wire bridge. A known resistance of 100 ohms is inserted at the 100-cm. end of the bridge (see Fig. 137, p. 161). A balance is obtained when the slider reads 18.5 cm. What is the value of the unknown resistance?

274. If a 10-ohm resistance be used as the known resistance in Prob. 273, what will be the reading on the slide wire when a balance is obtained?

275. A Kelvin bridge, similar to that in Fig. 139, page 164, is used to measure the resistance of a sample of 00 stranded copper wire. The distance between the contact points a and b is 18 in. The temperature is 20° C. With $M = m = 1{,}800$ ohms, $N = n = 10{,}000$ ohms, as shown in Fig. 139, a balance is obtained when $P = 0.0006632$ ohm. (*a*) What is the resistance of the copper sample? (*b*) What is its resistance per centimeter cube? (*c*) What is its per cent. conductivity?

276. A cable 1,800 ft. long, wound on a reel, is known to have a fault in its insulation. It is immersed in a tank of water and the Murray-loop test is used to locate the fault. The slide-wire bridge, 100 cm. long, reads 22.3 cm. when the balance is obtained. What is the distance from one end of the cable to the fault?

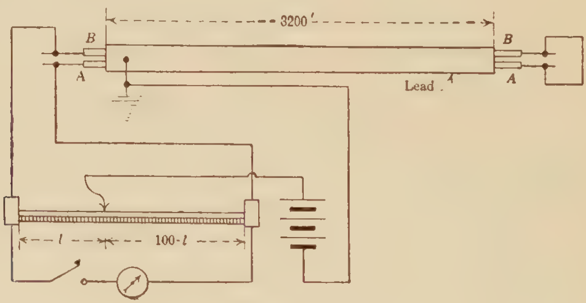


Fig. 277*A*.

277. A ground is known to exist in conductor B of a lead-covered No. 10 A.W.G. signal pair. The entire cable is 3,200 ft. long. To locate the fault, the far end of the pair is looped (Fig. 277*A*) and the Murray-loop test is made by means of a slide-wire bridge. A balance is obtained when $l =$ 18.3 cm. How far from the near end is the ground in B?

278. An installed two-conductor cable of 4/0 copper is 3,200 ft. long. Due to a burn-out both conductors are short-circuited and grounded at the same point. To locate the fault a single 00 conductor of another cable which parallels the faulty one is looped to one conductor of the faulty cable at the far end, as shown in Fig. 278*A*. The perfect conductor is connected to the

low reading end of the slide wire and one of the faulty conductors to the 100-cm. end. A balance is obtained at 89.4 cm. How far out on the faulty conductor is the burn-out located?

279. Both conductors of a No. 6 solid arc-lighting cable are known to be grounded in one 9,450-ft. section. To locate the fault a Varley-loop test is made. One of the grounded conductors is connected at the far end to one conductor of a similar cable which has no fault. Connections similar to those shown in Fig. 141, page 166 are used. With the switch at a and with $M = 10$ ohms and $N = 1{,}000$ ohms, a balance is obtained when $P =$ 2530 ohms. With the switch at b and with $M = 10$ ohms and $N = 100$ ohms, a balance is obtained when $P = 382$ ohms. How far from the home end is the fault?

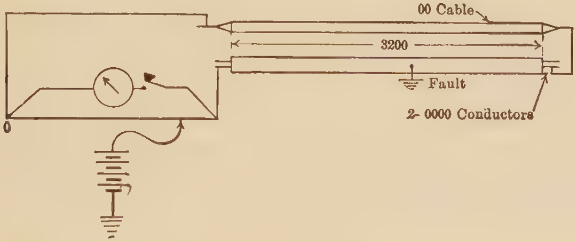


Fig. 278*A*.

280. Both wires of a pair of No. 19 signal wires have become grounded. A Varley-loop test is made to locate the fault. The only available return conductor is a single conductor of a No. 18 pair. The length of the wire between stations is 3,500 ft. The connections are the same as those shown in Fig. 141, page 166. With $M = N = 100$ ohms, and the switch at a, a balance is obtained with $P = 27.6$ ohms. With the switch at b, M and N remaining unchanged, a balance is obtained when $P = 52.4$ ohms. The ratio of the resistance of the No. 19 to the No. 18 wire may be taken as 1.26:1. How far is the fault from the home end?

281. One conductor in a cable containing two No. 14 wires a and b, each 8,000 ft. long, is known to be grounded. The two are looped at the far end and the Varley-loop test made. P is connected in series with conductor a. The two arms M and N (Fig. 141, p. 166) are each set at 100 ohms. A balance cannot be otained with P, as the galvanometer is found to deflect the same way with $P = 0$ and $P = \infty$. P is then shifted over in series with b, the other conductor, and a balance obtained when $P = 8.4$ ohms. The resistivity of the conductors is known to be 2.58 ohms per 1,000 ft. In which conductor is the fault and how far is it from the home end of the cable?

282. In an insulation test of a cable the connections are made as in Fig. 142, p. 167. When the cable is short-circuited and the Ayrton shunt is set at 0.0001, the galvanometer deflects 16.4 cm. The short-circuit is then removed, putting the cable in circuit, and the galvanometer deflects 19.8

cm. with the shunt set at 1.0 after the cable has been charged for 1 min. What is the insulation resistance of the cable?

283. The cable in Prob. 282 is 1,700 ft. long. What is its insulation resistance in megohms per mile?

284. In Prob. 282, the Ayrton shunt has a total resistance of 10,000 ohms and the galvanometer has a resistance of 1,200 ohms. The battery has an e.m.f. of 320 volts and its internal resistance is negligible. (*a*) Determine the battery current and the galvanometer current when the cable is short-circuited. (*b*) Repeat (*a*) with the cable in circuit.

285. In order to measure the insulation resistance of a cable in which the insulation resistance is known to be low, connections similar to those shown in Fig. 285*A* are made, the galvanometer, 0.1 megohm, and the cable insulation all being in series with the battery. The e.m.f. of the battery is 200 volts and its resistance is negligible. The resistance of the galvanometer is 470 ohms. With the cable short-circuited, the galvanometer is short-circuited and the switch *Sw* closed. The galvanometer short-circuit is then gradually removed by means of its shunt. After the shunt has become open-circuited the galvanometer deflection is 23.5 cm. (*a*) What current flows through the galvanometer? (*b*) With the cable short-circuit removed, the procedure is repeated and the galvanometer deflection after 1 min. of electrification is 0.4 cm. What current now flows in the galvanometer? (*c*) What is the resistance of the cable insulation?

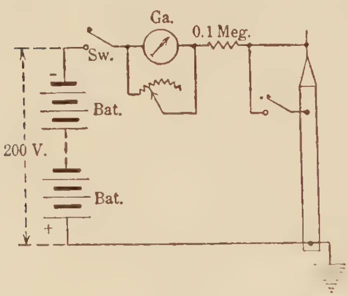


Fig. 285*A*.

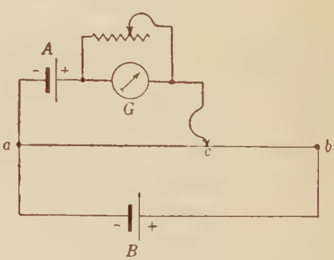


Fig. 286*A*.

286. Fig. 286*A* shows a 100-cm. manganin slide-wire *ab*, which has a total resistance of 0.54 ohm, connected across the terminals of the storage cell *B*, the end *a* being connected to the negative terminal of *B*. The storage cell *B* has an e.m.f. of 2.12 volts and an internal resistance of 0.08 ohm. *A* is a dry cell whose e.m.f. is 1.22 volts. Its negative terminal is also connected to *a* and its positive terminal is connected through a galvanometer *G* to a slider *c* on *ab*. For what position of *c*, in centimeters from *a*, will the galvanometer deflection be zero?

287. It is desired to measure the terminal voltage of a storage battery by means of a standard cell. The ratio and rheostat arms of a Wheatstone bridge (Fig. 287*A*) are connected across the terminals of the storage battery

and a standard cell having an electromotive force of 1.0183 volts is connected across a 1,000-ohm coil. The galvanometer in the standard-cell circuit stands at zero when 1,050 additional ohms are unplugged in P. What is the terminal voltage of the storage battery?

288. The storage battery of Prob. 287 is of such comparatively large capacity that its electromotive force and terminal voltage are sensibly the same when delivering the small current required by the resistance of 2,050 ohms. To measure the electromotive force of another cell which is not capable of delivering any appreciable current, its negative terminal is connected to the point a and its positive terminal to the point b through a key and galvanometer (Fig. 287A). P is then adjusted until this galvanometer reads zero. P is then read and found to be 780 ohms. What is the electromotive force of this cell?

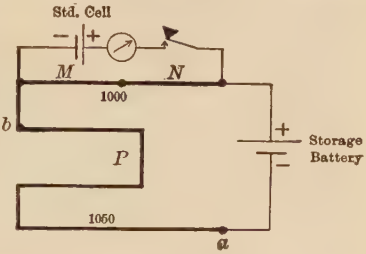


FIG. 287A.

289. It is desired to calibrate a 15-scale voltmeter near the upper end of its scale. The voltmeter is connected across a battery as shown in Fig. 289A. A resistance abc is connected in parallel with the voltmeter and battery. Between point b and the negative terminal of the battery a standard cell S, whose e.m.f. is 1.0186 volts, is connected in series with a key K and galvanometer, the negative terminal of the standard cell being connected to the negative side of the battery at c. The resistance bc is made equal to 100 ohms. When the resistance of ab is adjusted to 1,319 ohms the galvanometer does not deflect when the key K is depressed. At this instant the voltmeter reads 14.30 volts. What is the correction to the voltmeter scale for this reading?

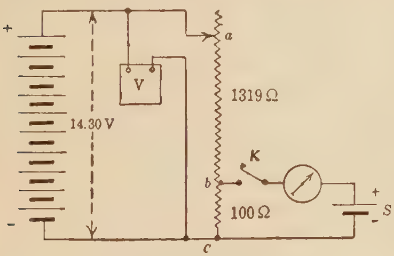


FIG. 289A.

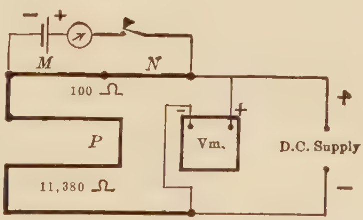


FIG. 290A.

290. It is desired to calibrate a voltmeter at the 115-volt point. No potentiometer is available. The voltmeter is connected in parallel with the arms of a bridge box (Fig. 290A) and the supply voltage is adjusted until the voltmeter reads just 115 volts. A standard cell, which is known to have an electromotive force of 1.0180 volts, is connected across the two ratio arms in series with a key and galvanometer, the proper polarity being observed

and 100 ohms are unplugged in these two arms. The galvanometer reads zero with the key depressed when 11,380 ohms are unplugged in P. What correction should be applied to the voltmeter at this point?

291. Fig. 291*A* shows a portion of a high-resistance potentiometer arranged on the Thomson-Varley slide principle. Between a and b are sixteen 50-ohm resistances, each corresponding to 0.1 volt. $a'b'$ consists of eleven 10-ohm resistances. $a''b''$ is a 20-ohm resistance divided into ten 2-ohm resistances. (a) What is the working current of the potentiometer? With the contacts in the positions shown, determine: (b) the potential-drop from a to c; (c) the current in the resistance $a'b'$; (d) the potential-drop from a' to c'; (e) the current in $a''b''$; (f) the potential-drop between a'' and c''; (g) the total potential-drop from a to c'' or the e.m.f. across ef. (Compare this potential-drop with the readings of the potentiometer.)

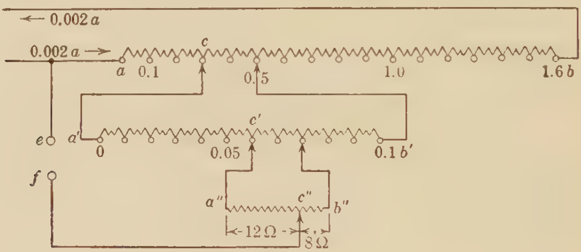


Fig. 291*A*.

292. The power to a 25-watt tungsten lamp is being measured with a voltmeter and an ammeter. The voltmeter, which has a resistance of 13,200 ohms, is connected directly across the lamp terminals. When the ammeter reads 0.206 amp. the voltmeter reads 118 volts. What is the true power taken by the lamp? What per cent. error is introduced if the instrument power be neglected?

293. The ammeter (Prob. 292) has a resistance of 0.06 ohm. The voltmeter is now connected directly across the line. (a) If the line voltage remains unchanged, what does the voltmeter read? (b) What is the true power now taken by the lamp? (c) What is the power given by the product of the voltmeter and ammeter readings? (d) What percentage error does the product in (c) give?

294. In measuring the power taken by a low-resistance rheostat, an ammeter having a resistance of 0.0008 ohm and a voltmeter having a resistance of 120 ohms are used. When the ammeter reads 70 amp. the voltmeter, which is connected directly across the resistance, reads 2.45 volts. What is the true value of the resistance? What per cent. error is introduced by the voltmeter current? What error is introduced by connecting the voltmeter outside the ammeter?

295. In a test of a direct-current 25-amp. watthour meter the average voltmeter reading is 118 volts and the average ammeter reading is 23.2 amp.

During the test interval thirty revolutions are counted and the time is found to be 39.1 seconds. If the meter constant is 1.0, what is the per cent. accuracy of the meter at this load? What adjustment should be made to bring it nearer the correct registration?

296. The meter load (Prob. 295) is dropped to 1.5 amp. but the voltage is still 118 volts. It takes 62.2 sec. for the disc to make three revolutions. What is the per cent. accuracy of the meter at this point? What adjustment should be made in order to bring it nearer the correct registration?

297. In order to make a laboratory test of a 2,000-amp., 220-volt, astatic watthour meter, of the type shown in Fig. 158, page 189, its current coils are supplied with current from a 4-volt storage battery and its armature circuit, which has a resistance of 2,200 ohms, is connected across the 235-volt mains, as show in Fig. 297*A*. A calibrated voltmeter is connected in

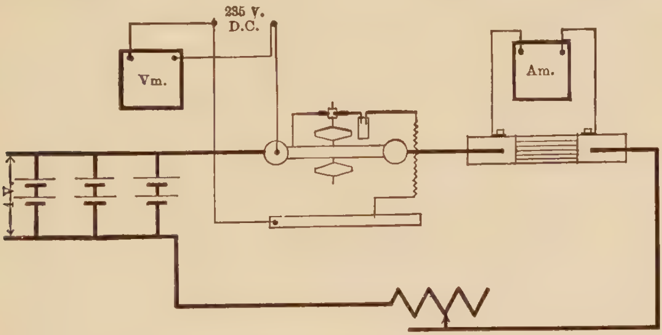


FIG. 297*A*.

parallel with the armature circuit and an external shunt ammeter is connected in series with the current terminals of the meter. The meter constant is 150. The corrected voltmeter reading is 235 volts and the corrected ammeter reading is 2020 amp. The meter makes 40 revolutions in 46.4 seconds. What is the per cent. accuracy of the meter at this load? How much power is required for this test? How much power would be required if the meter current were supplied at 235 volts?

QUESTIONS ON CHAPTER VIII

1. In what way does the magnetic circuit resemble the electric circuit? In what three respects do they differ from each other? Why cannot magnetic flux be readily confined to definite paths? In a general way how does the precision obtainable in magnetic calculations compare with that obtainable in electrical calculations?

2. Are ampere-turns dependent on current alone? On turns alone? What is the numerical relation between magnetomotive force and ampere-turns? Which is the larger unit, the gilbert or the ampere-turn? To what quantity in the electric circuit does magnetomotive force correspond?

What is reluctance and to what does it correspond in the electric circuit? What is the fundamental unit of reluctance? How is permeance related to reluctance and to conductance? How should permeances in parallel be added?

What is meant by permeability?

To what quantity in the electric circuit does magnetic flux correspond?

3. How is the reluctance of a magnetic path related to its length? To its cross-section? To its permeability? How are reluctances in series combined? In parallel?

4. Why is it usually necessary to represent the relation between magnetizing force and flux, in iron and steel, by curves? What general shape does the lower part of such curves have? The upper part? What is meant by saturation? How may a permeability curve be obtained from a *B-H* curve? How does the variation of permeability compare with such variation of electrical resistance as is due to heating?

5. State the simple law governing the relation among flux, mmf. and reluctance. To what law in the electric circuit does this law correspond?

6. Why is a method of trial and error sometimes necessary in solving magnetic problems?

7. Upon what three factors is the magnetomotive force acting upon a circuit dependent? How may the 0.4π be eliminated from computations in centimeter units? In inch units? How are magnetization curves plotted in order to reduce computations to the simplest basis?

8. Upon what two factors alone are the ampere-turns for an air-gap dependent? Derive the equations for such ampere-turns in both cm. and inch units.

9. Of what substances is permalloy composed? What remarkable magnetic characteristics has it? Compare its magnetic properties with those of a pure iron for both low and high flux densities.

10. If the magnetomotive force acting upon a sample of iron be increased from zero to some definite value and then decreased again to zero and the relation between magnetomotive force and flux be plotted, does the curve for increasing values of magnetomotive force correspond to that for decreasing values? If the excitation be decreased to zero, does the magnetic flux return to zero? How may the magnetic flux be brought back to zero? What is a cycle? A hysteresis loop? Remanence? Coercive force? What does hysteresis represent in terms of energy?

11. How is the hysteresis loss related to the loop area? How may the loss be calculated under practical conditions? How is the loss related to the maximum flux density? What is the Steinmetz law?

12. How is the geometrical position of the lines of induction related to the current in a circuit? Does this relation suggest the term "linkages?" How may these linkages be calculated? With constant permeability, what relation does inductance bear to the total linkages?

13. Describe a method of producing an electromotive force in a circuit which is insulated from everything else.

14. If an induced current is allowed to flow in a coil, what reaction will exist between this current and the inducing agent? If the inducing agent as, for example, a bar magnet, be withdrawn from a coil, will the induced electromotive force have the same direction as when the bar magnet was inserted in the coil? What reaction will be produced between the induced current and the inducing agent?

15. Upon what two factors does induced electromotive force depend? Is it possible to determine the value of this electromotive force in volts if these factors are known?

What is Lenz's law?

16. If the flux linking a coil be made to change by altering the value of the current in the coil itself, show that an electromotive force is induced. What is the relation of this electromotive force to the direction of the current flowing in the coil? How does this relation affect the rapidity with which the current builds up to its Ohm's law value?

17. What is the "time constant" of a circuit and by what two quantities may it be expressed? In a general way, what does it indicate as regards the circuit? What is the practical importance of the time lag of current due to self-inductance?

18. If an inductive circuit carrying a current be short-circuited, why does not the current die out immediately? To what is this tendency of the current to persist due?

What is the nature of inductance as regards its effect upon circuit changes? To what mechanical property does it correspond?

19. How does the effect of inductance manifest itself when the current of a circuit is interrupted? How can it be shown that this condition is not produced by the current alone? To what is the arc due? Under what conditions in practice may it become a menace? How may this menace be partially or wholly removed?

What personal dangers may result from opening inductive circuits?

20. Upon what three factors does the electromotive force of self-induction depend?

21. Does the establishment of a magnetic flux require an expenditure of energy? Is energy expended in maintaining this flux after it is once established? What becomes of the power required by electromagnet field coils? Give examples of electromagnetic energy manifesting itself.

22. State the equation which gives the electromagnetic energy stored in a circuit. How may the energy of generator fields be very materially reduced, before opening the circuit?

23. How does the gas-lighting spark coil utilize the electromotive force of self-induction in its operation? How is it connected in the circuit? Show that the spark coil can be considered as a reservoir in which magnetic energy is stored and later liberated. Explain why the coil produces a hot spark.

24. Show that it is possible for a magnetic flux produced by one coil to induce an electromotive force in another coil from which the first is insulated. Show that this is analogous to the production of electromotive force by the insertion of a bar magnet in the second coil. What is the relation of the

direction of the induced voltage in the secondary when the primary circuit is closed to its direction when the primary circuit is opened? Upon what three factors does this electromotive force depend?

25. Is it possible for *all* the flux produced by one coil to link another? What is the definition of the "coefficient of coupling" of two circuits?

How is mutual inductance defined? How may it be utilized to determine the induced voltage?

How may the mutual inductance of two circuits be materially increased?

26. Explain how the action of the induction coil depends upon mutual inductance. How is the primary current interrupted and why is it necessary that this current be interrupted?

27. Upon what two factors does the pull between magnetized surfaces depend? How does this pull vary with the flux density?

28. Describe the method by which flux may be measured by means of a ballistic galvanometer. Upon what factors does the ballistic throw depend?

29. Derive the equation which gives the flux in a ring-type solenoid having a cross-sectional area of A sq. cm., a mean circumferential length of l cm. and wound with n turns per centimeter (circumferential). Under what conditions does this equation give the flux within a long straight solenoid near its center?

30. How is the galvanometer calibrated by means of such a solenoid? Derive the Eq. (90) page 227 which gives the quantity Q_1 discharged through the galvanometer when the flux in the standard solenoid is caused to change. In a similar manner derive the Eq. (91) which gives the quantity Q_2 discharged through the galvanometer when the flux in the test specimen is changed from ϕ_1 to ϕ_2. By means of these two equations derive the galvanometer constant, Eq. (92).

31. Describe the yoke method of magnetic testing. What are its advantages? Name two important sources of error.

32. Describe the ring method of magnetic testing, comparing the procedure with that of the yoke method. State its advantages and disadvantages.

33. Upon what principle is the Koepsel permeameter based? Where is the test specimen placed? Describe the exciting coil and give the function of the compensating coils. What is the advantage of this instrument as a method of testing? What errors may exist in the instrument?

PROBLEMS ON CHAPTER VIII

298. Each of two similar exciting coils A and B (Fig. 298A) of an electromagnet has 2,500 turns and a resistance of 10 ohms. These two coils are connected in series across 120-volt mains, the connection being such that the coils are acting in conjunction on the magnetic circuit. (*a*) What are the total ampere-turns acting on the magnetic circuit? (*b*) Two more coils C and D, similar to A and B, are placed on the magnetic circuit and are connected in series electrically with A and B in such a way that their ampere-turns are acting in conjunction with those of A and B. What are the total ampere-turns now acting on the magnetic circuit? (*c*) What is the total

m.m.f. in gilberts? (*d*) Show with (*b*) and (*c*) that the ampere-turn is a larger unit of m.m.f. than the gilbert.

299. (*a*) In Prob. 298 determine the total ampere-turns when the current in both pairs of coils, *A* and *B* and *C* and *D* in series, is adjusted until the heating in each coil is the same as in (*b*). (*b*) Determine the total ampere-turns when the current is adjusted until the total heating is the same as the total heating in (*a*), Prob. 298.

300. An exciting coil having 400 turns designed for a 32-volt circuit is wound with No. 16 d.c.c. wire and has a resistance of 16 ohms. (*a*) Determine the number of turns and the resistance of another exciting coil of the same size and having the same winding space-factor (ratio of net copper section to total coil section), but which is wound with No. 19 wire having one-half the cross-section of No. 16. (*b*) With the same voltage, what are the ratios of the ampere-turns in the two cases? (*c*) Compare the heating in the two cases. (*d*) What voltage should be impressed across the second coil in order that the ampere-turns in the two cases may be the same? (*e*) In order that the heating may be the same?

301. Two coils *A* and *B* have respectively 3,600 and 2,400 turns and resistances of 25 and 20 ohms. Both coils are designed to operate at 120 volts without overheating. Both are placed on the same magnetic circuit. Determine the ampere-turns acting on the circuit: (*a*) with coil *A* alone; (*b*) with coil *B* alone; (*c*) coil *A* and *B* in series; (*d*) coil *A* and *B* in parallel. The voltage is 120 volts in each case, and in (*c*) and (*d*) the coils act in conjunction on the magnetic circuit.

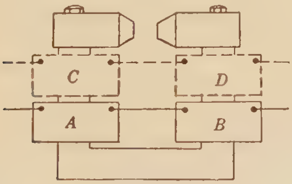


FIG. 298*A*.

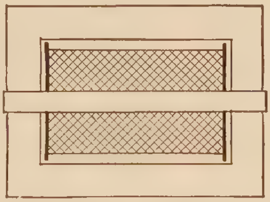


FIG. 302*A*.

302. In a certain iron-clad solenoid, Fig. 302*A*, the reluctance of the yoke is negligible compared with that of the plunger. When the plunger is inserted the lines of induction passing through the central core increase from 38 to 32,000. What is the permeability of the plunger at this flux density?

303. The steel field core of a dynamo is 6 in. in diameter and carries a magnetic flux of 2,160,000 lines. What is the flux density in lines per square inch and per square centimeter?

304. A magnet plunger, of circular cross-section and 4.0 in. diameter, carries a flux of 200,000 lines. What is the flux density in lines per square inch and per square centimeter?

305. A smooth-core, 2-pole dynamo armature has a diameter of 6 in. and an axial length of 10 in. The pole faces are square and cover 70 per cent. of

the armature surface. If the length of air-gap is $\frac{3}{16}$ in., what is the reluctance of each air-gap in oersteds? Neglect fringing.

306. The field core of a dynamo has a circular cross-section and is 4 in. long and 4 in. in diameter. At a flux density of 75,000 lines per square inch it has a permeability of 950. What is the reluctance between opposite ends of this field core at this flux density?

307. The two iron pole pieces of an electromagnet are 5 in. in diameter and are spaced $\frac{3}{8}$ in. apart, forming the air-gap. What is the reluctance of this gap? Neglect fringing.

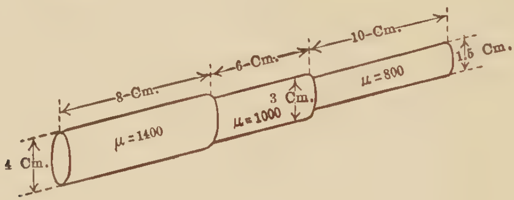


FIG. 308A.

308. Fig. 308A shows three portions of a magnetic circuit. Compute the reluctance of each portion and the total reluctance of the combination. Each portion is circular in cross-section.

309. Compute the reluctance of the magnetic circuit shown in Fig. 309A.

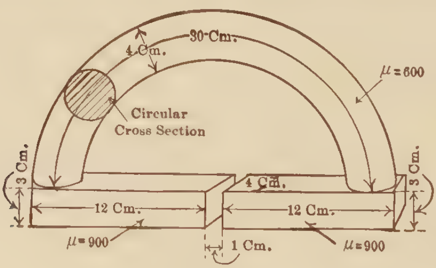


FIG. 309A.

310. Fig. 310A shows a magnetic circuit composed of two branches, which are similar and which are in parallel. Compute the reluctance of each half and the total reluctance of the circuit. The iron has a permeability of 250 throughout.

311. Two coils, each having 2,200 turns, are placed on the magnetic circuit shown in Fig. 309A, and are connected in series in such a way that they act in conjunction. What is the magnetomotive force acting on the circuit when 0.8 amp. flows in each coil? What is the resulting flux? What is the flux density in the gap, in lines per sq. cm.?

312. If a field coil of 2,500 turns is placed upon the central core of the magnetic circuit shown in Fig. 310A, and 1.8 amp. flow through the coil, what is

the resulting magnetomotive force? What are the total flux and the gap density in lines per square inch?

313. An anchor ring is shown in Fig. 313*A*. Determine its reluctance when the permeability of the iron is 1,000. What is the magnetomotive force if there are 160 turns wound on this ring and 1.25 amp. flow through the winding? What are the flux and the flux density?

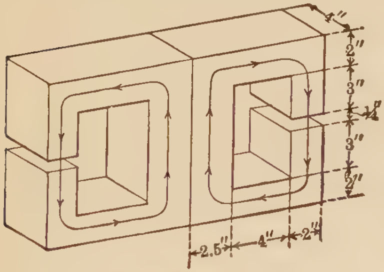


FIG. 310*A*.

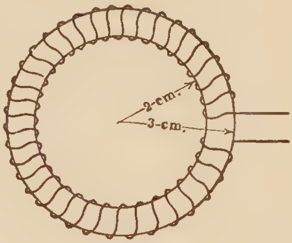


FIG. 313*A*.

314. A transverse gap 0.7 mm. long is cut in the anchor ring (Fig. 313*A*). The corresponding reduction of flux density in the iron increases its permeability to 1,050. Determine the magnetomotive force, the reluctance, the flux and the flux density.

315. Assume that the ring (Fig. 313*A*) is made of cast steel whose magnetization and permeability curves are shown in Figs. 161 and 162 (page 195), and that a transverse air-gap 0.7 mm. long is cut in it. Determine the flux and the flux density in the steel and air-gap when 2.0 amp. flows in the winding of 160 turns. (Use the trial and error method of Par. 151.)

316. Repeat problem 315, assuming the ring is made of cast iron and that 2.0 amp. flows in the winding (see Fig. 164, page 199).

317. Determine the ampere-turns necessary to send a total flux of 5,000 lines through the ring and the air-gap of Prob. 316.

318. Assuming that the magnet of Fig. 310*A* is made of cast steel whose permeability curve is shown in Fig. 162, page 195, determine the number of ampere-turns on the central core necessary to send a flux of 450,000 lines through each gap. Neglect fringing and leakage.

319. Repeat Prob. 318 using the cast-steel magnetization curve of Fig. 164, page 199.

320. The diameter of the non-slotted armature of a 4-pole generator is 12 in. and its axial length is 6½ in. The poles are square and cover 0.7 of the armature circumference. The air-gap is 0.18 in. Using the method of Par. 153, page 200, determine the ampere-turns necessary to send a flux of 1,700,000 maxwells across the gap.

321. The magnet shown in Fig. 321*A* has a yoke of cast steel and pole pieces of cast iron. Using the magnetization curves of Fig. 164, page 199, determine the ampere-turns necessary to send 120,000 lines through the air-gap. Neglect fringing.

322. Fig. 322*A* shows the magnetic circuit of a 2-pole dynamo. The field cores are of cast steel and are 4 in. by 5 in. The armature is of O.H. sheet steel and has a net axial length of 3.6 in. over the iron; the yoke is of cast steel and has a cross-section of 2 by 6 in. Using the magnetization curves of Fig. 164, page 199, determine the necessary field ampere-turns for an average flux density of 40,000 lines per sq. in. in the air-gap.

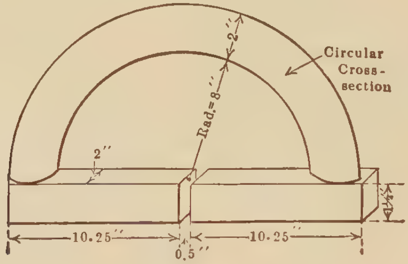


FIG. 321*A*.

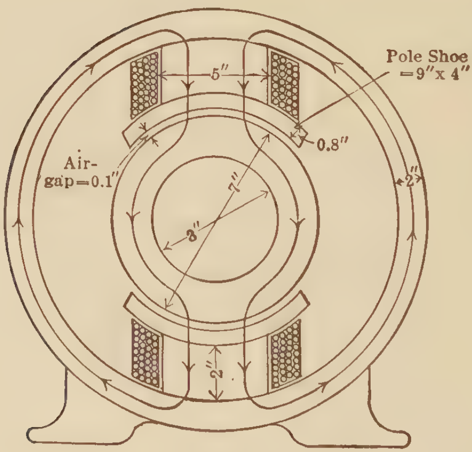


FIG. 322*A*.

323. Repeat problem 322 assuming the air-gap to be 0.075 in. and that only 80 per cent. of the flux in the yoke and field cores enters the armature. (Leakage factor = $^{100}/_{80}$ = 1.25.)

324. A core built up of silicon sheet steel has a total volume of 4,500 cu. cm. and a gross cross-sectional area of 100 sq. cm. The core operates with an alternating flux which varies between positive and negative values of 500,000 maxwells at a frequency of 50 cycles per second. Owing to the

oxide on the surfaces of the laminations, the ratio of the net steel to the gross cross-section is 0.9. Using the data of Par. 158, page 208, determine: (*a*) the ergs loss per cubic centimeter per cycle; (*b*) the total loss in ergs per cycle; (*c*) the ergs loss per second; (*d*) the power loss in watts due to hysteresis.

325. A transformer core of silicon steel has a volume of 600 cu. in., a gross cross-sectional area of 12 sq. in., and a ratio of net iron to total core cross-section of 0.9. What is the hysteresis loss in ergs per cubic inch per cycle if the maximum flux density is 50,000 lines per square inch? If the frequency is 60 cycles per second, what is the hysteresis loss in watts?

326. A demagnetizing coil with no iron in its magnetic circuit is wound with 2,280 turns. When the current is 2.5 amp. the flux linking the coil is 32,000 maxwells. Determine; (*a*) the c.g.s. flux linkages; (*b*) the c.g.s. linkages per ampere, divided by 10^8; (*c*) the inductance of the coil in henrys.

327. A plunger-type tractive solenoid (see p. 24) is wound with 2,800 turns of No. 12 d.c.c. wire. When the current is 4.2 amp., the net flux linking the turns is 50,000 maxwells. Assuming constant permeability, determine: (*a*) the c.g.s. linkages; (*b*) the c.g.s. linkages per ampere divided by 10^8; (*c*) the inductance of the solenoid in henrys. (*d*) Would the inductance of such a solenoid be constant, even if the permeability of the iron remained constant?

328. The solenoid (Prob. 327) is rewound with 5,600 turns of No. 15 d.c.c. wire having half the cross-section of the No. 12 wire. It now operates at double voltage so that the current is 2.1 amp. Determine: (*a*) the c.g.s. linkages; (*b*) the inductance in henrys. (*c*) What conclusion may be drawn as to the relation of inductance to number of turns, other factors remaining unchanged?

329. Assuming that the permeability of the iron does not change, determine the inductance of the solenoid (Prob. 328) when the current is doubled. In general, how would the inductance of a coil having a core of ordinary iron vary with increase of current?

330. The two exciting coils of a bipolar dynamo are wound with 1,600 turns each. When the exciting current is 2.6 amp., the flux crossing the air-gap is 2,400,000 maxwells. The coefficient of leakage is 1.25 (see Prob. 323). Assuming constant permeability for the iron, determine the inductance of this field circuit.

331. A steel anchor ring having a cross-sectional diameter of 1¼ in. and a mean diameter of 8 in. is wound uniformly with 180 turns of No. 12 d.c.c. copper wire. Using the magnetization curve for cast steel (Fig. 164, p. 199) and assuming the curve for the lower flux densities is a straight line, determine its inductance when the current is 2.8 amp.

332. If the field current, 2.6 amp., of the bipolar dynamo, Prob. 330, is interrupted in 0.05 sec., (*a*) what is the average induced e.m.f.? (*b*) What is the total stored energy before the circuit is interrupted? (*c*) What is the average power during the period of interruption?

333. When the exciting current of an electromagnet is flowing, there are 1,800,000 lines of induction linking the circuit. The exciting coil has 4,800

turns. (*a*) If the exciting current is interrupted, requiring 0.2 sec. to rupture completely the arc, what is the average induced voltage across the ends of the exciting coil? (*b*) If the exciting current is 0.8 amp., what is the inductance of the exciting coil? (*c*) How much energy is stored in the magnetic field?

334. Recompute Prob. 333 assuming that the circuit is interrupted in 0.05 sec.

335. A relay coil wound with 2,400 turns of No. 28 A.W.G. wire has a resistance of 120 ohms and the flux linking these turns is 8,000 maxwells when the current is 0.9 amp. Neglecting any change of permeability, determine: (*a*) the total c.g.s. linkages; (*b*) the self-inductance in henrys; (*c*) the time constant of the coil; (*d*) with 110 volts across the coil the value of the current when a time equal to the time constant has elapsed after the closing of the circuit. (*e*) Make a sketch showing the change of current with time.

336. The windings of an electromagnet have an inductance of 3.4 henrys and a resistance of 6.2 ohms. (*a*) What is the time constant of the magnet? (*b*) When these windings are connected across 120-volt mains, determine the value of current at the instant when the elapsed time after the closing of the switch is equal to this time constant.

337. A resistance of 4 ohms is connected in series with the electromagnet windings (Prob. 336). (*a*) What is the corresponding value of the time constant? (*b*) What is the value of the current at the instant when the elapsed time after the closing of the switch is equal to this time constant? (*c*) Compare (*a*) and (*b*) with (*a*) and (*b*) of Prob. 336.

338. Sketch on the same plot the curve of current and time, in the two Probs. 336 and 337, and compare the curves.

339. Determine the stored energy in Probs. 336 and 337, after the current has reached its Ohm's law value. If the circuit is completely opened in 0.04 sec. in each case, what is the average value of the power dissipated during the period of opening?

340. In Prob. 339, compute the average induced e.m.f. when the circuit is opened in 0.05 sec. using; (*a*) Eq. (74), page 211; (*b*) Eq. (75), page 216.

341. The field winding of an alternator has an inductance of 6.8 henrys and a resistance of 48 ohms. (*a*) What is the energy stored in the magnetic field when this field winding is connected across the 120-volt exciting bus? (*b*) How much additional resistance must be connected in series with the winding in order that the stored energy may be reduced to one-half this value?

342. A resistance of 25 ohms is connected across the field of the alternator (Prob. 341) by means of a field-discharge switch. Determine: (*a*) the time constant of the field circuit and this resistance in series; (*b*) the value of the current at the instant when the time, after the connecting of the field-discharge resistance across the field, is equal to the time constant.

343. Determine the average induced e.m.f. which would result if the current in Prob. 342 were reversed in 0.1 sec. after having reached its Ohm's law value.

344. A certain generator field circuit has an inductance of 4 henrys and carries 150 amp. The induced voltage across the field terminals must not exceed 1,000 volts. What is the minimum time which can be allowed for opening the field? How much energy is liberated in opening this field? What is the average power during the opening period?

345. Repeat Prob. 344 with the total field resistance doubled by means of the field rheostat.

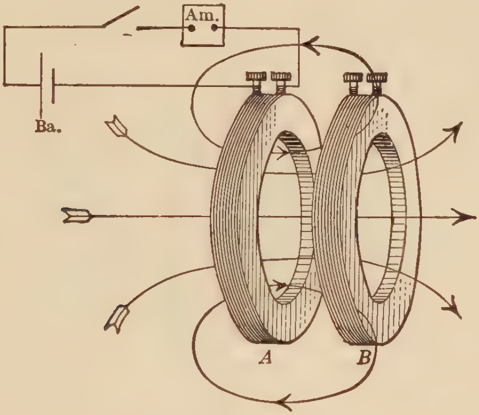


FIG. 346*A*.

346. Two coils *A* and *B*, Fig. 346*A*, are insulated electrically but are so placed that 70 per cent. of the flux produced by one of the coils links the other. Coil *A* has 150 turns and coil *B* has 250 turns. When 2.5 amp. flow in coil *A*, 200,000 lines link the coil. (*a*) How many lines link B? (*b*) What is the coefficient of coupling of the two circuits? (*c*) If the current in *A* is interrupted in $\frac{1}{20}$ sec. what induced voltage results in *B*? In *A*?

347. The same flux that was produced in *A* by 2.5 amp. is produced in *B* by 1.5 amp. What voltage is induced in *A* upon interrupting the 1.5 amp. of *B* in 0.1 second? What voltage is induced in *B*?

348. Determine the mutual inductance in henrys of coils *A* and *B*, in Probs. 346 and 347 using Eq. (80), page 220. What is the self-inductance of *A*? Of *B*? Determine the mutual inductance between coils *A* and *B* using Eq. (81) page 221.

349. Coils *A* and *B* of Prob. 346 are now linked magnetically by an iron core as shown in Fig. 349*A*, so that practically all the flux of one links with the other. 0.1 amp. in *A* now produces 180,000 lines in the joint magnetic circuit. How many amperes in *B* will produce this same flux? What is the self-inductance of *A*? Of *B*?

350. If the 0.1 amp. in *A* of Prob. 349 is interrupted in 0.05 sec., what electromotive force is induced in *A*? In *B*? What is the mutual inductance of the circuits? The self-inductance of *A*? Of *B*?

At what rate must the current in B be interrupted to induce 10 volts in A? If the current in B is 0.05 amp., in what time should the circuit be opened?

351. The flat pole pieces of an electromagnet are in contact with each other and a total flux of 2,500,000 lines passes from one to the other. If the face of each pole is circular and 5 in. in diameter what force in pounds is necessary to pull these pole pieces apart? Assume that the flux is distributed uniformly over the pole faces.

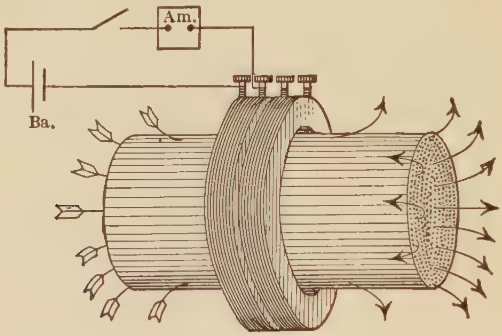


FIG. 349A.

352. A straight air-core solenoid 40 in. long has a single-layer primary winding of d.c.c. wire, with 16 turns to the inch. The inside diameter of solenoid is 1.40 in. A secondary of 1,000 turns is wound over this primary winding near the center. (a) When the primary current is 8.2 amp., determine the flux density in the solenoid. (b) Determine the total flux. (c) What are the c.g.s. flux linkages with the secondary?

353. A secondary coil used in connection with the yoke method, and having 12 turns, and a ballistic galvanometer, are connected in series with the secondary of the solenoid of Prob. 352. When the current of 0.82 amp. in the primary of the standard solenoid is suddenly reduced to zero by opening the switch, the ballistic deflection of the galvanometer becomes 14 cm. (a) Determine the galvanometer constant (see Eq. (92), p. 228). (b) The current in the primary of the yoke specimen is increased from zero to 0.4 amp. and one section of the test specimen is suddenly withdrawn, thus reducing the flux suddenly to zero. The corresponding galvanometer deflection is 18.2 cm. What is the value of the flux in the specimen? (c) If the yoke specimen has a diameter of 1 cm., what is the flux density?

354. In Prob. 353 the current in the yoke specimen is increased to 1.5 amp., decreased to zero, and then increased to 0.9 amp. in the reverse direction. At this point, the half of the yoke is again withdrawn and the corresponding galvanometer deflection is 19 cm. reversed. What is the flux density corresponding to this value of current? Make a sketch of the hysteresis loop showing this point and the one given in Prob. 353.

QUESTIONS ON CHAPTER IX

1. Discuss the probable character of "dynamic" electricity as based on our present knowledge of atomic structure. Compare dynamic and static electricity from this same point of view. Why do the two frequently appear to be different in character?

2. If two insulated ellipsoids near each other are connected to the terminals of an influence machine, upon what portions of the ellipsoids will the density of charge be greatest? Would any considerable change be observed in these charges if the wires to the machine were disconnected? How can it be shown that charges are "bound."

3. What is the character of the force existing between the ellipsoids? The charges? Explain.

4. If a positive charge is brought into the neighborhood of an insulated and uncharged ellipsoid or sphere, what phenomenon occurs? What is the relation of the induced charge to the inducing charge? Distinguish between free and bound charges. How may it be shown experimentally that free and bound charges behave differently?

5. State three laws governing electrostatic attraction and induction.

6. Define a unit charge. State Coulomb's law.

7. What effect does an electric charge produce in the immediate surrounding medium? Compare this condition with that surrounding a magnet. How many lines of force must emanate from each unit charge?

8. State three laws which govern the configuration of a dielectric field of force. Compare the dielectric field with current flow; with magnetic lines of force and magnetic lines of induction. Show that dielectric lines must ordinarily be normal to a conducting surface where they enter or leave such a surface.

9. Why must a positive charge on an isolated sphere reside on the surface of the sphere and be uniformly distributed over it? Prove by two methods that a charge distributed uniformly over the surface of a sphere behaves, so far as its external effects are concerned, as if the charge were concentrated at the center of the sphere.

10. Distinguish between the dielectric properties and the insulating properties of a medium. In what one important respect do dielectric lines differ from current and magnetic lines? Give examples of excellent insulators and excellent dielectrics. In what terms is dielectric strength measured?

11. Discuss the probable mechanism of dielectric rupture according to modern electron theory. How may ions be produced by collision? What is meant by *ionization?* Sketch an arrangement of electrodes which will permit a high degree of ionization at one electrode in the surrounding air without complete rupture following. State the properties of ionized air.

12. Define a condenser. What is the effect of applying a voltage to an electric condenser? What is the order of magnitude of the time required by a current to charge such a condenser? Why does the current cease to flow? To what hydraulic phenomenon can this be compared?

13. How can it be shown that electricity is actually stored in a condenser? How does the quantity which can be stored in a condenser vary with the voltage? What simple relation does this give among charge, capacitance and voltage? State the equation, written in three different ways, which gives the relation among charge, capacitance, and voltage.

14. What is the usual effect of inserting some dielectric medium other than air between condenser plates? What is "specific inductive capacity" and to what magnetic property is it analogous? What is the dielectric constant of glass? Of mica? Of rubber?

15. Determine the equivalent capacitance of condensers connected in parallel. To what electric circuit condition is this analogous?

16. Determine the equivalent capacitance of condensers connected in series. What is the relation among the electric charges on each of a number of condensers connected in series? To what equation in the electric circuit is the equation relating to the equivalent capacitance of condensers in series similar?

17. Derive the relationship among the voltages across a number of condensers in series, assuming that the line voltage and the individual capacitances are known. Are these voltage relations dependent at all upon the insulating properties of the dielectrics? In the case of leaky condensers, upon what does the ultimate voltage distribution depend?

18. How may it be shown that electric energy can be stored in a condenser? Upon what factors does this energy depend? What is the relation of energy to voltage if the capacitance is fixed?

19. Upon what factors does the capacitance of a parallel plate condenser depend? What is the effect upon the capacitance of changing the area of the plates? Of decreasing the distance between them? Of substituting hard rubber or glass for air?

20. What two methods are commonly employed in the measurement of capacitance? Upon what fact does the ballistic galvanometer method depend? What relation exists between the quantity passing through the galvanometer and its maximum ballistic throw?

Should the measurement be made upon "charge" or upon "discharge?" Explain. How is the galvanometer calibrated?

21. Describe the bridge method of capacitance measurement. Compare it with the Wheatstone bridge method of resistance measurement. How does the bridge formula for capacitance differ from the formula employed when resistance is measured? What is the source of power and what simple detector is used in the capacitance bridge?

22. How may a disconnection in a cable be located? Upon what principle does this method of measurement depend? Is this method applicable if the fault is grounded?

PROBLEMS ON CHAPTER IX

355. Each of two small spheres, spaced 20 cm. between centers in air, has a charge of 12 e.s.u. (statcoulombs), one charge being positive and the

other negative. (*a*) What is the force in dynes acting between the two spheres? (*b*) If these two spheres are immersed in oil, having a dielectric constant of 2.3, what is the force acting between the spheres? The distance between spheres and the charges remain unaltered.

356. Determine the force between two small spheres in air, spaced 4 in. between centers and charged with 7 and 12 positive statcoulombs. Determine the force between these spheres if they are immersed in an oil whose dielectric constant is 2.1. The spacing and the charges remain unaltered.

357. Determine the dielectric flux emanating from each of the spheres (Prob. 356) when they are in air.

358. An isolated sphere in air, 5 cm. in diameter, is charged with 30 statcoulombs of positive electricity. Determine: (*a*) the total dielectric flux emanating from this sphere; (*b*) the density of the dielectric flux in lines per square centimeter at the surface of the sphere; (*c*) the force on a unit charge at the surface of the sphere, determined by two methods.

359. Two spheres, each 0.5 cm. in diameter, are spaced 22 cm. between centers in air. One is given a positive charge of 18 statcoulombs and the other a negative charge of 30 statcoulombs. Determine: (*a*) the force acting between the spheres; (*b*) the dielectric flux emanating from each; (*c*) the force in dynes on a unit charge at the surface of each; (*d*) the force at a point midway between the centers of the spheres; (*e*) the force at a point on the line joining the centers of the spheres and 5 cm. from the center of the second sphere.

360. A condenser having a capacitance of 25 μf. is connected across a 200-volt source. (*a*) What charge in coulombs does it take? (*b*) If 320 volts are impressed across the condenser, what charge does it take? (*c*) The condenser in each case is charged for 0.2 sec. with a current whose value is maintained constant. What is the value of the current in each case?

361. A condenser has a capacitance of 12 μf. What is its counter e.m.f. when its charge is 3,200 microcoulombs?

362. For how long must a steady current of 0.002 amp. flow into a 42-μf. condenser before its e.m.f. is raised to 380 volts?

363. A ballistic galvanometer shows that a charge of 420 microcoulombs is taken by a condenser when the potential difference is 150 volts. What is the capacitance of the condenser?

364. A Leyden-jar type of condenser has a capacitance of 0.003 μf. How many coulombs are necessary to raise its potential to 10,000 volts?

365. An air condenser consists of three equidistant parallel plates, the two outer plates being connected together to form one electrode and the center plate forming the other electrode (see page 254, Fig. 203 (*b*)). This condenser has a capacitance of 0.00016 μf. When it is immersed in hot paraffin, and the paraffin allowed to cool so that the space between the plates is entirely filled, the capacitance is found to be 0.000362 μf. What is the dielectric constant of the paraffin?

366. A certain condenser consisting of three parallel plates, with air as dielectric, has a capacitance of 0.00014 μf. A slab of glass is placed between each outer plate and the center plate occupying the entire space between the

plates. The capacitance is now found to be 0.00072 μf. What is the specific inductive capacity of the glass?

367. The condenser of problem 366 with air as a dielectric is charged to a potential of 320 volts between plates and the supply then disconnected. The glass is then inserted between the plates, completely filling the space. This insertion of the glass in no way changes the value of the electric charge on the plates. What is the condenser voltage after the insertion of the glass? What does the charge become if the 320-volt supply is again connected?

368. A plate condenser, with air as dielectric, has a capacitance of 0.0014 μf. and 320 volts is impressed across its terminals. The condenser is then immersed in a bath of transformer oil having a dielectric constant of 2.4, the voltage supply remaining connected. What is the charge on this condenser before and after immersion in the oil? The voltage supply is disconnected while the condenser is immersed in the oil, and the condenser is then removed from the oil, so that the dielectric is now air. What does the voltage across the condenser become? Neglect any leakage.

369. The charges taken by four condensers are 1,200, 1,500, 2,100, and 2,500 microcoulombs when they are connected in parallel across 240 volts. What is the equivalent capacitance of the combination?

370. Three condensers having capacitances of 4, 2, and 1.5 μf. are connected in series across a 50-volt battery: (*a*) What single capacitance will replace the three? (*b*) Assuming negligible leakage, what is the charge on each? (*c*) What is the voltage across each?

371. The three condensers of Prob. 370, after being charged in series, are disconnected and then connected in parallel, with plates of like polarity together. (*a*) What is the total charge on the combination? (*b*) What is the charge on each condenser? (*c*) What is the voltage across the parallel combination of condensers?

372. Determine the equivalent capacitance of four condensers in series having capacitances of 15, 20, 25, and 35 μf. If 220 volts are impressed across the four in series, determine the charge on each and the voltage across each, assuming negligible leakage. .

373. The four condensers of Prob. 372, after being charged at 220 volts in series, are disconnected and connected in parallel. The last three are connected with terminals of like polarity together and the 15-μf. condenser is connected in parallel with them, but with its positive terminal to their negative terminal. Determine: (*a*) the net charge on the entire combination; (*b*) the charge on each condenser; (*c*) the voltage of the combination.

374. When four condensers are connected in series across a 250-volt source, their voltages are 35, 75, 80, and 60 volts, and the charge on each is 1,240 microcoulombs. (*a*) What is the capacitance of each? (*b*) What is the capacitance of the series combination?

375. Two condensers in parallel have charges of 1,500 and 3,200 microcoulombs when connected across a 120-volt source. Determine the energy on each and the total energy.

376. Four condensers having capacitances of 20, 25, 32, and 40 μf. are connected in series across a 300-volt source. (*a*) What is the total energy

of the combination? (*b*) How much energy is stored in each of the condensers?

377. If the four condensers, of Prob. 376, are disconnected after being charged and are then connected in parallel with terminals of like polarity together, what is their total energy? Account for the loss in the energy of the system because of the transfer.

378. A 10-μf. condenser is connected in series with a 4-μf. and an 8-μf. condenser which are in parallel. The combination is then connected across a 250-volt source. Determine; (*a*) the total energy; (*b*) the energy in the 10-μf. condenser; (*c*) the energy in the 4-μf. condenser; (*d*) the energy in the 8-μf. condenser.

379. Repeat Prob. 378 with the voltage increased to 500 volts. How does the stored energy in a condenser vary with the voltage?

380. A condenser consisting of six Leyden jars, used for radio telegraphy sending, has a total capacitance of 0.018 μf. The condenser is charged to 12,000 volts when it discharges into the oscillating circuit. (*a*) How much energy does the condenser discharge each time into the oscillating circuit? (*b*) If the condenser discharges this amount of energy 1,000 times each second, how much power is it discharging?

381. Determine the energy stored in a 12-μf. condenser when the charge is 4,000 microcoulombs.

382. An air condenser consists of three plates. The two outer ones are connected together as one terminal and the other terminal is connected to the intermediate plate between the two outers. The dimensions of each plate are 14 by 14 in. and the plates are spaced $\frac{3}{64}$ in. apart. What is the capacitance of this condenser?

383. If the space between the plates of the condenser of Prob. 382 is filled with paraffin, having a dielectric constant of 2.1, what does the capacitance become?

384. A high-voltage condenser is to be made of alternate layers of glass and tin foil, the glass having a dielectric constant of 7.6. The glass is $\frac{3}{64}$ in. thick and the tin foil is 2 mils thick and its dimensions are 3 by 4 in. How many plates and sheets of tin foil are necessary to make a condenser having a capacitance of 0.025 μf.? If the glass plates are 5 by 6 in., what is the size of the completed condenser?

385. It is desired to construct a 2-μf. condenser from sheets of tin foil, 1 mil thick and 4 in. square, by pressing them between alternate layers of thin paraffined paper, 1.5 mils thick and having a dielectric constant of 2.0. How many sheets of paper and how many sheets of tin foil are necessary? What are the dimensions of the finished condenser if the sheets of paper are 5 in. square?

386. Determine the capacitance in $\mu\mu$f. of a cylindrical air condenser, 3 m. long, the outer cylinder of which has an inside diameter of 10 cm. and the inner cylinder a diameter of 4 cm. Neglect end effect.

387. A single-conductor cable consists of a solid No. 6 A.W.G. copper conductor, having a cross-section of 26,200 cir.-mils, and a wall of rubber whose outside diameter is 0.377 in. and the dielectric constant of which is 4.5.

The rubber is surrounded by a $\frac{5}{16}$-in. lead sheath. What is the capacitance of a mile length of this cable?

388. In a 500,000-C.M. single-conductor, impregnated-paper cable the diameter over the conductor is 0.815 in. The conductor is wrapped with an $\frac{8}{32}$ in. wall of impregnated paper having a dielectric constant of 2.4. The paper is surrounded by a $\frac{1}{8}$ in. lead sheath. Determine the capacitance in microfarads of a 5-mile length of this cable.

389. In a ballistic measurement of capacitance (see p. 257, Fig. 205) the ballistic deflection of the galvanometer is 19.3 cm. when the unknown condenser is connected, an Ayrton shunt used in connection with the galvanometer being set at 0.1. The galvanometer deflection becomes 11.3 cm. with the Ayrton shunt set at 1.0 when the galvanometer is calibrated with a 1-μf. standard condenser. The same battery is used in each case. (*a*) What is the galvanometer constant? (*b*) What is the value of the unknown capacitance?

390. In a bridge measurement of condenser capacitance, the bridge is connected as shown in Fig. 206 (*a*), page 258. When a balance is obtained, $R_1 = 100$, $R_2 = 976$, $C_2 = 0.6$ μf. What is the value of C_x, the unknown capacitance?

391. In a test for a cable fault, the apparatus is connected as shown in Fig. 207, page 259. In the capacitance measurement of the part x, the galvanometer has a ballistic throw of 6.4. In the measurement of the capacitance of the perfect cable plus the looped end of the faulty cable, the deflection is found to be 18.3 cm. If the length of each conductor is 2,200 ft., how far from the point of test is the cable broken?

QUESTIONS ON CHAPTER X

1. In what way is the flux linking any one coil of a generator armature made to vary? Describe the manner in which this variation of the flux causes a voltage to be induced. How does this induced voltage vary with the speed? The flux? The number of turns in the coil?

2. If instead of regarding this voltage as due to the change of flux linking a coil, it is considered as being due to the individual conductors cutting flux, is the ultimate result in any way affected? If the voltage is considered as being due to the cutting of lines by individual conductors how does this voltage vary with the length of conductor? The flux density? The velocity of the conductor?

3. What definite relation exists among the direction of the induced e.m.f., the direction in which the conductor moves and the direction of the flux? What simple rule enables one to determine these relations?

4. What is the value of the e.m.f. induced in a rotating coil; (*a*) when the coil is in the plane perpendicular to the flux? (*b*) When its plane lies parallel to the flux? Does the voltage ever reverse its sign? Explain.

5. How may the alternating current produced in a coil be changed to direct current? What is the effect of adding coils to the rotating member? To what are the "ripples" in a voltage wave due?

6. In what way is the open-coil type of armature different from the closed-coil type? Which type is the gramme-ring armature (Fig. 215)? Show that the resultant electromotive force is different in the two types, even though the number of coils and turns be the same.

7. Name two serious objections to the ring winding. How are these objections overcome in the drum winding? What two methods are used to fasten conductors on the surfaces of armatures? Which is the better method and why?

8. What is meant by "coil pitch" and what is its relation to pole pitch? What relative positions in the slots do the two sides of any one coil occupy? Why? What is meant by a "winding element?" May it consist of more than one conductor? Explain.

9. What is "front pitch?" "Back pitch?" "Average pitch?" What is the relation between the number of winding elements and the number of coils? The number of commutator segments?

10. In a simplex lap winding how many commutator segments does the winding advance each time that a coil is added? What three fundamental conditions must be fulfilled by a winding? What is a winding table and what is its practical value?

11. Why is it sometimes desirable to place more than two winding elements in a slot? In what type of generator is this necessary? Is the conductor numbering and are the winding relations in any way affected? What one condition should be imposed upon this type of winding and why?

12. What is meant by "paths through an armature?" How is the current output of a machine affected by increasing the number of paths? How is the voltage affected? The power output? How many paths are there in all simplex lap windings?

13. What is meant by a duplex winding? Show that such a winding may be composed of two simplex windings each lying in alternate slots. How many closures may such a winding have and what is its degree of re-entrancy in each case?

14. If a duplex winding does not close after one passage around the armature, is the number of segments even or odd? When does such a winding close? How many times does it close and therefore what is its degree of re-entrancy?

15. In a winding whose multiplicity is m, how many winding elements separate a given element from the next returning element? How many armature paths are there in a 6-pole machine having a simplex lap winding? A duplex lap winding? A triplex lap winding?

16. To what causes are unequal voltages in different paths of an armature winding due? Do equalizing connections do away with these inequalities? What is the purpose of equalizing connections? What care should be taken regarding the number of slots per pole when equalizing connections are used? Why?

17. What is the fundamental difference between a lap and a wave winding? Does the direction of induced e.m.f. in opposite sides of a coil differ in the two types? Explain.

18. After a wave winding has passed under every pole, in passing around an armature, what relation should it bear to its starting position if the winding is simplex? What would a closure after one passage around the armature mean?

19. Show that the definitions of front pitch and back pitch in a wave winding do not differ from the similar definitions in a lap winding. Can the front pitch be even? Odd? Can the back pitch be even? Odd? Can the two be equal? Can the average pitch be even? Odd? When is a winding progressive? Retrogressive? Explain.

20. Is it always possible to fit a wave winding to an armature having a fixed number of slots if all the slots are utilized? Explain. What makeshift may be used to accomplish the desired result?

21. If the number of pairs of poles is even, is the number of commutator segments even or odd? Answer if the number of pairs of poles is odd?

22. What is the minimum number of brush sets that can be used in a wave winding? What is the maximum number that it is possible to use? When would two sets be used and why? Why is the maximum number usually desirable?

23. How many paths are there in a simplex wave winding? In what way is the number of such paths affected by the number of poles? How many paths in a duplex wave winding? A triplex wave winding?

24. When is it desirable to use a wave winding and why? A lap winding? Give specific reasons.

25. In addition to forming a part of the magnetic circuit, what other function does the yoke of a generator perform? Of what two materials is it made and why? Describe a process whereby the yoke is made without casting.

26. Of what materials are the field cores made? The pole shoes? What are the two general shapes of the core sections? Where is each used?

27. Is the armature a solid casting? If not, how is it built up? By what two methods are the stampings produced? How are they held in position when placed upon the armature? What is the purpose of the ventilating ducts?

28. Sketch two general types of slot. Where is each used? What two methods are used to prevent the conductors from being affected by centrifugal forces?

29. Of what is the commutator made? What insulation is used between segments? How are the segments clamped together? How are the coil connections made?

30. What is the purpose of the brushes? Of what material are brushes usually made? What pressure is used to hold the brush on the commutator? What is the purpose of the plating on the brush? What is the purpose of the pig-tail?

PROBLEMS ON CHAPTER X

392. A coil 20 cm. square, having 40 turns, lies perpendicular to a uniform magnetic field having a density of 600 gausses. This coil is made to revolve

a quarter turn about its axis in 0.1 sec., so that it lies parallel to the direction of the field. (*a*) What is the average induced e.m.f. during the period? (*b*) If the coil revolves at 2.5 r.p.s., what is the average induced e.m.f.? (*c*) What is the induced e.m.f. when the coil is perpendicular to the field? When it is parallel to the field?

393. A coil 25 cm. square, having 50 turns, rotates at a speed of 600 r.p.m. in a uniform magnetic field having a density of 250 lines per square centimeter. (*a*) What is the average e.m.f. induced in the coil? (*b*) If the flux and the speed are both doubled, what average e.m.f. is obtained?

394. (*a*) In Prob. 393, determine the e.m.f. per conductor when the coil is parallel to the direction of the field. (*b*) What is the induced e.m.f. per coil-side at this instant? (*c*) What is the total e.m.f. induced in the coil at this instant? (*d*) Show that the ratio of the maximum induced e.m.f. to the average e.m.f. under these conditions is $2/\pi$.

395. Fig. 395*A* shows a conductor 30 cm. long on the surface of an armature whose diameter is 25 cm. The armature rotates at a speed of 20 r.p.s. What e.m.f. is induced in the conductor when it is in the position shown, directly under the pole, where the flux density is uniform at 6,000 gausses?

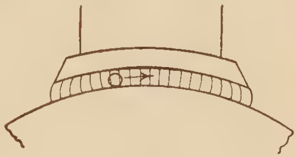


Fig. 395*A*.

396. A uniform magnetic field having an intensity of 1,000 gausses is just sufficient in cross-section to pass perpendicularly through a coil 40 in. × 18 in. The coil has 120 turns. If the coil slides perpendicularly out from this field in 0.001 second and at a uniform rate, what voltage is induced due to the change in flux linking the coil? What voltage is generated by the cutting of the flux by the individual conductors? (Work with the coil sliding in the two directions, one parallel to the 18-in. side and one parallel to the 40-in. side.)

397. An armature has 44 slots. Design a 2-layer, 4-pole, simplex lap winding, in which the back pitch is 21 and the front pitch is 19. Make a winding table.

398. Repeat Prob. 397 making the front pitch 21 and the back pitch 19. Which winding is progressive and which is retrogressive?

399. Design a 2-layer, simplex lap winding for a 6-pole, 63-slot machine, choosing the proper pitches.

400. An 8-pole armature has 106 slots and 6 winding elements per slot. Determine a correct value of back and front pitch for a simplex lap winding. Choose the value of back pitch which is nearest the value of the average pitch and yet fulfils the conditions which such a winding necessitates. Sketch a few slots with their winding elements and connections. How many commutator segments are necessary?

401. Repeat Prob. 400 for a winding with 8 elements per slot.

402. A 6-pole, simplex, lap-wound armature delivers a total current of 402 amp. at 230 volts. How many amperes per path through the armature? How many volts per path? What is the kilowatt rating of the machine?

403. Determine the current per parallel path in a 6-pole, 250-kw., 120-volt, electro-plating generator, which has a duplex lap winding. What is the current per brush set?

404. Sketch a duplex, doubly re-entrant, progressive, lap winding for a 4-pole, 64-slot armature, making the back pitch 31. There are two winding elements per slot.

405. Design a duplex, doubly re-entrant, progressive, lap winding for a 4-pole, 64-slot armature in which there are four winding elements per slot.

406. Repeat Prob. 405 for an armature having 63 slots, which gives a singly re-entrant winding.

407. A 4-pole armature has 37 slots. Sketch a simplex wave winding making $y_b = 19$ and $y_f = 17$. Check with Eq. (116), page 287. Is this winding progressive or retrogressive?

408. Repeat Prob. 407 making $y_b = 19$ and $y_f = 19$. Check with Eq. (116), page 287. Is this winding progressive or retrogressive?

409. A 6-pole armature has 63 slots and there are to be four winding elements per slot. Attempt to place a wave winding on this armature making $y_b = 43$ and $y_f = 41$. Omit a single coil (dummy coil) and investigate as to whether or not the winding is possible. Use Eq. (117), page 288.

410. If the generator (Prob. 409) is rated at 220 volts, 20 kw., how many amperes per path are there? What is the average e.m.f. between adjacent commutator segments?

411. Determine the amperes per path (Prob. 410) for a duplex wave winding.

412. The armature of an 8-pole, 200-kw. 230-volt generator is wound with a duplex wave winding. There are eight brush sets. (*a*) What is the current per armature path? (*b*) What is the current per brush holder?

QUESTIONS ON CHAPTER XI

1. What fundamental relationship gives the e.m.f. in a single armature conductor, while it is cutting the flux under the pole of a generator?

2. Show that in a simplex lap winding, the e.m.f. between brushes at any instant is equal to the sum of the e.m.fs. in all the conductors between any two commutating planes. Show from this that the e.m.f. between brushes is equal to the average e.m.f. induced in any single conductor during the time that it is passing between any two commutating planes, multiplied by the number of conductors between commutating planes. Why is the curve of e.m.f. and time for any one conductor identical in shape with the curve of flux density along the air-gap?

3. Derive the equation giving the e.m.f. in a generator in terms of flux per pole, speed, number of armature conductors, number of poles, and number of paths through the armature.

4. A certain armature has a fixed number of conductors on its surface. What are the separate effects on the induced voltage of; (*a*) doubling the speed of the armature; (*b*) doubling the flux entering the armature; (*c*) reconnecting the armature so that the number of paths through the armature is doubled?

5. In a given generator, upon what two factors does the induced voltage depend? If the speed of the generator be maintained constant, upon what one factor does the induced voltage depend?

6. Show that a similarity should exist between two curves plotted as follows:

(*a*) The field ampere-turns of a generator as abscissas and the flux leaving one of its north poles as ordinates.

(*b*) The field current of the same generator as abscissas and the induced armature voltage at constant speed as ordinates.

7. In the curve relating ampere-turns of the field and the flux of one north pole, why does not the flux start at zero value? Why is the first part of the curve practically a straight line? At the higher values of field current why does the induced voltage increase less and less rapidly for any given increase in field current?

8. Discuss any difference that may exist between the saturation curve obtained with *increasing* values of field current and that obtained with *decreasing* values.

9. Sketch the connections used in determining a saturation curve. (*a*) Using a simple field rheostat. (*b*) Using a drop wire with the field. Give two reasons why the generator should be separately excited.

10. Show that Ohm's law can be expressed graphically. What two quantities are plotted when expressing Ohm's law in this manner?

11. Sketch the connections of a shunt generator. Is the field of comparatively low resistance or of high resistance? Explain.

12. Explain in detail how a shunt generator "builds up." What limits the voltage to which a machine can build up?

13. What is meant by critical field resistance? Give three causes, each of which may prevent the generator building up. What tests and remedies should be used for each?

14. What is the general direction of the flux produced by the current in the armature conductors? What effect does this have upon the resultant flux in a machine? How does it affect the position of the neutral plane? What effect does the change in position of the neutral plane have upon the brush position?

15. What is the relation of the direction of the armature field to the brush axis? When the brushes are moved forward in a generator what is the resulting direction of the armature field? Into what two components can this be resolved? What is the effect of each component upon the resultant flux?

16. Which conductors on an armature produce a demagnetizing effect? Which produce a cross-magnetizing effect?

17. Sketch the conductors on the armature, together with the poles, for a loaded multipolar machine, indicating the current directions in the various conductors. Sketch a curve showing the values of armature magnetomotive force along the armature surface. Show the flux produced by this magnetomotive force when acting alone.

18. Show the effect of the above flux on the distribution of the total flux along the armature surface. How is the neutral zone affected? What change must be made in the brush position?

19. Name four methods by which armature reaction is either practically eliminated or reduced. State the principle of each method.

20. Sketch an ideal commutation curve assuming uniform current distribution over the brush.

21. What is the effect of having voltages induced in a coil during the time that it is being short-circuited by the brush? What limits the current in such a coil? How do these currents affect the uniform distribution of current over the brush?

22. Sketch commutation curves for the following conditions: (*a*) Brush advanced too far; (*b*) brush too far back; (*c*) brush too wide.

23. Why does an armature coil have self-inductance? What is the effect of this self-inductance during the commutation period? What effect does the voltage of self-induction have upon the relation of the brush position to the neutral zone?

24. What is the order of magnitude of the voltages induced in a coil undergoing commutation? If the voltages are low what makes them so objectionable?

25. What is the advantage of copper brushes over carbon? Why are carbon brushes used almost universally?

26. What evidence points to the fact that the taking of current from the commutator by the brushes is not pure conduction? To what is "high mica" due? How may it be reduced or even eliminated? Name two methods.

27. In general, what is the effect of arcing on the commutator? Why should any appearance of arcing be a reason for eliminating the cause of the arcing as soon as possible? Why is it not desirable to use emery paper or cloth in grinding brushes or smoothing the commutator?

28. What changes occur in the flux at the geometrical neutral of a generator as load is applied? What is the effect of these changes upon the brush position? Why is it necessary to move the brushes ahead of the load neutral plane?

29. Show that instead of moving the brushes forward in order to obtain the proper commutating flux, the same result may be obtained by the use of commutating poles.

30. Why are the commutating poles connected in series with the armature? Why have they an unusually long air-gap?

31. What is the relation of the polarities of the main poles and of the commutating poles to the direction of rotation, in a generator? In practice how are the commutating poles adjusted to the proper strength?

32. Sketch the connections used in obtaining the shunt characteristic. Sketch the characteristic. Why does the machine finally "break down?" Why does the return curve from short-circuit not follow the curve obtained with decreasing values of load resistance?

33. Give three reasons why the voltage of a shunt generator drops as load is applied. Why are these three reactions cumulative? What prevents a generator from "unbuilding" as load is applied?

34. What effect does running a generator at higher than rated speed have upon its characteristic, provided that the field current is so adjusted that the no-load volts are the same in each case?

35. What is meant by generator regulation? Does a large value of the regulation indicate that a generator is a desirable one for supplying lamp loads? Explain.

36. What is meant by the "total characteristic" of a generator? What is its relation to the shunt characteristic? How may the total power developed within an armature be determined?

37. Determine graphically the shunt characteristic of a generator, given the saturation curve, the field-resistance, the armature resistance, and the armature reaction: (*a*) neglecting armature reaction; (*b*) taking the demagnetizing effect of armature reaction into consideration. Show the effects of speed and degree of saturation on the shunt characteristic.

38. How may the objectionable drooping characteristic of the shunt generator be improved? How are the additional turns connected and in what way do they differ from the shunt-field turns?

39. Show the difference between "long shunt" and "short shunt" connection. What is the effect of the connection upon the characteristic? Sketch the characteristics of an over-compounded, a flat-compounded and an under-compounded generator. Where is each used and why?

40. How is the degree of compounding in a generator adjusted? When do generators have two separate series fields?

41. What is the effect of speed upon the degree of compounding, if the no-load voltage is the same in each case? Compare this with the effect of speed upon the shunt characteristic and explain.

42. Determine graphically the compound characteristic of a generator, given the saturation curve, the shunt, series-field and armature resistances, the armature reaction and the shunt- and series-field turns.

43. Show how the number of series turns for a desired degree of compounding may be determined experimentally. What is the armature characteristic and how may it be utilized?

44. How does the series generator differ fundamentally from the shunt generator in construction? In the type of load that it supplies?

45. Describe the external characteristic of the series generator and show its relation to the saturation curve.

46. In what way does the machine "build up?" What is meant by the critical external resistance? Why is it desirable to operate upon the right-hand side of the external characteristic?

47. Name a very common use of the series generator. Name two common types of machines. Why are special commutators necessary?

48. What is the "Thury system" of power transmission? Where is it used?

49. How may series generators be used to control the voltage at the end of a feeder? Upon what portion of the characteristic does such a generator operate? Sketch the connections. What precautions must be taken in the installation and operation of such a booster?

50. How may the speed of a prime mover affect the generator characteristic? Is such a drop in speed chargeable to the generator? How may it be taken into consideration?

51. State one essential difference between a unipolar generator and the ordinary type. What design is necessary to prevent the armature being short-circuited on itself? What is the advantage of this type of machine over the ordinary type and for what type of work is it best adapted? What are its disadvantages?

52. What is the basic principle of the Tirrill regulator? What is the function of the main control magnet? Of the relay magnet? Why cannot this regulator be applied directly to the fields of machines of large capacity? How may it be applied to these machines?

53. Draw the wiring diagram and describe the operation of the counter-e.m.f. voltage regulator. Analyze the reactions which follow a rise in bus-bar voltage; a drop in bus-bar voltage.

PROBLEMS ON CHAPTER XI

413. Fig. 413*A* shows the flux leaving a north pole and entering the armature of a 4-pole generator. Directly under the pole the flux density is 7,000 gausses, as shown by the graph. The diameter of the armature is 30 cm. and the speed is 1,600 r.p.m. The axial length of active conductor is 15 cm. Determine: (*a*) the peripheral velocity in cm. per sec. of a conductor *a* on the surface of the armature; (*b*) the e.m.f. induced in this conductor when it is directly under the pole where the flux density is constant at 7,000 gausses.

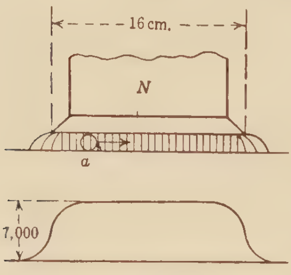


Fig. 413*A*.

414. In Prob. 413 the pole arc is 16 cm. The *average* value of the flux density directly under the pole and included in a distance equal to the pole arc is now 7,200 gausses. (*a*) What is the average value of the e.m.f. induced in the conductor during the time that it is directly under the pole? (*b*) What is the average value of the e.m.f. induced in the conductor during the time that it is passing between two brushes? (*c*) If there are 132 such conductors in one path between any pair of brushes, what is the induced e.m.f. between brushes?

415. The pole-faces of a 4-pole, 220-volt, 12-kw. generator have an axial length of 4.25 in. and the pole arc is 6 in. The armature has a diameter of 10.5 in. and the speed is 1,200 r.p.m. The flux density directly under the pole-faces is 40,000 lines per square inch and the flux density curve may be considered as a rectangle having a length equal to the pole arc. Determine: (*a*) the e.m.f. induced in a single conductor on the surface of the arma-

ture when it is directly under a pole-face; (*b*) the average e.m.f. induced in the conductor during the time that it is passing between any two brushes; (*c*) the induced e.m.f. between brushes. There are 47 slots on the armature and 12 conductors per slot. The armature is lap wound, giving the same number of parallel paths as there are poles.

416. The pole-faces of a shunt generator are 8 in. square and the average flux density *under the poles* is 38,000 lines per square inch. The machine has 4 poles and there are 336 surface conductors on the armature. The machine is wave wound, giving two parallel paths through the armature. What is the induced voltage when the armature rotates at 800 r.p.m.? 1,000 r.p.m.?

417. If the current per path in Prob. 416 is 25 amp., what is the rating of the machine in kilowatts?

418. Repeat Probs. 416 and 417 for a simplex lap winding, the number of conductors, the speed, etc. remaining the same.

419. In an 8-pole, 220-volt generator, the pole-faces are 12 in. square. The average flux density under the poles at no load is 47,500 lines per square inch. There are 16 slots per pole. The speed of the machine is 750 r.p.m. If the armature is lap wound, how many conductors per slot are necessary to give the rated voltage at no load?

420. The following data are given for the saturation curve of a 60-kw., 220-volt, 720-r.p.m., d.-c. shunt generator, the data being taken at 720 r.p.m. and for increasing values of field current:

Field current......	0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	10	12
Electromotive force	10	80	118	140	160	178	190	204	224	240

Plot the saturation curve for speeds of 720 r.p.m. and 650 r.p.m.

421. The generator (Prob. 420) has six poles and 280 shunt-field turns per pole. The armature is simplex lap wound and there are 65 slots and 8 conductors per slot. Plot a curve with the field ampere-turns as abscissas and flux per pole as ordinates. What is the flux per pole when the machine op erates at 720 r.p.m. and is generating 220 volts?

422. Determine the critical field resistance (Prob. 420) at both speeds, 720 and 650 r.p.m. To what value must the field resistance be adjusted to give 220 volts at 720 r.p.m.?

423. When the generator of Prob. 420 has been brought up to a speed of 720 r.p.m. and the adjustment of the field resistance is such that the machine builds up to 220 volts, what initial current flows through the field due to the residual magnetism? What induced e.m.f. results from this field current? What field current results from this last e.m.f.? Trace the successive increments of field current and e.m.f. which follow each other until the machine reaches the condition of stability.

424. A generator fails to build up. When the shunt field is connected across the armature, a voltmeter across the armature shows 4 volts. When this field circuit is opened the voltmeter reads 7 volts. What is the probable reason that the machine does not build up and what remedy is suggested?

425. The no-load flux per pole of a bipolar generator is 2,500,000 lines. When the generator is carrying its rated load and the brushes are in the

no-load neutral plane, the armature itself produces a flux of 800,000 lines. Neglecting the effect of saturation, determine the *resultant* flux.

426. Repeat Prob. 425 when the brushes are advanced 30 deg.

427. There are 240 conductors on the surface of the armature of a bipolar generator. The generator delivers 60 amp., giving 30 amp. in each conductor. If the brushes are advanced 15 deg. how many demagnetizing and cross-magnetizing ampere-conductors are there? How many demagnetizing and cross-magnetizing ampere-turns are there?

428. The brushes of a 4-pole generator are advanced 12 space degrees. The armature is simplex lap wound and has 456 surface conductors. (*a*) How many demagnetizing and cross-magnetizing ampere-*turns* are there on the armature when the generator delivers 160 amp.? (*b*) There are 400 shunt turns per pole and the shunt-field current is 4.8 amp. Draw the m.m.f. vector diagram (see Fig. 265 p. 319). Determine the result graphically.

429. Repeat Prob. 428 for a generator having the same number of poles and armature conductors and delivering the same current, but with a simplex wave-wound armature. There are 600 turns on each pole and the field current is 5.4 amp. What is the ratio of the kilowatt capacities of the two machines?

430. The armature of a 20-kw., 220-volt generator has a simplex lap winding and there are 8 conductors per coil, making 16 conductors per slot. The commutator is 7 in. in diameter and the peripheral width of each brush is $\frac{5}{8}$ in. The speed is 1,500 r.p.m. When the generator is carrying its rated load, 12,800 magnetic lines link the conductors of each slot (see Fig. 277, p. 329). Determine: (*a*) the peripheral velocity of the commutator in inches per second; (*b*) the time in which the current in any one coil is reversed from its full positive to its full negative value; (*c*) the total flux linking each coil before commutation begins; (*d*) the total change of flux during the commutation period; (*e*) the e.m.f. induced in a single coil during the commutation period, assuming straight-line commutation.

431. Repeat Prob. 430 with the speed reduced to 1,200 r.p.m. and the current to 0.75 of its value in Prob. 430.

432. A commutating-pole circuit has a resistance of 0.02 ohm. The rated full-load current of the generator is 160 amp. The most satisfactory condition of commutation is obtained when 125 amp. flow in the commutating-pole circuit. (*a*) What must be the resistance of a shunt to be connected across the commutating-pole circuit? (*b*) What is the voltage drop in the commutating-pole circuit and in the shunt at this load? (*c*) What power is lost in each?

433. The terminal voltage of a shunt generator is 600 volts when the armature delivers 225 amp. If the armature resistance is 0.12 ohm, what voltage is being induced in the armature?

434. An 80-kw., 230-volt, shunt generator has 238 volts induced in its armature when it is delivering its rated load at 230 volts. At the same time 12 amp. are taken by the shunt field. (*a*) What is the armature resistance? (*b*) The no-load voltage of the generator is 244 volts. What is the regula-

tion? (*c*) Why is the induced voltage at rated load not equal to the no-load volts?

435. The induced e.m.f. in the armature of a shunt generator is 628 volts when the load current is 125 amp. and the field current is 3.4 amp. The armature resistance is 0.27 ohm. (*a*) What is the terminal voltage of the machine? (*b*) How much power is being generated? (*c*) What is the power output? (*d*) What is the *electrical* efficiency of the machine?

436. The data for the saturation curve of a 60-kw., 4-pole, 220-volt, compound generator is taken at 720 r.p.m. with the generator operating as a simple shunt machine with decreasing values of field current as follows (see Prob. 420):

Field current......	11.4	10	9	7	6	4	3	2	1	0
Electromotive force	235	226	218	196	183	148	124	92	50	10

The armature resistance of the machine is 0.033 ohm. The field rheostat is adjusted to give 235 volts at no load. (*a*) Neglecting armature reaction, determine and plot a characteristic with armature current as abscissas and terminal volts as ordinates (see Par. 227). (*b*) From (*a*) also plot load current as abscissas and terminal voltage as ordinates. (*c*) What is the terminal voltage when the load current is equal to the rated value?

437. The machine (Prob. 436) has 280 shunt turns per pole. There are 65 slots on the armature, 8 conductors per slot, and the armature is simplex lap wound (see Problem 421). The brushes are moved ahead 4 space degrees. (*a*) Determine the demagnetizing ampere-turns per pole when the machine delivers rated current. (*b*) Using the data of (*a*), determine the characteristic with armature current as abscissas and terminal voltage as ordinates. (*c*) Plot a curve giving load current as abscissas and terminal voltage as ordinates.

438. A 400-kw., 600-volt, 600-r.p.m., compound generator has the following constants: armature resistance 0.03 ohm; series-field resistance 0.012 ohm; shunt-field resistance 48 ohms. The machine is connected long shunt. When the machine is delivering its rated load at rated terminal voltage, determine; (*a*) the power lost in the shunt field; (*b*) the power lost in the series field; (*c*) the power lost in the armature; (*d*) the total power loss; (*e*) the total power generated; (*f*) the electrical efficiency of the machine (output divided by (*e*)).

439. Repeat Prob. 438 with the load current at one-half its rated value and the terminal voltage 580 volts.

440. The following are the constants of a 15-kw., 115-volt, 1,200-r.p.m., compound generator: armature resistance 0.078 ohm; series-field resistance 0.03 ohm; series-field diverter resistance 0.048 ohm; shunt-field resistance 28 ohms. The generator is connected short shunt. With the machine delivering its rated current at rated voltage, determine the power loss in the following: (*a*) the armature; (*b*) the series field; (*c*) the diverter; (*d*) the shunt field. Determine: (*e*) the induced e.m.f.; (*f*) the total power generated within the armature.

441. Repeat Prob. 440 with the generator delivering one-half its rated load at 115 volts.

442. A 250-kw., 550 to 600-volt, compound railway generator is so compounded that it maintains the voltage at a point 3 miles away constant at 550 volts at all loads. The overhead system consists of a 4/0 hard-drawn trolley wire, having a resistance of 0.27 ohm per mile, paralleled by two 500,000-C.M. feeders in parallel, having a resistance of 0.118 ohm per mile each. The resistance of the track return is 0.03 ohm per mile. (*a*) What should be the rated-load voltage of this generator? (*b*) What is the efficiency of transmission, if it be assumed that the generator is delivering its entire rated load at this point?

443. A compound generator has a no-load voltage of 230 volts. It supplies a 400-kw. load, situated 800 ft. distant, over a 1,500,000-C.M. cable. It is desired to maintain the voltage at the load constant at 230 volts from no load to full load of 400 kw. What must be the no-load and the full-load voltage rating of the generator? Assume that a C.-M.-ft. has a resistance of 11 ohms at the operating temperature of the cable.

444. Repeat Prob. 443 for the condition in which it is desired that the voltage at the load rise from 230 to 240 volts from no load to full load.

445. The generator of Prob. 443 is connected long shunt. Its armature resistance is 0.004 ohm, the shunt-field resistance is 7.6 ohms, the series field resistance is 0.0015 ohm and the diverter resistance is 0.0009 ohm. (*a*) What is the voltage induced in the armature with the 400-kw. load of Prob. 443? (*b*) How much power is lost in the armature, in the shunt field, in the series field, in the diverter and in the cable? Of the total power generated, what percentage reaches the load?

446. Repeat Prob. 445 for the 400-kw. load of Prob. 444. It is now necessary to remove the series-field diverter.

447. The generator of Probs. 436 and 437 has 6½ series turns per pole. The entire resistance of the series field is 0.010 ohm. The generator is connected long shunt. (*a*) From the saturation curve taken with increasing values of field current (see Prob. 420), determine the current in the series field which is necessary to make the machine flat compounded at rated load. (*b*) What should be the resistance of the series-field diverter? (See Par. 230.)

448. Using the method described in Par. 230, determine the terminal voltage of the generator in Prob. 447 when the armature current is equal to the rated-load current and the diverter is removed. The no-load voltage of the generator is 220 volts.

449. It is desired to add series turns to a shunt generator, so that its rated-load voltage is the same as the no-load voltage. There are 320 shunt turns per pole. It is found necessary to increase the shunt-field current from 8.0 to 11.52 amp. in order to keep the rated-load voltage equal to the no-load voltage. The rated current of the machine is 250 amp. (*a*) How many series turns per pole should be added? (*b*) If 7½ turns are added, what should be the ratio of diverter to series-field resistance?

450. It is desired that the voltage of a 500-kw., 600-volt, compound generator increase from 550 volts at no load to 600 volts at rated load. With the series field out of circuit and the shunt field excited from an external source, it is found that this increase of voltage may be obtained by increasing the shunt-field current from 10.5 to 18.0 amp. There are 220 shunt turns per pole and 6½ series turns per pole. The total series field resistance is 0.015 ohm. What must be the resistance of a shunt or diverter to be connected across the series field?

451. A series generator supplies power to forty 6.6-amp., 500-watt, series magnetite arc lamps. The armature resistance is 20 ohms and the field resistance 25 ohms. The line resistance is 10 ohms. (*a*) What is the terminal voltage of the generator? (*b*) What is the voltage induced in the armature?

452. A 150-kw. load is situated 2,000 ft. distant from the 230-volt bus-bars of a station. The load is supplied over a 750,000-C.M. feeder. It is desired that when the load is 150 kw. the load voltage shall be not less than 225 volts. What must be the current and voltage rating of a series booster designed to maintain this voltage at the above value? Assume that a circular-mil-foot has a resistance of 11 ohms at the operating temperature of the cable.

453. The booster in Prob. 452 has an efficiency of 80 per cent. It is driven by a shunt motor connected across the bus-bars, the motor efficiency being 80 per cent. What is the over-all efficiency of the feeder?

QUESTIONS ON CHAPTER XII

1. In what way does a motor differ from a generator in the work which it performs? In general construction?

2. What effect is noted when a conductor carrying a current is placed in a magnetic field? How can this action be explained by two elementary laws of magnetism? What is the effect of reversing the current in the conductor?

3. To what three factors is this force proportional? If the flux is doubled how is the force affected? If the current is doubled?

4. State a convenient rule by which the relation among the direction of the current, the direction of the field and the direction of the force can be determined. What other simple method enables one to determine this relation?

5. What is torque? In what units is it expressed? In the British system? In the metric system?

6. Show that a coil carrying current when placed in a magnetic field may develop a torque. In what position of the coil is the torque a maximum? When is it zero? If continuous rotation is desired what change in the connection to the armature coil should be made when the torque reaches its zero value?

7. Why are a large number of conductors upon the armature desirable? To what three factors is the torque of an armature proportional? In any one machine, to what two factors is the torque proportional?

8. How can it be shown that resistance alone does not determine the amount of current taken by a motor armature? Why must a motor of necessity be generating a voltage when it is rotating? What is the relation of this voltage to the direction of the current? To the direction of the applied voltage?

9. Is the counter electromotive force greater or less than the applied voltage? Why? By what quantity do the two voltages differ from each other?

10. Show that the mechanical power developed by a motor armature is equal to the product of the current and the counter e.m.f.

11. Fundamentally, upon what two quantities does the speed of a motor depend?

12. In what direction is the flux of a motor distorted by armature reaction? In what direction should the brushes be moved as the load is applied to a motor? What general effect on the field flux does this movement of the brushes have? What is the effect upon the speed?

13. What is the relation among the main poles, the interpoles and the direction of rotation of a motor? How does this relation compare with the similar one for a generator?

14. When load is applied to a motor what is its first tendency? In the case of the shunt motor, how does this tendency affect the back electromotive force? The current flowing into the armature?

15. What two characteristics are very important in considering the suitability of a motor for commercial work?

16. How does the torque of the shunt motor vary with the load? Why? How does the speed vary with the load? Demonstrate. Ordinarily is its change of speed with load excessive? What effect does armature reaction have upon the speed? What is meant by "speed regulation?" Does the per cent. speed regulation have any significance as regards a motor's performance? To what general type of work is a shunt motor adapted and why?

17. How does the flux in a series motor vary with the load? How does this affect the variation of torque with load? To what extent is the speed of a series motor affected by the application of load? By the removal of load? What precautions should be taken when the series motor is being installed for industrial purposes?

18. To what general types of load is a series motor adapted and why? For what reasons is it especially adapted to street-railway work?

19. What factors are plotted in giving the characteristics of a street car motor? Why?

20. In what way do the windings of a compound motor differ from those of a shunt motor? A series motor? In what two ways, with respect to the shunt winding, may the series winding be connected?

21. What is the speed characteristic of the cumulative compound motor? The torque characteristic? What advantage has it over the series motor? For what general type of work is it best adapted?

22. What is the nature of the speed and torque characteristics of the differentially-compounded motor? Is this type of motor in general use? Explain. What precaution is necessary in starting this type of motor?

23. How may a motor be reversed? What is the effect of reversing the line terminals?

24. Why is a starting rheostat necessary for direct-current motors? In what circuit is the starting resistance connected? Why should it not be connected in the line?

25. What two additions to the starting resistance of Fig. 328, page 387, are incorporated in a 3-point starting box? Why? Sketch the connections of a 3-point box. Show that the starting resistance which is in series with the shunt field when the arm is in the running position has little effect upon the field current.

26. Under what conditions of motor operation is a 3-point box undesirable? Why? Show that this objection is overcome by the use of a 4-point box. Sketch the connections of a 4-point box. What is the principal advantage of having the hold-up magnet in series with the shunt field?

27. Sketch the connections of a starting box containing the field resistance. Why is it necessary to short-circuit this resistance on starting? How is this accomplished?

28. How should a shunt motor be stopped? Give reasons. What is the effect of stopping the motor by throwing back the starting arm?

29. Sketch the connections of series-motor starters. What is the advantage of the no-load release over the no-voltage release?

30. When are controllers used and why? What two functions may a controller perform outside actual starting duty?

31. What are two advantages of automatic starters in medium sizes of motors? In the larger sizes of motors? What limits the rate of cutting out resistance in the type shown in Fig. 334, page 392?

32. Upon what principle do the plungers and solenoids of the E.C. & M. controller operate? Why do the plungers remain down when the current is large? Why do they rise and close the contacts when the current decreases?

33. What is the principle of the magnetic blow-out? When is it used?

34. Of what material are resistance units for the smaller types of starting boxes made? The larger types?

35. What two factors only can be varied in obtaining speed control of a motor? In the armature resistance control method, which of these factors is varied? What are the advantages of this method of control? Name two serious disadvantages.

36. What is the principle of the multivoltage system? How are coarse adjustments of speed obtained? Fine adjustments? What is the objection to this system?

37. What factor in the speed equation is varied in the Ward-Leonard system of speed control? How many machines are necessary in this system? What is its chief advantage and where has it been used extensively? Name two disadvantages.

38. What factor in the speed equation is varied in the field-control method? Name two distinct advantages of this method. What limits the range of speed obtainable? What type of motor is especially adapted to this type of speed control?

39. What principle is involved in the speed control of the Stow motor? Why can a wide range of speed, with good commutation, be obtained with this motor?

40. Upon what principle does the Lincoln motor operate? What are its advantages?

41. What is meant by series-parallel control of railway motors? Why is such control desirable? Sketch the half speed and the maximum speed connections in a 2-motor car. In a 4-motor car.

42. Give three reasons why it is objectionable to place the main controller on the platform in the larger sizes of electric-car equipment. How are these objections overcome? Give two other reasons why automatic control is desirable.

43. What is the general principle underlying the multiple-unit control? What is the train line?

44. Name briefly the sequence of closing of the contactors in starting a train.

45. What is meant by "dynamic braking?" Where is it used? Can a motor armature be brought to a standstill by this method of braking? Explain. What is regenerative braking and where is it used?

46. Under what circumstances is it desirable to know the efficiency of a motor? What type of brake is often used for loading motors? Does this type lend itself to ready calculation of torque and power output of the motor? Explain. What is meant by the dead weight of the brake arm and how can it be determined and correction be made?

47. Describe a simple type of rope brake. How many balances are necessary in this type? What is a common method of cooling prony brakes?

48. In what way does a speed counter differ from a tachometer? Upon what principle is the magneto-voltmeter method of measuring speed based?

49. Define a dynamotor. What factors determine the voltage ratio between the two commutators? Discuss the effect on the voltage ratio of varying the field excitation.

PROBLEMS ON CHAPTER XII

454. The flux density directly under a north pole of a motor is 6,000 gausses. The active length of the armature conductors is 15 cm.; each conductor carries a current of 12 amp. (*a*) What is the force in dynes acting on each conductor when under the poles? (*b*) In grams? (*c*) In kilograms?

455. A flat coil 20 cm. square and having 16 turns is so placed in a uniform magnetic field that the plane of the coil is parallel to the field and two oppo-

site sides of the coil have the same direction as the field (see Fig. 455*A*). The field intensity is 600 gausses and is uniform. The current is 8.0 amp. (*a*) What force in kilograms acts on each side of the coil? (*b*) What is the turning couple in kilogram-meters acting on the coil when it is in this position? (*c*) In pound-feet?

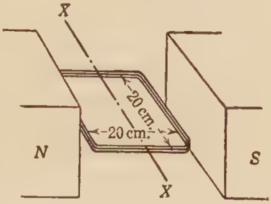


Fig. 455*A*.

456. Determine the turning couple in pound-feet on the coil of Prob. 455 when it is turned an angle of 45 deg. about its axis *xx*.

457. A gear having 120 teeth drives another having 50 teeth. The distance from the center of the first gear to the point of contact of the teeth is 6.0 in., the pitch circle having a diameter of 12 in. The pressure between the teeth at the point of contact is 400 lb. What is the torque in pound-feet developed by each of the gears?

458. A pulley having a diameter of 16 in. drives a 40-in. pulley with a 6-in. belt. The respective tension in the tight and loose sides of the belt are 1,400 and 350 lb. respectively. What net torque in pound-feet is developed by each pulley?

459. The motor (Prob. 454) has four poles, and the diameter of the armature is 28 cm. There are 34 slots on the armature and 12 conductors per slot. There are 72 of these conductors under each pole at any one instant. The flux density under each pole may be considered as uniform and fringing may be neglected. Determine: (*a*) the force in kilograms developed under each pole; (*b*) the total force developed by the armature conductors; (*c*) the torque in kilogram-meters developed by the armature; (*d*) the torque in pound-feet.

460. Repeat Prob. 459 with 15 amp. per conductor and the flux density reduced to 5,000 gausses.

461. A coil consisting of 16 turns of wire lies parallel to a magnetic field having a strength of 30,000 lines per square inch (see Fig. 315 (*a*), p. 369). The distance across this coil parallel to the field is 14 in., and 16 in. of active conductor lie in the magnetic field. What torque in kilogram-meters is developed by the coil when the current per conductor is 6 amp.? Sketch the coil and the magnetic field, indicating the directions of the forces acting.

462. Repeat Prob. 461 with the coil turned 90 deg. in its own plane so that the 14-in. side is now perpendicular to the field and the 16-in. side is parallel to the field.

463. When the flux density in the air-gap of a shunt motor is 40,000 lines per square inch and the armature current is 70 amp., the motor develops 90 lb.-ft. torque. What is the torque developed when the armature takes 40 amp., the flux remaining constant? 50 amp.?

464. When the load is entirely removed from the armature of Prob. 463, the motor armature requires 8 amp. to keep it running. What torque is required to overcome the motor losses? What is the torque available at the pulley in each case of Prob. 463, assuming that the no-load torque remains constant?

465. Determine the torque developed in the motor (Prob. 463) when the flux density is 50,000 lines per square inch and the current is 75 amp.

466. The armature of a shunt motor has a resistance of 0.044 ohm. When this motor is connected across 115-volt mains, it develops a counter electromotive force of 111 volts. What current does the armature take? What current would it take if it were connected across the same mains while stationary?

467. What counter electromotive force does this motor armature develop when it is taking 75 amp. from the mains? If this same machine were running as a generator what would be its internal electromotive force when the armature is delivering 75 amp. at 115 volts?

468. A shunt machine having an armature resistance of 0.12 ohm is connected across the 230-volt bus-bars. Determine its induced e.m.f. when the armature takes 60 amp., operating both as a motor and as a generator.

469. The armature of a 4-pole shunt motor has 450 surface conductors and is wave wound. What back electromotive force does it develop when rotating at 1,200 r.p.m.? The flux is 2,700,000 lines per pole. Its armature resistance is 0.2 ohm. What is its terminal voltage when the motor takes 50 amp. from the line if the speed and the flux remain constant?

470. What current does the armature of the motor in Prob. 469 take from the line when its speed drops to 1,170 r.p.m. if the terminal voltage and flux remain constant?

471. The armature of a 15-hp., 4-pole, 115-volt motor has 45 slots, 8 conductors per slot; the winding is simplex wave and the armature resistance, including brushes, is 0.078 ohm. The poles have an axial length of 4.25 in. and the pole arc is 6 in. Determine the counter e.m.f. of this armature when the speed is 1,200 r.p.m. and the average flux density under the pole is 30,500 lines per square inch.

472. The armature of a 15-hp., 220-volt shunt motor has a resistance of 0.27 ohm. When running without load, the motor takes 6.4 amp., the field current is 2.2 amp., and the speed is 980 r.p.m. (*d*) What is the speed when the motor takes 72 amp. from the line, the field current remaining unchanged? Neglect armature reaction. (*b*) If the rated current of the motor is 60 amp., what is its speed regulation?

473. (*a*) Determine the speed of the motor (Prob. 471) when the armature takes 80 amp. and due to armature reaction the average flux density under a pole becomes 29,000 lines per square inch. (*b*) What is the speed regulation of the motor under these conditions? (*c*) Find the ratio of the torque which the motor develops to that developed under the conditions of Prob. 471.

474. A compound winding on the motor of Prob. 472 is connected long shunt and it aids the shunt field. Its resistance is 0.05 ohm. When the motor is taking its rated current at 220 volts, this compound winding increases the flux per pole 20 per cent. Assume that the increase in flux is proportional to the armature current and neglect armature reaction. (*a*) Find the speed when the motor takes 6.4 amp. and when it takes 72 amp. The shunt-field current remains unchanged. (*b*) Compare the torques at

72 amp. with and without the series field. At no load the flux density in Prob. 472 is 31,500 lines per square inch.

475. If it is desired that the motor (Probs. 472 and 474) have the same speed at no load and when its armature takes 75 amp., what must be the flux density under the poles when the current is 75 amp.?

476. Determine the mechanical power in kilowatts and horsepower developed in the armature of Prob. 472 when the motor takes 72 amp. What is the internal torque under these conditions?

477. A 50-hp., 230-volt, 800-r.p.m. motor is taking 170 amp. at 230 volts. The field current is 4.8 amp. and the armature resistance is 0.044 ohm. (*a*) What internal power in kilowatts and horsepower is it developing? (*b*) If the motor speed is 800 r.p.m. when it takes 12 amp., what is its internal torque when it takes 170 amp.? Neglect armature reaction.

478. A 220-volt shunt motor has a field resistance of 80 ohms and an armature resistance of 0.12 ohm. (*a*) The total line current is 70 amp. What is the back electromotive force of the armature? (*b*) When the armature is stationary, the line current is adjusted to 50 amp. by means of a series resistance. Determine the armature current and the field current. (*c*) Compare the division of current in (*b*) with that in (*a*) and account for the difference.

479. A 550-volt series motor has a series-field resistance of 0.08 ohm and an armature resistance of 0.25 ohm. When taking 90 amp. from the line, its speed is 420 r.p.m. What is its speed when it takes 50 amp. from the line? Assume that the saturation curve is a straight line and neglect armature reaction.

480. Determine the speed of the motor (Prob. 479) when the current is 120 amp., the other conditions remaining unchanged.

481. A shunt motor takes 74 amp. at 230 volts and develops 112 lb.-ft. internal torque at a speed of 980 r.p.m. The armature resistance is 0.12 ohm and the field resistance is 104 ohms. What internal torque and speed does it develop when the motor current is 45 amp.? Neglect armature reaction.

482. What internal power does the motor (Prob. 481) develop under both conditions?

483. The rated current of a 10-hp., 230-volt, 1,350-r.p.m., shunt motor is 27.5 amp. The field current is 1.1 amp. At rated current and speed, the internal torque is 41.5 lb.-ft. (*a*) Determine the torque when the line current is 13.2 amp.; (*b*) when the line current is 34.0 amp.; (*c*) if the armature resistance is 0.41 ohm, find the speed in (*a*) and (*b*). Neglect armature reaction. (*d*) What is the speed regulation of this motor?

484. The field of the motor, Prob. 483, is reduced to 0.88 of the value in Prob. 483. The field current is now 0.9 amp. Find: (*a*) the torque and speed at rated current; (*b*) the torque and speed when the motor current is 15 amp. Neglect armature reaction.

485. A 230-volt, 5-hp., series motor develops 12 lb.-ft. torque when its current is 11 amp. Determine the torque when the current is 20 amp. Neglect armature reaction and assume a straight-line saturation curve.

486. When a street car, operating with two series motors in parallel, takes 120 amp., it develops a tractive effect of 1,600 lb. What tractive effort will be developed when the car is starting with the two motors in series and the car is taking 105 amp.? Neglect armature reaction and assume a straight-line saturation curve.

487. The armature of the motor (Prob. 485) has a resistance of 0.11 ohm and the field resistance is 0.04 ohm. The speed when the motor takes 20 amp. is 400 r.p.m. (*a*) What is the speed when the current is 11 amp.? (*b*) What internal horsepower does the motor develop when the current is 20 amp.? (*c*) When the current is 11 amp.?

488. A 20-hp., 230-volt, cumulative compound motor runs at 840 r.p.m. when running light and the line current is 4.40 amp. Under these conditions, the motor develops 7.7 lb.-ft. internal torque. The shunt-field resistance is 140 ohms; the series-field resistance is 0.05 ohm; and the armature resistance is 0.23 ohm. The motor is connected long shunt. The line current at rated-load is 55 amp. and the corresponding speed is 520 r.p.m. Assume that the increase in flux is proportional to the armature current. Determine: (*a*) the counter e.m.f. at rated load speed; (*b*) the ratio of rated to no-load flux; (*c*) the rated-load speed; (*d*) the torque at rated load.

489. The rated current of a 15-hp., 220-volt, shunt motor is 59 amp. The no-load speed of the motor is 1,000 r.p.m. The armature resistance is 0.27 ohm and the field resistance is 108 ohms. It is desired that the starting torque of the motor be equal to rated-load torque. What should be the total initial resistance of the starting box?

490. Determine the current taken by the armature of the motor, (Prob. 489) when it reaches 25 per cent. of its no-load speed, with the entire starting resistance still in circuit.

491. When the motor (Prob. 489) has reached 25 per cent. no-load speed, as in Prob. 490, it is desired that the armature current again be brought to its rated value by moving the starting arm to the second contact. How much resistance remains in the starting box?

492. The motor (Prob. 489) reaches one-half the no-load speed with the resistance determined in Prob. 491 still in circuit. (*a*) What current is the armature now taking? (*b*) If it is desired to bring the armature current back to its rated value by moving the starting arm to the next contact, how much resistance must be left in the starting box?

493. The resistance of the armature of a 50-hp., 550-volt, shunt motor is 0.35 ohm and the field resistance is 320 ohms. The rated current is 76 amp. It is desired that the motor develop 150 per cent. rated torque on starting. What should be the total resistance of the controller?

494. A 600-volt series motor has an armature resistance of 0.3 ohm and a field resistance of 0.16 ohm. The rated current is 70 amp. It is desired that the motor develop 200 per cent. rated torque on starting. What should be the total resistance of the starting controller? Neglect armature reaction and assume a straight-line saturation curve.

495. A 220-volt shunt motor has an armature resistance of 0.12 ohm. When the armature takes 4.4 amp. from the line it runs at 1,040 r.p.m. It

is desired to obtain 600 r.p.m. at 64 amp. by inserting resistance in the armature circuit. What external resistance is necessary? With this external resistance in circuit, at what speed will the motor run when the armature takes 22 amp.?

496. Repeat Prob. 495 for 300 r.p.m. What percentage of the line power is delivered to the armature at 64 amp. in Probs. 495 and 496?

497. A motor when connected across the 110-volt mains of Fig. 339, page 398 runs at 450 r.p.m. What speeds can be obtained by the use of this system if the shunt field is kept constant? Neglect the I_aR_a drop in the motor armature.

498. In a Ward-Leonard system of speed control the efficiencies of the machines are as follows: M_1 (Fig. 340, p. 399), 86 per cent.; G, 84 per cent.; M_2, 82.5 per cent. The line voltage is 230 volts. When M_2 delivers 12 hp. how much current is being supplied by the line? What is the over-all efficiency of the system?

499. In a brake similar to that shown in Fig. 347, page 406, the length L is 2 ft. The balance reading is 47.4 lb.; the dead weight of the arm is 2.8 lb.; the speed of the armature is 1,120 r.p.m. (*a*) What horsepower does this motor develop? (*b*) The motor input is 70.8 amp. at 230 volts. What is its efficiency at this load?

500. Repeat Prob. 499 for a balance reading of 36 lb. and a speed of 1,130 r.p.m. The motor input is now 54.5 amp. at 230 volts. The dead weight of the arm remains unchanged.

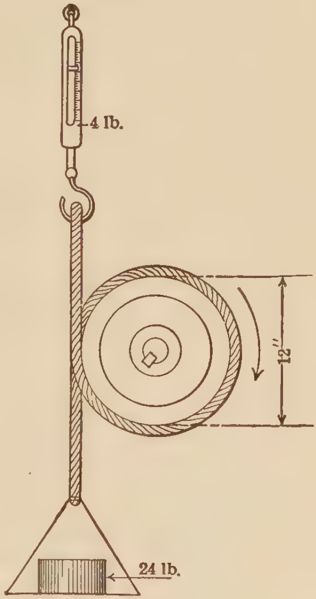


FIG. 502*A*.

501. In a brake similar to that shown in Fig. 349, page 409, the diameter of the drum is 10 in. The speed is 1,100 r.p.m. One balance reads 14.5 lb. and the other reads 4.8 lb. (*a*) What torque does the motor develop at this load? (*b*) What is the horsepower output? (*c*) If the input is 9.2 amp. at 115 volts, what is its efficiency at this load?

502. Calculate the horsepower output developed by the rope brake shown in Fig. 502*A*. The speed of the drum is 1,450 r.p.m.

QUESTIONS ON CHAPTER XIII

1. Discuss the disposition of the energy which is lost within electrical apparatus. Explain in detail its effect on the apparatus.

2. Into what three groups can the losses in either a motor or a generator be classified? Name the losses under the first group, indicating how they are determined. Are they readily determinable?

3. What constitutes the losses of the second group? How are these losses supplied, electrically or mechanically? Upon what do they depend? How is the eddy-current loss made small? What is meant by pole-face loss and to what is it due? How is it reduced?

4. Why are all the losses except the copper loss grouped as one? Upon what factors do they all depend? If it is desired to duplicate stray-power losses under two different conditions of load, what two factors must also be duplicated?

5. Show that if the losses in a machine are known at any particular load, its efficiency at that load can be calculated. Why is the formula for generator efficiency different from that for motor efficiency?

6. How may the efficiency of a generator be measured directly? What practical conditions make such measurements difficult? What effect do errors in the measurements have upon the precision of the results? What other objections are there to direct measurements of efficiency?

7. How is a machine ordinarily operated in order to measure its stray power? What measurements are made? To what is the stray power then equal?

8. In stray-power measurements, how is the flux adjusted to the proper value? How is the speed adjusted? Does the flux adjustment have any effect upon the speed and if so how are any readjustments made?

9. For what purpose is a set of stray-power curves desirable? Why cannot the stray power over the entire operating range of the machine be shown with one curve? What errors are introduced by using the field current as a measure of the flux and how is one of these errors partially neutralized?

10. For determining losses what is the advantage of the opposition method over the stray-power method? Upon what principle does this method depend?

11. What assumption is it necessary to make in the opposition method? Does this assumption introduce appreciable error? In this method how are the two machines started and then adjusted? What instruments are used and what measurements are necessary? State the disadvantages of this method.

12. What determines the rating of a steam engine? A steam turbine? A gas engine? An electric machine? Give reasons in each case.

13. State the effects of excessive temperature on the insulation of electric machinery. What insulating materials can withstand the highest temperatures?

14. What is the "hottest spot" temperature and what difficulties accompany its measurement? Name one method by which an approximation of the "hottest spot" temperature may be reached.

15. What other well-known principle is utilized to determine the temperature rise? What is meant by "ambient" temperature?

16. For what length of time should a temperature test be run? How may the temperature rise be accelerated? In what way may a machine's approach to constant temperature be determined? Why does the temperature of a machine rise more rapidly at the beginning of a test than at the

end? What relations exist between the heat supplied and the heat dissipated when a constant temperature is reached?

17. Why must care be taken not to include the brush and contact resistance when measuring the armature resistance for temperature determinations? Where must the voltmeter leads be held?

18. What difficulties arise when the resistances of multipolar armatures are measured for temperature determinations? How may these difficulties be eliminated? What precautions should be taken when the field temperature is being determined by resistance measurements? Why must the temperature measured in this way be still further increased?

19. Give five reasons why it is either necessary or desirable to operate shunt generators in parallel. What, in their characteristic, makes them especially adapted to parallel operation?

20. Analyze the reactions which follow when one generator starts to take more than its share of the load. What is meant by "stable equilibrium?"

21. State in detail the steps necessary to connect a machine in service. If the generator is connected in service with its voltage equal to that of the bus, why does it not take load? What must be done in order that it may take its share of the load?

22. Describe the steps necessary to remove a machine from service. Why is it undesirable to open the generator switch when the machine is delivering load? What is necessary as regards the generator characteristics in order that the machines may properly divide the load over their entire range of operation?

23. Show that over-compounded generators in parallel are in unstable equilibrium. What simple connection makes their operation stable?

24. What two conditions are necessary for two compound generators to divide the load proportionately over their entire range of operation?

25. Why does not a diverter change the division of load between two compound generators in parallel? What adjustment can be made to change the load division?

26. How many equalizers may be necessary in certain types of compound generators? How many poles must the switch have for such a generator?

27. Compare circuit breakers and fuses, stating the advantages and disadvantages of each. Which has the higher first cost? Which ordinarily has the lower maintenance cost? Which requires the more space? Which operates the faster? Under what conditions should each be used?

28. Upon what principle do circuit breakers in general operate? How is a "wiping contact" secured? What is the purpose of the carbon contacts? Why should breakers always be mounted at the top of a switchboard?

PROBLEMS ON CHAPTER XIII

503. The eddy-current loss in a generator is 360 watts when it is running at 900 r.p.m. and with a flux of 1,000,000 lines per pole. (*a*) What is the eddy-current loss when the speed is raised to 1,000 r.p.m., the flux remaining unchanged? (*b*) What is the loss at 900 r.p.m. with a flux of 1,200,000 lines? (*c*) With the same flux as in (*b*) what is the loss at 1,000 r.p.m.?

504. The hysteresis loss in the generator of Prob. 503 is 600 watts when the speed is 900 r.p.m. Find the hysteresis loss under the conditions of (*a*), (*b*) and (*c*).

505. When the induced e.m.f. in a 50-kw., 220-volt, shunt generator is 230 volts and the speed is 1,200 r.p.m., the hysteresis loss in the armature iron is 750 watts and the eddy-current loss is 450 watts. Determine the hysteresis loss under the following conditions: (*a*) Induced e.m.f. 250 volts, speed 1,200 r.p.m.; (*b*) induced e.m.f. 230 volts, speed 1,020 r.p.m.; (*c*) induced e.m.f. 240 volts, speed 1,020 r.p.m.

506. Find the eddy-current loss in Prob. 505, under the conditions of (*a*), (*b*) and (*c*).

507. A 50-kw., 230-volt, 900-r.p.m., shunt generator delivers 215 amperes at 225 volts and rated speed. The total losses are 5,270 watts. (*a*) What is the power input in horsepower? (*b*) What torque is required to drive the generator? (*c*) Determine its efficiency.

508. When the generator of Prob. 507 delivers its rated load at rated speed, the total losses are 5,500 watts. Repeat (*a*), (*b*), and (*c*).

509. The armature of a 15-hp., 220-volt, 1,200-r.p.m., shunt motor has a resistance of 0.22 ohm and the field resistance is 96 ohms. The stray power, when it delivers rated load at rated voltage, is 540 watts. Determine: (*a*) the input at rated load; (*b*) the efficiency; (*c*) the per cent. error in the efficiency caused by a 10 per cent. error in determining the stray power. (Hint: A simple quadratic equation involving the armature current I_a may be used to find I_a. A trial and error method is an alternative.)

510. When the motor (Prob. 509) takes 30 amp. from the line, the stray power is 560 watts. Find (*a*) and (*b*). (*c*) Determine the torque at the pulley.

511. The armature resistance of a 400-kw., 600-volt, 750-r.p.m., shunt generator is 0.028 ohm; the shunt-field resistance is 52 ohms; and the stray power at rated load and speed is 10.5 kw. Determine at rated load and speed: (*a*) the total losses; (*b*) the efficiency; (*c*) the torque necessary to drive the generator.

512. A 25-kw., 220-volt, 800-r.p.m., shunt generator is running light as a motor. Its armature takes 6.4 amp. from 220-volt mains. The armature resistance is 0.12 ohm. What is the stray power loss of the machine under these conditions?

513. A 35-hp., 115-volt, 1,140-r.p.m., shunt motor, running light, takes 27 amp. from 115-volt mains. Its shunt-field resistance is 14 ohms and its armature resistance is 0.018 ohm. What is the stray-power loss in this motor under these conditions?

514. It is desired to determine the stray power of the shunt generator in Prob. 507 under the conditions given in that problem. The generator is run light as a motor, the connections being those shown in Fig. 358, page 419. To what value should the voltage V_1 across the armature be adjusted and how is this adjustment made? What adjustment of the field is necessary?

515. The stray-power curves in Fig. 360, page 425, were taken for a 20-kw., 220-volt, 960-r.p.m., shunt generator, whose armature resistance is

0.096 ohm. When the machine delivers its rated output at rated voltage and speed, the field current is 2.24 amp. Determine: (*a*) the total losses; (*b*) the generator efficiency; (*c*) the losses in per cent. of input; (*d*) the driving torque which is applied to the shaft.

516. The generator (Prob. 515) runs as a motor with load and takes 75 amp. at 220 volts from the line. The speed is 1,050 r.p.m. and the field current is 1.85 amp. Determine: (*a*) the value of E/S; (*b*) the stray power; (*c*) the total losses; (*d*) the output of the motor in horsepower; (*e*) the efficiency; (*f*) the torque; (*g*) the losses in per cent. of input.

517. Two similar 10-kw., 230-volt generators are connected for the Kapp opposition test, shown in Fig. 361, page 426, for the purpose of having their losses measured. When the machine operating as a generator is delivering its rated armature current of 45.0 amp., the line current I is found to be 7.5 amp. The generator field current is 1.8 amp. and the motor field current is 1.2 amp. Each machine has an armature resistance of 0.2 ohm. What is the efficiency of each machine at this load?

518. When the current in the generator armature of Prob. 517 is 24 amp. (half load), the line current I is 5 amp. The generator field current is now 1.6 amp. and the motor field current is 1.0 amp. Determine the stray power and the efficiency of each machine at this load. Why are the field currents different in the two machines?

519. The armature and field resistances of a 550-volt, shunt generator were measured after the machine had been standing idle for some time in an engine room, whose temperature is 30° C. The voltage across the field winding, exclusive of the rheostat, is found to be 420 volts and the field current is 4.8 amp. The armature resistance between two marked commutator segments is found to be 0.21 ohm. After the machine had been running under load for 2 hr., these same measurements were repeated. The field voltage is now 450 volts and the field current 4.8 amp. The armature resistance is now 0.225 ohm. What is the temperature rise of the armature winding and of the field winding? Are these maximum temperatures safe for untreated cotton insulation?

520. A 230-volt, 7.5-hp., 1,140-r.p.m., shunt motor has been standing idle for some time in a room whose temperature is 23° C. The armature resistance between marked segments is measured and found to be 0.351 ohm; the resistance of the field coils themselves is measured and found to be 348 ohms. After the machine runs at rated load for 4 hr., these measurements are repeated. The armature resistance between the same two segments is again measured and found to be 0.394 ohm; the resistance of the field copper is found to be 371 ohms. Find: (*a*) the temperature rise in the armature; (*b*) the temperature rise in the field.

521. Two 50-kw., 220-volt generators are operating in parallel. They both are adjusted to 230 volts at no load and are then paralleled. When operating alone the terminal voltage of generator No. 1 drops uniformly from 230 volts at no load to 224 volts at rated load; in a similar manner the terminal voltage of generator No. 2 drops uniformly from 230 volts at no load to 220 volts at rated load. When the aggregate load on the system is

360 amp., what current does each generator deliver? What kilowatt load does each deliver?

522. Repeat Prob. 521 for an aggregate load of 400 amp.

523. It is desired to operate a 100-kw., 230-volt, shunt generator and a 60-kw., 230-volt, shunt generator in parallel. The terminal voltage of the first drops 8 volts from a no-load voltage of 238 when its rated load is applied. What should be the drop in terminal voltage of the second generator from no load to full load in order that each generator may take its proportionate share of the load at all times? Assume that the voltage-load characteristic is a straight line in each case. How much current does each deliver when the system demand is 700 amp.? What is the kilowatt output of each generator under these conditions?

524. Two 230-volt, compound generators are operating in parallel. One has a rating of 150 kw. and the other a rating of 100 kw. The resistance of the series field of the first is 0.001 ohm. What should be the resistance of the series field of the second machine for proper division of the load?

QUESTIONS ON CHAPTER XIV

1. Why cannot the ordinary direct-current voltages be used for transmitting considerable amounts of power over long distances? Under what conditions is direct current utilized for general light and power distribution? What are its advantages under these conditions?

2. What is the general scheme for transmitting large amounts of power from a remotely situated power station to the consumers' premises? What are the ranges of transmission voltages? Of distribution voltages? What part does the substation play in the system?

3. How does the weight of conductor vary with the transmission voltage? If the transmission voltage were doubled how would the weight of copper be affected, the other factors remaining unchanged?

4. What five conditions in general determine the size of conductor to be used? For what conditions does the question of heating particularly apply? How may the economics of the problem determine the size of conductor? What is the disadvantage of having too large a conductor? Too small a conductor?

5. Why is 110 volts most convenient for incandescent lighting? Why is a higher voltage undesirable? What are the advantages and disadvantages of a lower voltage for this purpose?

6. What are the common trolley voltages? Why are these voltages so chosen?

7. What is meant by distributed loads? Where do such loads occur? Where are conductors of uniform cross-section throughout most commonly used?

8. Theoretically, what type of conductor is most economical for uniformly distributed loads? What is the practical condition that most nearly approaches this theoretical condition?

9. Why is the "return loop" system of distribution used? What is its one disadvantage?

10. What system overcomes the disadvantage of the return loop system? Make a sketch and show how this system may be further modified to form a still more efficient system.

11. What advantage is gained by connecting 110-volt loads in series groups of two and utilizing 220-volt supply? What are the disadvantages of so grouping the loads?

12. How are the objections to the series-parallel system overcome? What are the relations existing among the voltages of the Edison 3-wire system?

13. If the neutral wire be of the same size as the two outers, what are the relative weights of copper in the 3-wire system with 220 volts across outers and in the simple 110-volt system, other conditions being the same?

14. What is meant by balanced loads? Under this condition how much current flows through the neutral?

15. In what direction does the neutral current flow if the positive load is the greater? The negative load? What relation does the neutral current bear to the current in the outer wires? What type of ammeter should be used in the neutral? What are the commercial limits of unbalancing?

16. State briefly the effect of opening the neutral with (*a*) balanced loads and (*b*) unbalanced loads. Why is the neutral usually grounded?

17. What in general is the effect of putting too heavy a load on one side of a 3-wire system upon the voltage on that side of the system? Upon the voltage on the other side of the system?

18. Sketch a method of obtaining a neutral by the use of two shunt generators. What is the principal disadvantage of this method?

19. How may a storage battery be used for obtaining a neutral? In general how does the current in the neutral wire divide when it reaches the center of the battery?

20. Upon what principle does the balancer set operate? What determines which machine shall operate as a motor? As a generator? What two methods are used to accentuate the motor and the generator actions?

21. Upon what principle does the 3-wire generator operate? Where does the alternating current flow? The returning direct current from the neutral? How is the direct current able to pass so readily back into the armature?

22. How in general is power supplied to direct-current loads in the more congested districts? What is the function of the feeders? The mains? The junction boxes? Where are the house services connected? How are the voltages at feeding points generally determined?

23. A load is supplied from constant-voltage bus-bars over a line of constant resistance. Under what conditions is the power taken by the load a maximum? What is the efficiency of transmission under these conditions? When is the current a maximum? What is the efficiency of transmission under these conditions?

24. What type of generator is most commonly used to supply power for railways? How are such generators connected to the system?

25. Under what conditions does a single trolley suffice for transmitting the power to the car? If a single trolley of the ordinary size is of insufficient

cross-section, what means can be taken to assist it in supplying the required power? Why is the size of trolley not increased?

26. Under what conditions are multiple feeders employed? What is the disadvantage of their use? How may this disadvantage be overcome?

27. Why does the return current from a trolley car leave the track? What determines the paths which it follows? What damage, if any, occurs at the point where the current enters a pipe? Where it leaves the pipe?

28. Name two methods by which electrolysis may be reduced. What measurements give a good idea of the magnitude of stray currents between pipes and track?

29. Sketch a typical central station load curve. Show how the habits of a community determine the general shape of such a curve. Why is such a load curve far more undesirable than a uniform load curve having the same total kilowatt-hours?

What is meant by load factor? Is a high or a low load factor desirable? Why?

30. How may a storage battery smooth out a station load curve? When should the battery be charged? Discharged? Why are storage batteries not more generally used for this purpose?

31. Where can storage batteries be used efficiently to carry the load in off-peak times? For what purposes are they now commonly used by central stations? Where should they be located? Under what conditions is a battery very useful to a central station?

32. What difficulty is met when an attempt is made to operate storage batteries in conjunction with a power plant? What simple method may be used to control the battery load? What is the objection to this method?

33. Upon what simple principle do the counter-electromotive force cells operate? What is the chief advantage of this method of control over the resistance method?

34. What is meant by end-cell control? How is such a battery charged? In what manner is the connection changed from one cell to the next without opening the circuit or dead-short circuiting the batteries?

35. What is meant by a "floating" battery? What is the purpose of such a battery? Why is it often necessary to install auxiliary means for accentuating the battery charge and discharge with change of load? Sketch the connections of one simple method for accomplishing this purpose.

36. Why will a battery placed at the end of a long feeder tend to equalize the station load without auxiliary apparatus for charging and discharging? Under what conditions does such a battery "float?"

37. What is the essential difference between the series system and the parallel system of distribution? In the series system what is the effect of attempting to remove a load by opening the circuit? How is a load cut out in a series system?

38. By what devices is a series system supplied? What are the advantages of the series system? Where does its field of application lie? Sketch the layout of two different systems of series lighting distribution. Name the advantages of each.

PROBLEMS ON CHAPTER XIV

525. A motor takes 200 amp. at 220 volts over a 1,000-ft. length of 250,000-C.M. feeder having a resistance of 0.0431 ohm per 1,000 ft. (*a*) How many kilowatts are transmitted? (*b*) What is the voltage at the sending end of the line? (*c*) Determine the power loss. (*d*) Determine the efficiency of transmission. (*e*) Determine the weight of copper. A 1,000-ft. length of No. 10 wire, having a cross-section of 10,400 C.M., weighs 31.4 lb.

526. In Prob. 525, assume that the power had been transmitted to the motor at 110 volts and the power loss and efficiency had remained the same. Determine: (*a*) the current; (*b*) the total resistance of the feeder; (*c*) the resistance per 1,000 ft.; (*d*) the circular milage of the feeder; (*e*) the weight of the copper; (*f*) the sending-end voltage. (*g*) Compare the weight of copper with that in Prob. 525.

527. A 10-hp. motor is fed from a switchboard, the bus-bars of which are maintained at 115 volts. The motor is located at a distance of 600 ft. from the switchboard and it is desired to have a potential difference of 110 volts at the motor terminals when the motor is carrying its full load of 10 hp. What must be the diameter in mils:

(*a*) of the copper wire used to connect the motor to the switchboard? The resistance of a circular-mil foot at the operating temperature is 11.0 ohms. 1 hp. = 746 watts. The efficiency of the motor at full load is 86 per cent.

(*b*) If copper weighs 0.32 lb. per cubic inch, what will be the weight of the wire in (*a*)?

(*c*) Repeat (*a*) and (*b*) for a switchboard voltage of 230 volts and the same per cent. drop to the motor.

(*d*) Repeat (*a*) and (*b*) for a switchboard voltage of 550 volts and the same per cent. drop to the motor.

528. It is desired to transmit 100 kw. a distance of 1,600 ft. from 230-volt bus-bars and the voltage at the load must not be less than 220 volts. The resistance of a circular-mil-foot of copper may be assumed to be 10 ohms. Determine: (*a*) the circular milage of the feeder; (*b*) its weight (see Prob. 525).

529. Repeat Prob. 528 for a distance of 3,200 ft. How does the weight of copper vary with the distance if the power and the efficiency remain fixed?

530. A certain street is 2,000 ft. long. It is illuminated by eleven 200-watt multiple-connected lamps, placed 200 ft. apart. No. 4 A.W.G. conductors are used to supply this system. The voltage at the feeding end of the street is 120 volts. What are the voltage drops between adjacent lamps? What is the voltage at the last lamp? Assume that each lamp takes 2.0 amp.

531. If the lamps of Prob. 530 are fed by the anti-parallel system (see Fig. 374 (*a*), p. 447), No. 4 wire still being used, determine the voltage at the lamps on the two ends of the street. Compare their absolute voltage and their difference of voltage with the results of Prob. 530.

532. A load of 140 amp. is situated 800 ft. from 600-volt bus-bars. 1,000 ft. beyond, a second load of 75 amp. is located. A 300,000-C.M. annealed

copper feeder runs from the bus-bars to the first load. No. 0 wires run from the 140 amp. load to the 75 amp. load. Determine: (*a*) the voltage at each load; (*b*) the weight of copper used; (*c*) the efficiency of transmission.

533. (*a*) Determine the size of a uniform feeder which will have the same weight as the two feeders of Prob. 532. (*b*) Determine the voltage at each load with this uniform feeder. (*c*) Under which condition is the copper more effectively utilized?

534. It is desired to operate a group of fifty 75-watt, 110-volt lamps, located 500 ft. from a 230-volt source of supply. Two No. 6 A.W.G. wires, having a resistance of 0.4 ohm per 1,000 ft., are used to transmit the power. Assume that each lamp takes 0.7 amp. and that the lamps are connected in series-parallel with two in series. Determine: (*a*) the voltage at the lamps; (*b*) the power loss in the line; (*c*) the efficiency of transmission. (*d*) If rubber-insulated wire is used, verify its carrying capacity (see Appendix F).

535. Determine the size of wire required to supply the lamps in Prob. 534 with the lamps all in parallel and taking their power from a 115-volt source. The distance and power loss remain unchanged. What is the ratio of the weight of copper to that in Prob. 534? Verify the carrying capacity by Appendix F.

536. Find the weight of copper in Prob. 534, using a neutral which has one-fourth the cross-section of that of the outer wires.

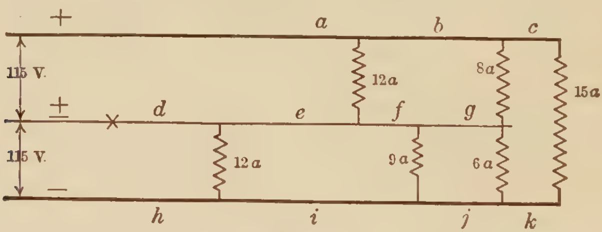


Fig. 537*A*.

537. Fig. 537*A* shows an Edison 3-wire system with various loads. Indicate the current and its direction at each of the points *a–k* inclusive.

538. If the neutral is cut at point *X* (Fig. 537*A*) find the voltages across the two sides of the system, assuming that the load resistances do not change. Neglect the drop in the mains themselves.

539. Find the voltages across loads *A* and *B* (Fig. 539*A*) if loads *A* and *B* are each 50 amp.

540. Repeat Prob. 539 when load *A* is 70 amp. and load *B* is 30 amp.

541. Find the voltage across each load (Prob. 540) which would occur if the neutral were opened.

542. Determine the voltage across each of the loads *AB* and *BC* (Fig. 542*A*) and also the voltage across the motor. Use 11 ohms as the resistance of a circular-mil foot.

543. Repeat Prob. 542 with the motor connected between the neutral and the negative conductor. Owing to the fact that the voltage is halved

the motor must now take approximately 200 amp. to develop its former power.

544. Find the current in each machine of the balancer set of Fig. 544*A* and indicate which machine is the motor and which is the generator. There are 115 volts across each machine and the efficiency of each machine is 84 per cent. How much current does the main generator deliver?

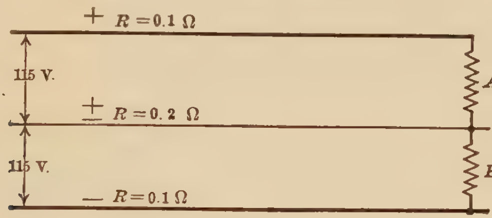


FIG. 539*A*.

545. Solve Prob. 544 when there is a total load of 180 amp. connected between the neutral and negative main and there is no load on the positive side.

546. A direct-current load is located 400 ft. from the direct-current bus-bars and is supplied by a 2-conductor feeder. When the load current is 40.5 amp. the voltage at the load is 104.7 volts; when the load current is 25.8 amp. the voltage at the load is 109.6 volts. Determine: (*a*) the voltage at the bus-bars; (*b*) the resistance of the feeder; (*c*) the circular milage of

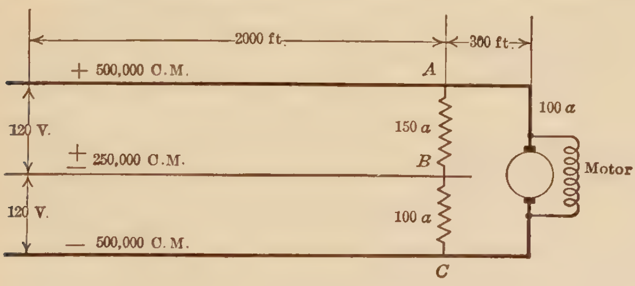


FIG. 542*A*.

the feeder; (*d*) the size (A.W.G.) of the feeder. The resistance of a circular-mil-foot may be taken as 10.8 ohms.

547. An electric car on a suburban line, in climbing a grade 8 miles from the power house, requires 60 kw. The bus-bars at the power house are maintained at 600 volts. The No. 00 hard-drawn trolley wire (= 133,100 C.M.) is paralleled by a 400,000-C.M. single conductor. The total resistance of the rail and ground return is 0.6 ohm. The resistance of a circular mil-foot, for both trolley and feeder, may be taken as 11 ohms. Determine: (*a*) the current; (*b*) the voltage at the car; (*c*) power delivered to the line at

the station; (*d*) power lost in overhead system and ground return; (*e*) efficiency of transmission.

548. Repeat Prob. 547, attempting to obtain 80 kw. at the car.

549. What is the maximum power which the car, Probs. 547 and 548, can take? If the combined resistance of the motors is 0.6 ohm, what is the maximum current which they can take? Plot a curve with power at the car as ordinates and current as abscissas (see Fig. 388).

550. Repeat Prob. 547 with 750 volts at the bus-bars.

551. Repeat Prob. 548 with 750 volts at the bus-bars.

552. Repeat Prob. 549 with 750 volts at the bus-bars.

553. A 4/0 hard-drawn copper trolley wire runs from 600-volt bus-bars to a station 6 miles out. For 4 miles it is paralleled by a 400,000-C.M. feeder which feeds it every quarter mile (see Fig. 389 (*b*), p. 462). The 4/0 wire has a resistance of 0.26 ohm per mile and the 400,000-C.M. feeder has a resistance of 0.143 ohm per mile. The resistance of the track and ground return is 0.05 ohm per mile. Find the voltage at a car 5 miles out and taking 70 amp. What is the voltage at the end of the line?

554. (*a*) Find the voltage at the car in Prob. 553, when the car is 4 miles from the power station and taking the same current. (*b*) When the car is 2.5 miles from the station and taking 60 amp.

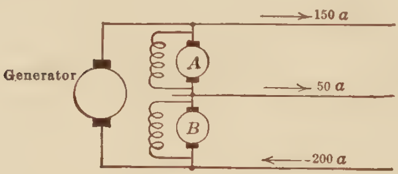


Fig. 544*A*.

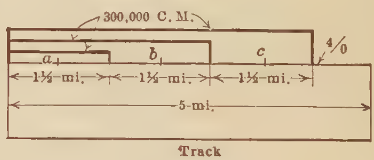


Fig. 555*A*.

555. Fig. 555*A* shows a 5-mile length of hard-drawn 4/0 copper trolley wire (211,000 C.M.). This is fed by three 300,000 C.M. multiple feeders, feeding at points 1½ miles apart. Find the equivalent resistance of the trolley and feeders to the end of the line. The resistance of a circular-mil-foot may be taken as 11 ohms.

556. Find the voltage at a car (Fig. 555 *A*) when it is at the end of the line and taking 110 amp. The station voltage is 600 volts and the ground and track resistance is 0.04 ohm per mile.

557. Find the voltage at the car of Prob. 556 when the car is 3 miles from the station and taking 110 amp.

558. Repeat Probs. 556 and 557 for a sectionalized trolley, the insulated sections being at *a*, *b*, *c*, (Fig. 555 *A*). Which system of feeding is the more economical of copper?

559. A central station has a peak load of 9,400 kw. It delivers 71,000 kw.-hr. over a 24-hr. period. What is its daily load factor?

560. A storage battery helps the station of problem 559 by taking 1,700 kw. off the peak, it being necessary to use this battery for an hour and a half.

If the battery efficiency is 85 per cent. and it is charged off peak, what is the new load factor of the station?

561. A storage battery consists of 68 cells connected in series, each cell having an electromotive force of 2.0 volts and a resistance of 0.003 ohm. It is desired to allow this battery to discharge at a 40-amp. rate into 115-volt bus-bars. (*a*) What series resistance is necessary? (*b*) How much power is developed within the battery? (*c*) How much is lost in the battery resistance? (*d*) How much is lost in the series resistance?

562. Determine the discharge current of the battery (Prob. 561) when the e.m.f. per cell of the battery drops to 1.95 volts and the other factors remain unchanged. To what value must the series resistance be adjusted to bring the discharge rate back to 40 amp.?

563. If counter-electromotive force cells, each having an electromotive force of 2.14 volts, were used in problem 561 how many would be necessary? Neglect the internal resistance of the counter-electromotive force cells.

564. If end-cell control were used in Prob. 561 how many cells would be cut off the end of the battery?

565. A 4/0 hard-drawn copper trolley, having a resistance of 0.26 ohm per mile, extends 5 miles from 600-volt bus-bars. The track return has a resistance of 0.05 ohm per mile. A storage battery consisting of 270 cells floats at the end of the line. Each cell has an electromotive force of 2.01 volts and a resistance of 0.0015 ohm. For what value of current at the battery terminals will the battery "float?"

566. What current must a car 3 miles from the station of Prob. 565 take in order that the battery may "float?"

567. At what rate does the battery charge when there is no car at all on the system?

568. When a car at the battery, problem 565, takes 110 amp., how much current does the station supply and how much does the battery supply? How much power is supplied by each?

569. Repeat Prob. 568 for a car taking 120 amp., 3 miles from the station.

570. A series-arc generator supplies 58 500-watt 6.6-amp. magnetite arcs over a No. 6 cable, which has a resistance of 0.395 ohm per 1,000 ft. The length of the arc circuit is 10 miles. What is the terminal voltage of the generator and what is the efficiency of transmission?

571. Repeat Prob. 570 for a 78-lamp circuit in which the length of the circuit is 14 miles.

INDEX

E

F

G

H

I

J

K

L

M

1 mil = .001

$(1\text{ mil})^2 = 1$ Cir mil

$R = \rho \frac{L}{D^2} = \frac{\text{ohms CM}}{\text{ft}} \cdot \frac{\text{ft}}{\text{CM}} = \text{Oh}$

ρ for Cu = $1.724 \times 10^{-6} \times \frac{\text{ft} \times 48 \times 2.54 \times 10^{6}}{D^2 \times \frac{\pi}{4} (2.54)^2}$ $\frac{D}{1000}$ = inch

$R = \frac{1.724 \times 48}{\pi \times 2.54}$